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SIGNALS,  
SYSTEMS,  
and INFERENCE

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Class Notes for  
6.011: Introduction to  
Communication, Control and  
Signal Processing  
Spring 2010

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