6.012 Microelectronic Devices and Circuits

Formula Sheet for Exam Two, Fall 2009

Parameter Values:

$q = 1.6x10^{-19}$ *Coul*

$\varepsilon_o = 8.854x10^{-14}$ *F/cm*

$\varepsilon_{r,Si} = 11.7, \quad \varepsilon_{Si} \approx 10^{-12}$ *F/cm*

$n_i[Si@R.T] \approx 10^{10}\ cm^{-3}$

$kT/q \approx 0.025\ V; \quad (kT/q)\ln 10 \approx 0.06\ V$

$1\ \mu m = 1x10^{-4}\ cm$

Periodic Table:

III	IV	V
B	C	N
Al	Si	P
Ga	Ge	As
In	Sn	Sb

Drift/Diffusion:

Drift velocity: $\bar{s}_x = \pm\mu_m E_x$

Conductivity: $\sigma = q(\mu_e n + \mu_h p)$

Diffusion flux: $F_m = -D_m \dfrac{\partial C_m}{\partial x}$

Einstein relation: $\dfrac{D_m}{\mu_m} = \dfrac{kT}{q}$

Electrostatics:

$$\varepsilon \frac{dE(x)}{dx} = \rho(x) \qquad E(x) = \frac{1}{\varepsilon}\int \rho(x)dx$$

$$-\frac{d\phi(x)}{dx} = E(x) \qquad \phi(x) = -\int E(x)dx$$

$$-\varepsilon \frac{d^2\phi(x)}{dx^2} = \rho(x) \qquad \phi(x) = -\frac{1}{\varepsilon}\iint \rho(x)dxdx$$

The Five Basic Equations:

Electron continuity: $$\frac{\partial n(x,t)}{\partial t} - \frac{1}{q}\frac{\partial J_e(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\cdot p(x,t) - n_i^2\right]r(T)$$

Hole continuity: $$\frac{\partial p(x,t)}{\partial t} + \frac{1}{q}\frac{\partial J_h(x,t)}{\partial x} = g_L(x,t) - \left[n(x,t)\cdot p(x,t) - n_i^2\right]r(T)$$

Electron current density: $$J_e(x,t) = q\mu_e n(x,t)E(x,t) + qD_e\frac{\partial n(x,t)}{\partial x}$$

Hole current density: $$J_h(x,t) = q\mu_h p(x,t)E(x,t) - qD_h\frac{\partial p(x,t)}{\partial x}$$

Poisson's equation: $$\frac{\partial E(x,t)}{\partial x} = \frac{q}{\varepsilon}\left[p(x,t) - n(x,t) + N_d^+(x) - N_a^-(x)\right]$$

Uniform doping, full ionization, TE

n - type, $N_d >> N_a$

$$n_o \approx N_d - N_a \equiv N_D, \quad p_o = n_i^2/n_o, \quad \phi_n = \frac{kT}{q}\ln\frac{N_D}{n_i}$$

p - type, $N_a >> N_d$

$$p_o \approx N_a - N_d \equiv N_A, \quad n_o = n_i^2/p_o, \quad \phi_p = -\frac{kT}{q}\ln\frac{N_A}{n_i}$$

Uniform optical excitation, uniform doping

$$n = n_o + n' \qquad p = p_o + p' \qquad n' = p' \qquad \frac{dn'}{dt} = g_l(t) - (p_o + n_o + n')n'r$$

Low level injection, $n', p' << p_o + n_o$: $$\frac{dn'}{dt} + \frac{n'}{\tau_{\min}} = g_l(t) \quad \text{with} \quad \tau_{\min} \approx (p_o r)^{-1}$$

Flow problems (uniformly doped quasineutral regions with quasi-static excitation and low level injection; p-type example):

Minority carrier excess: $$\frac{d^2 n'(x)}{dx^2} - \frac{n'(x)}{L_e^2} = -\frac{1}{D_e} g_L(x) \qquad L_e \equiv \sqrt{D_e \tau_e}$$

Minority carrier current density: $$J_e(x) \approx qD_e \frac{dn'(t)}{dx}$$

Majority carrier current density: $$J_h(x) = J_{Tot} - J_e(x)$$

Electric field: $$E_x(x) \approx \frac{1}{q\mu_h p_o}\left[J_h(x) + \frac{D_h}{D_e} J_e(x)\right]$$

Majority carrier excess: $$p'(x) \approx n'(x) + \frac{\varepsilon}{q}\frac{dE_x(x)}{dx}$$

Short base, infinite lifetime limit:

Minority carrier excess: $$\frac{d^2 n'(x)}{dx^2} \approx -\frac{1}{D_e} g_L(x), \quad n'(x) \approx -\frac{1}{D_e}\iint g_L(x)\,dx\,dx$$

Non-uniformly doped semiconductor sample in thermal equilibrium

$$\frac{d^2\phi(x)}{dx^2} = \frac{q}{\varepsilon}\left\{n_i\left[e^{q\phi(x)/kT} - e^{-q\phi(x)/kT}\right] - \left[N_d(x) - N_a(x)\right]\right\}$$

$$n_o(x) = n_i e^{q\phi(x)/kT}, \quad p_o(x) = n_i e^{-q\phi(x)/kT}, \quad p_o(x) n_o(x) = n_i^2$$

Depletion approximation for abrupt p-n junction:

$$\rho(x) = \begin{cases} 0 & \text{for} \quad x < -x_p \\ -qN_{Ap} & \text{for} \quad -x_p < x < 0 \\ qN_{Dn} & \text{for} \quad 0 < x < x_n \\ 0 & \text{for} \quad x_n < x \end{cases}$$

$$N_{Ap} x_p = N_{Dn} x_n$$

$$\phi_b \equiv \phi_n - \phi_p = \frac{kT}{q}\ln\frac{N_{Dn}N_{Ap}}{n_i^2}$$

$$w(v_{AB}) = \sqrt{\frac{2\varepsilon_{Si}\left(\phi_b - v_{AB}\right)}{q}\frac{\left(N_{Ap} + N_{Dn}\right)}{N_{Ap}N_{Dn}}} \qquad \left|E_{pk}\right| = \sqrt{\frac{2q\left(\phi_b - v_{AB}\right)}{\varepsilon_{Si}}\frac{N_{Ap}N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

$$q_{DP}(v_{AB}) = -AqN_{Ap}x_p\left(v_{AB}\right) = -A\sqrt{2q\varepsilon_{Si}\left(\phi_b - v_{AB}\right)\frac{N_{Ap}N_{Dn}}{\left(N_{Ap} + N_{Dn}\right)}}$$

Ideal p-n junction diode i-v relation (large signal model):

$$n(\text{-}x_p) = \frac{n_i^2}{N_{Ap}} e^{qv_{AB}/kT}, \quad n'(\text{-}x_p) = \frac{n_i^2}{N_{Ap}}\left(e^{qv_{AB}/kT} - 1\right); \qquad p(x_n) = \frac{n_i^2}{N_{Dn}} e^{qv_{AB}/kT}, \quad p'(x_n) = \frac{n_i^2}{N_{Dn}}\left(e^{qv_{AB}/kT} - 1\right)$$

$$i_{\mathrm{D}} = Aqn_i^2\left[\frac{D_h}{N_{Dn}w_{n,eff}} + \frac{D_e}{N_{Ap}w_{p,eff}}\right]\left[\mathrm{e}^{qv_{AB}/kT} - 1\right] \qquad w_{m,eff} = \begin{cases} w_m - x_m & \text{if } \mathrm{L_m} >> w_m \\ L_m\tanh\left[\left(w_m - x_m\right)/L_m\right] & \text{if } \mathrm{L_m} \sim w_m \\ L_m & \text{if } \mathrm{L_m} << w_m \end{cases}$$

$$q_{\mathrm{QNR},p\text{-}side} = Aq\int_{-w_p}^{-x_p} n'(x)\,dx, \qquad q_{QNR,n\text{-}side} = Aq\int_{x_n}^{w_n} p'(x)\,dx, \qquad \text{Note: } p'(x) \approx n'(x) \text{ in QNRs}$$

Large signal BJT Model in Forward Active Region (FAR):

(npn with base width modulation)

$$i_B(v_{BE},v_{CE}) \; = \; I_{BS}\left(e^{qv_{BE}/kT}-1\right)$$

$$i_C(v_{BE},v_{BC}) \; = \; \beta_F \; i_B(v_{BE},v_{CE})\left[1+\lambda v_{CE}\right] = \beta_F I_{BS}\left(e^{qv_{BE}/kT}-1\right)\left[1+\lambda v_{CE}\right]$$

$$\text{with:} \quad I_{BS} \equiv \frac{I_{ES}}{(\beta_F+1)} = \frac{Aqn_i^2}{(\beta_F+1)}\left(\frac{D_h}{N_{DE}w_{E,eff}}+\frac{D_e}{N_{AB}w_{B,eff}}\right), \quad \beta_F \equiv \frac{\alpha_F}{(1-\alpha_F)}, \text{ and} \quad \lambda \equiv \frac{1}{V_A}$$

$$\text{Also,} \quad \alpha_F = \frac{(1-\delta_B)}{(1+\delta_E)} \quad \text{and} \quad \beta_F \approx \frac{(1-\delta_B)}{(\delta_E+\delta_B)} \quad \text{with} \quad \delta_E = \frac{D_h}{D_e}\cdot\frac{N_{AB}}{N_{DE}}\cdot\frac{w_{B,eff}}{w_{E,eff}} \quad \text{and} \quad \delta_B = \frac{w_{B,eff}^2}{2L_{eB}^2}$$

$$\text{When} \quad \delta_B \approx 0 \quad \text{then} \quad \alpha_F \approx \frac{1}{(1+\delta_E)} \quad \text{and} \quad \beta_F \approx \frac{1}{\delta_E}$$

MOS Capacitor:

Flat - band voltage: $V_{FB} \equiv v_{GB}$ at which $\phi(0) \; = \; \phi_{p-Si}$ $\qquad [\Delta\phi = 0 \quad \text{in Si}]$

$$V_{FB} \; = \; \phi_{p-Si} - \phi_m$$

Threshold voltage: $V_T \equiv v_{GC}$ at which $\phi(0) \; = \; -\phi_{p-Si} - v_{BC}$ $\qquad \left[\Delta\phi = |2\phi_{p-Si}| - v_{BC} \quad \text{in Si}\right]$

$$V_T(v_{BC}) = V_{FB} - 2\phi_{p-Si} + \frac{1}{C_{ox}^*}\left\{2\varepsilon_{Si}qN_A\left[|2\phi_{p-Si}| - v_{BC}\right]\right\}^{1/2}$$

Depletion region width at threshold: $x_{DT}(v_{BC}) = \sqrt{\dfrac{2\varepsilon_{Si}\left[|2\phi_{p-Si}| - v_{BC}\right]}{qN_A}}$

Oxide capacitance per unit area: $C_{ox}^* = \dfrac{\varepsilon_{ox}}{t_{ox}}$ $\qquad \left[\varepsilon_{r,SiO_2} = 3.9, \quad \varepsilon_{SiO_2} \approx 3.5x10^{-13} \; F/cm\right]$

Inversion layer sheet charge density: $q_N^* = -C_{ox}^*\left[v_{GC} - V_T(v_{BC})\right]$

Accumulation layer sheet charge density: $q_P^* = -C_{ox}^*\left[v_{GB} - V_{FB})\right]$

Gradual Channel Appoximation for MOSFET Characteristics:

(n-channel; strong inversion; with channel length modulation; no velocity saturation)
Only valid for $v_{BS} \le 0$, $v_{DS} \ge 0$.

$$i_G(v_{GS},v_{DS},v_{BS}) \; = \; 0, \qquad i_B(v_{GS},v_{DS},v_{BS}) \; = \; 0$$

$$i_D(v_{GS},v_{DS},v_{BS}) \; = \begin{cases} 0 & \text{for} \quad \dfrac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right] < 0 < v_{DS} \\ \dfrac{K_o}{2\alpha}\left[v_{GS} - V_T(v_{BS})\right]^2\left[1+\lambda\left(v_{DS} - v_{DS,sat}\right)\right] & \text{for} \quad 0 < \dfrac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right] < v_{DS} \\ K_o\left\{v_{GS} - V_T(v_{BS}) - \alpha\dfrac{v_{DS}}{2}\right\}v_{DS} & \text{for} \quad 0 < v_{DS} < \dfrac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right] \end{cases}$$

$$\text{with} \quad V_T(v_{BS}) \equiv V_{FB} - 2\phi_{p-Si} + \frac{1}{C_{ox}^*}\left\{2\varepsilon_{Si}qN_A\left[|2\phi_{p-Si}| - v_{BS}\right]\right\}^{1/2}, \quad v_{DS,sat} \equiv \frac{1}{\alpha}\left[v_{GS} - V_T(v_{BS})\right]$$

$$K_o \equiv \frac{W}{L}\mu_e C_{ox}^*, \qquad C_{ox}^* \equiv \frac{\varepsilon_{ox}}{t_{ox}}, \qquad \alpha \equiv 1 + \frac{1}{C_{ox}^*}\left\{\frac{\varepsilon_{Si}qN_A}{2\left[|2\phi_{p-Si}| - v_{BS}\right]}\right\}^{1/2}, \quad \lambda \equiv \frac{1}{V_A}$$

Large Signal Model for MOSFETs Operated Sub-Threshold (weak inversion):

(n-channel) Only valid for $v_{GS} \le V_T$, $v_{DS} \ge 0$, $v_{BS} \le 0$.

$$i_G(v_{GS},v_{DS},v_{BS}) = 0, \qquad i_B(v_{GS},v_{DS},v_{BS}) \approx 0$$

$$i_{D,s-t}(v_{GS},v_{DS},v_{BS}) \approx I_{S,s-t}\, e^{q\{v_{GS}-V_T(v_{BS})\}/nkT}\left(1-e^{-qv_{DS}/kT}\right) \text{ where } I_{S,s-t} \equiv \frac{W}{2L}\mu_e\left(\frac{kT}{q}\right)^2\sqrt{\frac{2\varepsilon_{Si}qN_A}{\left|2\phi_p\right|-v_{BS}}} = \frac{K_o V_t^2\gamma}{2\sqrt{\left|2\phi_p\right|-v_{BS}}}$$

$$\text{with } V_t \equiv \frac{kT}{q},\quad K_o \equiv \frac{W}{L}\mu_e C_{ox}^*,\quad \gamma \equiv \frac{\sqrt{2\varepsilon_{Si}qN_A}}{C_{ox}^*},\quad n \approx 1+\frac{\gamma}{2\sqrt{\left|2\phi_p\right|-v_{BS}}}$$

Large Signal Model for MOSFETs Reaching Velocity Saturation at Small v_{DS}:

(n-channel) Only valid for $v_{BS} \le 0$, $v_{DS} \ge 0$. Neglects $v_{DS}/2$ relative to $(v_{GS}-V_T)$.

$$\text{Saturation model: } s_y(E_y) = \mu_e E_y \text{ if } E_y \le E_{crit},\ s_y(E_y) = \mu_e E_{crit} \equiv s_{sat} \text{ if } E_y \ge E_{crit}$$

$$i_G(v_{GS},v_{DS},v_{BS}) = 0, \qquad i_B(v_{GS},v_{DS},v_{BS}) = 0$$

$$i_D(v_{GS},v_{DS},v_{BS}) \approx \begin{cases} 0 & \text{for } (v_{GS}-V_T) < 0 < v_{DS} \\ W\ s_{sat}C_{ox}^*\left[v_{GS}-V_T(v_{BS})\right]\left[1+\lambda\left(v_{DS}-\mathrm{E}_{crit}L\right)\right] & \text{for } 0 < (v_{GS}-V_T),\ \mathrm{E}_{crit}L < v_{DS} \\ \dfrac{W}{L}\mu_e C_{ox}^*\left[v_{GS}-V_T(v_{BS})\right]v_{DS} & \text{for } 0 < (v_{GS}-V_T),\ v_{DS} < \mathrm{E}_{crit}L \end{cases}$$

$$\text{with } \lambda \equiv 1/V_A$$

CMOS Performance

Transfer characteristic:

$$\text{In general: } V_{LO} = 0, \quad V_{HI} = V_{\mathrm{DD}}, \quad I_{ON} = 0, \quad I_{OFF} = 0$$

$$\text{Symmetry: } V_M = \frac{V_{\mathrm{DD}}}{2} \text{ and } NM_{LO} = NM_{HI} \Rightarrow K_n = K_{\mathrm{p}} \text{ and } \left|V_{Tp}\right| = V_{Tn}$$

$$\text{Minimum size gate: } L_{\mathrm{n}} = L_p = L_{\min},\quad W_n = W_{\min},\quad W_p = \left(\mu_n/\mu_p\right)W_n \quad \left[\text{or } W_p = \left(s_{sat,n}/s_{sat,p}\right)W_n\right]$$

Switching times and gate delay (no velocity saturation):

$$\tau_{Charge} = \tau_{Discharge} = \frac{2C_L V_{DD}}{K_n\left[V_{DD}-V_{Tn}\right]^2}$$

$$C_L = n\left(W_n L_n + W_p L_p\right)C_{ox}^* = 3nW_{\min}L_{\min}C_{ox}^* \qquad \text{assumes } \mu_e = 2\mu_h$$

$$\tau_{Min.Cycle} = \tau_{Charge} + \tau_{Discharge} = \frac{12nL_{\min}^2 V_{DD}}{\mu_e\left[V_{DD}-V_{Tn}\right]^2}$$

Dynamic power dissipation (no velocity saturation):

$$P_{dyn@f_{\max}} = C_L V_{DD}^2 f_{\max} \propto \frac{C_L V_{DD}^2}{\tau_{Min.Cycle}} \propto \frac{\mu_e W_{\min}\varepsilon_{ox}V_{DD}\left[V_{DD}-V_{Tn}\right]^2}{t_{ox}L_{\min}}$$

$$PD_{dyn@f_{\max}} = \frac{P_{dyn@f_{\max}}}{\text{InverterArea}} \propto \frac{P_{dyn@f_{\max}}}{W_{\min}L_{\min}} \propto \frac{\mu_e\varepsilon_{ox}V_{DD}\left[V_{DD}-V_{Tn}\right]^2}{t_{ox}L_{\min}^2}$$

Switching times and gate delay (full velocity saturation):

$$\tau_{Charge} = \tau_{Discharge} = \frac{C_L V_{DD}}{W_{\min} s_{sat} C_{ox}^* \left[V_{DD} - V_{Tn}\right]}$$

$$C_L = n\left(W_n L_n + W_p L_p\right) C_{ox}^* = 2nW_{\min} L_{\min} C_{ox}^* \qquad \text{assumes } s_{sat,e} = s_{sat,h}$$

$$\tau_{Min.Cycle} = \tau_{Charge} + \tau_{Discharge} = \frac{4nL_{\min} V_{DD}}{s_{sat}\left[V_{DD} - V_{Tn}\right]}$$

Dynamic power dissipation per gate (full velocity saturation):

$$P_{dyn\,@\,f_{\max}} = C_L V_{DD}^2 f_{\max} \propto \frac{C_L V_{DD}^2}{\tau_{Min.Cycle}} \propto \frac{s_{sat} W_{\min} \varepsilon_{ox} V_{DD}\left[V_{DD} - V_{Tn}\right]}{t_{ox}}$$

$$PD_{dyn\,@\,f_{\max}} = \frac{P_{dyn\,@\,f_{\max}}}{\text{InverterArea}} \propto \frac{P_{dyn\,@\,f_{\max}}}{W_{\min} L_{\min}} \propto \frac{s_{sat} \varepsilon_{ox} V_{DD}\left[V_{DD} - V_{Tn}\right]}{t_{ox} L^2}$$

Static power dissipation per gate

$$P_{static} = V_{DD}\, I_{D,off} \approx V_{DD}\, \frac{W_{\min}}{L_{\min}} \mu_e\, V_t^2 \sqrt{\frac{\varepsilon_{Si} q N_A}{2\left|V_{BS}\right|}} \quad e^{\{-V_T\}/nV_t}$$

$$PD_{static} = \frac{P_{static}}{\text{Inverter Area}} \propto \frac{V_{DD}}{L_{\min}^2} \mu_e\, V_t^2 \sqrt{\frac{\varepsilon_{Si} q N_A}{2\left|V_{BS}\right|}} \quad e^{\{-V_T\}/nV_t}$$

<u>CMOS Scaling Rules</u> - Constant electric field scaling

Scaled Dimensions: $L_{\min} \to L_{\min}/s \qquad W \to W/s \qquad t_{ox} \to t_{ox}/s \qquad N_A \to sN_A$

Scaled Voltages: $V_{DD} \to V_{DD}/s \qquad V_{BS} \to V_{BS}/s$

Consequences: $C_{ox}^* \to sC_{ox}^* \qquad K \to sK \qquad V_T \to V_T/s$

$\tau \to \tau/s \qquad P_{dyn} \to P_{dyn}/s^2 \qquad PD_{dyn\,@\,f_{\max}} \to PD_{dyn\,@\,f_{\max}}$

$PD_{static} \to s^2\, e^{(s-1)V_T/s\,n\,V_t}\, PD_{static}$

<u>Device transit times</u>

Short Base Diode transit time: $\tau_b = \dfrac{w_B^2}{2D_{\min,B}} = \dfrac{w_B^2}{2\mu_{\min,B} V_{thermal}}$

Channel transit time w.o. velocity saturation: $\tau_{Ch} = \dfrac{2}{3} \dfrac{L^2}{\mu_{Ch}\left|V_{GS} - V_T\right|}$

Channel transit time with velocity saturation: $\tau_{Ch} = \dfrac{L}{s_{sat}}$

Small Signal Linear Equivalent Circuits:

- p-n Diode (n^+-p doping assumed for C_d)

$$g_d \equiv \left.\frac{\partial i_D}{\partial v_{AB}}\right|_Q = \frac{q}{kT} I_S\, e^{qV_{AB}/kT} \approx \frac{qI_D}{kT}, \qquad C_d = C_{dp} + C_{df},$$

where $C_{dp}(V_{AB}) = A\sqrt{\dfrac{q\varepsilon_{Si}N_{Ap}}{2\left(\phi_b - V_{AB}\right)}}$, and $C_{df}(V_{AB}) = \dfrac{qI_D}{kT}\dfrac{\left[w_p - x_p\right]^2}{2D_e} = g_d\tau_d$ with $\tau_d \equiv \dfrac{\left[w_p - x_p\right]^2}{2D_e}$

- BJT (in FAR)

$$g_m = \frac{q}{kT}\beta_o I_{BS}\, e^{qV_{BE}/kT}\left[1 + \lambda V_{CE}\right] \approx \frac{qI_C}{kT}, \qquad g_\pi = \frac{g_m}{\beta_o} = \frac{qI_C}{\beta_o kT}$$

$$g_o = \beta_o I_{BS}\left[e^{qV_{BE}/kT} + 1\right]\lambda \approx \lambda I_C \quad \left(\text{or} \approx \frac{I_C}{V_A}\right)$$

$C_\pi = g_m\tau_b +$ B-E depletion cap. with $\tau_b \equiv \dfrac{w_B^2}{2D_e}$, C_μ : B-C depletion cap.

- MOSFET (strong inversion; in saturation, no velocity saturation)

$$g_m = K\left[V_{GS} - V_T(V_{BS})\right]\left[1 + \lambda V_{DS}\right] \approx \sqrt{2K I_D}$$

$$g_o = \frac{K}{2}\left[V_{GS} - V_T(V_{BS})\right]^2\lambda \approx \lambda I_D \quad \left(\text{or} \approx \frac{I_D}{V_A}\right)$$

$$g_{mb} = \eta\, g_m = \eta\sqrt{2K I_D} \qquad \text{with} \quad \eta \equiv -\left.\frac{\partial V_T}{\partial v_{BS}}\right|_Q = \frac{1}{C_{ox}^*}\sqrt{\frac{\varepsilon_{Si}qN_A}{\left|q\phi_p\right| - V_{BS}}}$$

$C_{gs} = \dfrac{2}{3}W\,L C_{ox}^*$, $\quad C_{sb}, C_{gb}, C_{db}$: depletion capacitances

$C_{gd} = W\,C_{gd}^*$, where C_{gd}^* is the G-D fringing and overlap capacitance per unit gate length (parasitic)

- MOSFET (strong inversion; in saturation with full velocity saturation)

$$g_m = W\, s_{sat} C_{ox}^*, \qquad g_o = \lambda I_D = \frac{I_D}{V_A}, \qquad g_{mb} = \eta\, g_m \quad \text{with} \quad \eta \equiv -\left.\frac{\partial V_T}{\partial v_{BS}}\right|_Q = \frac{1}{C_{ox}^*}\sqrt{\frac{\varepsilon_{Si}qN_A}{\left|q\phi_p\right| - V_{BS}}}$$

$C_{gs} = W\,L C_{ox}^*$, $\quad C_{sb}, C_{gb}, C_{db}$: depletion capacitances

$C_{gd} = W\,C_{gd}^*$, where C_{gd}^* is the G-D fringing and overlap capacitance per unit gate length (parasitic)

- MOSFET (operated sub-threshold; in forward active region; only valid for $v_{bs} = 0$)

$$g_m = \frac{qI_D}{n\,kT}, \qquad g_o = \lambda I_D = \frac{I_D}{V_A}$$

$C_{gs} = W\,L\,C_{ox}^* \Big/ \sqrt{1 + \dfrac{2C_{ox}^{*2}\left(V_{GS} - V_{FB}\right)}{\varepsilon_{Si}qN_A}}$, $\quad C_{db}$: drain region depletion capacitance

$C_{gd} = W\,C_{gd}^*$, where C_{gd}^* is the G-D fringing and overlap capacitance per unit gate length (parasitic)

6.012 Microelectronic Devices and Circuits
Fall 2009