

6.013 – Electromagnetics and Applications

Problem Set 1 (six problems)

Suggested Reading: Text (Staelin), Sections 1.1 – 1.4 plus the prerequisite concepts in Appendix C (complex vector identities); also Sections 2.5 – 2.6, 3.1.1 – 3.1.3, and 3.3.1.

In general, the readings and homework cover material presented within a single week (Monday – Friday).

This problem set first reviews some of the basic prerequisites so that they can be refreshed as necessary. If several of these problems are too challenging, please discuss this issue with one of the faculty.

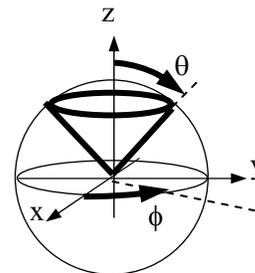
Problem 1.1

Consider the vectors $\bar{A} = 2\hat{x} - 2\hat{y} + \hat{z}$ and $\bar{B} = \hat{x} + 2\hat{y} - 2\hat{z}$.

- Find: (a) $\bar{A} + \bar{B}$ (b) $\bar{A} \cdot \bar{B}$ (c) $\bar{A} \times \bar{B}$ (d) $|\bar{A}|$
(e) The angle between \bar{A} and \bar{B}
(f) The area of the parallelogram defined by \bar{A} and \bar{B} .

Problem 1.2

The illustrated cone subtends a solid angle Ω of one steradian. What is the angle θ in degrees?



Problem 1.3

- (a) An infinite straight wire carries one ampere of direct current. What is the minimum radius R_{\min} from the wire that a compass must be located so that the earth's magnetic field (~ 1 gauss or 10^{-4} Tesla) dominates, i.e., is 100 times larger?
- (b) Two parallel straight wires a distance D apart each carry one ampere in opposite directions (this could be standard lamp cord). In terms of D , at what radius R_{\min} in the plane occupied by the two wires do the two currents produce the maximum allowed field $B_{\max} = 10^{-4}$ Tesla? Assume $R \gg D$, and drop any higher order terms to arrive at a simple answer. Superposition of fields applies.

- (c) Discussion question for tutorial only: qualitatively, how would twisting the pair of wires help reduce such disturbing fields, and how much twisting is required?

Problem 1.4

We should like to estimate the maximum voltage we can use in a particular integrated circuit before electrical breakdown occurs. The observed minimum radius of curvature in the pictured circuit is one micron and the minimum distance between two wires of different voltage is 4 microns.

- (a) If we roughly assume the breakdown electric field strength E_b for this planar integrated circuit is the same as that for two concentric spheres of one and three microns radius, then what is the maximum allowed circuit voltage difference V_{\max} ? Tutorial discussion question: is this a reasonable model?
- (b) If $E_b = 10^6$ [V/m] (approximately the breakdown voltage of air), then what is V_{\max} ?

Image of integrated circuit removed due to copyright restrictions.

Problem 1.5

A planar boundary without a gap or any free charge separates two dielectrics having permittivities of ϵ_a and ϵ_b where $\epsilon_a > \epsilon_b$. What are the upper and lower bounds for the ratio at the boundary of the magnitudes of the electric field vectors $|\bar{E}_a|/|\bar{E}_b|$ in the two media adjacent to the boundary? This ratio depends on the vector directions.

Problem 1.6

A cylindrical coaxial resistor has an inner conductor of radius r_i and an outer conductor of radius r_o , with the space between being filled with a material characterized by conductivity σ and permittivity ϵ . The device is open at the ends and has length L .

- (a) When charged to V_o , what is the surface charge density ρ_i [C/m²] on the surface of the inner conductor?
- (b) What is the capacitance C [farads] of this device?
- (c) What is the resistance R [ohms] of this device?
- (d) If it is open-circuited with an initial voltage V_o , what is $V(t)$ for $t > 0$?

MIT OpenCourseWare
<http://ocw.mit.edu>

6.013 Electromagnetics and Applications
Spring 2009

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.