

**MASSACHUSETTS INSTITUTE OF TECHNOLOGY**  
Department of Electrical Engineering and Computer Science

**6.013 – Electromagnetics and Applications**

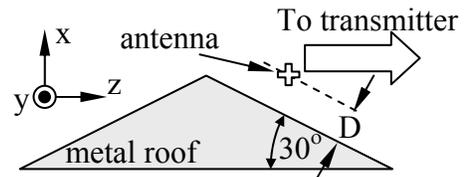
**Problem Set 11 (five problems)**

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**Suggested Reading:** Course notes, Sections 10.3.3-10.3.4; 4.2, 10.4.2, 10.4.5, 11.3.1-11.3.3; 11.1; 10.4.5, 11.3.4

**Problem 11.0**

- (a) Some homeowners have flat metal roofs sloped at  $30^\circ$  directly toward the transmitter, as illustrated, and use a single half-wave dipole antenna mounted a distance  $D$  from the roof (see figure).



If the transmitted signal is horizontally polarized and arrives from the horizon, what should  $D$  be in order to optimize reception, and how should the antenna be oriented? Assume the surface of the earth is absorbing.

- (b) Repeat (a) for a vertically polarized transmitter. This is tricky but not computationally difficult. You might want to think of the antenna as transmitting toward the horizon.
- (c) What is the increase or decrease  $R$  in receiver antenna gain  $G$  (dB) toward the transmitter relative to the gain for (b) if the roof were removed and the dipole antenna were reoriented in an optimum manner? A power ratio  $R$  in dB is  $10 \log_{10} R$ .

**Problem 11.1**

- (a) What is the maximum Thevenin open-circuit voltage  $V$  produced in a short dipole antenna of length  $L = 1$  m by a 1-kW 1-MHz transmitter at distance  $r = 1$  km away? Neglect all reflections.
- (b) What is this maximum open-circuit voltage  $V(h)$  for Part (a) if both the transmitting and receiving antennas are horizontally polarized short dipoles located at height  $h$  above a parallel perfectly conducting infinite flat surface? Assume  $h \ll r = 1$  km.
- (c) Repeat (b) for vertical polarization.
- (d) If we assume a damp earth conducts fairly well, what do your answers to (b) and (c) suggest concerning the best choice of polarization from AM broadcast radio stations operating near 1 MHz? Does this polarization argument apply to cell phones operating above 1 GHz? Explain briefly.

**Problem 11.2**

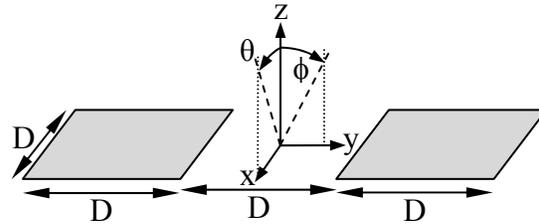
- (a) A 100-MHz digital high-definition television signal arrives at a receiver via two equal-strength paths in phase, one path being 10 km long and the other 11 km. This is called “multipath” propagation whenever more than one delay is involved. Over approximately what maximum contiguous bandwidth  $B$  [Hz] near 100 MHz can this

channel be used without the two rays adding to produce a null, i.e., what is the frequency separation between nulls? What does this suggest about the vulnerability to multipath of over-the-air HDTV broadcasting of 6-MHz wide TV signals? Discuss briefly.

- (b) An alternate HDTV broadcast format (not in the USA) uses OFDM (orthogonal frequency domain multiplexing) in which the digital signal is divided into many separate digital streams that are each broadcast within one of  $N$  distinct adjacent narrow frequency bands that together make up the total frequency allocation for an HDTV channel, say 6 MHz. If we assume the geometry of part (a), then what bandwidth  $B'$  would you recommend for these individual sub-bands, and why? Equipment costs generally increase with  $N$ .

**Problem 11.3**

Two square uniformly illuminated aperture antennas of length  $D$  are spaced  $D$  apart as illustrated, and transmit at wavelength  $\lambda = D/100$ .



- (a) At what magnitude angle  $|\theta|$  in the x-z plane does the first null occur?
- (b) At what magnitude angles  $|\phi|$  in the y-z plane do the first three nulls occur?
- (c) Returning to Part (a), approximately what is the gain of the first sidelobe  $G_{SL}$  relative to the on-axis gain  $G_o$  (i.e.,  $G_{SL}/G_o \cong ?$ )? Hint: the far-field electric vector magnitude  $|\underline{E}(\theta)|$  is the sum of phasor contributions from each of the sub-apertures, each sub-aperture being at a slightly different distance from the observer. Each such phasor has the same magnitude, and their phases are equally spaced at an angle that depends on  $\theta$ . At maximum gain all phasors have the same phase, so they add linearly; at other angles  $\theta$  their phases differ so they sum to a smaller total. Briefly sketch your phasor addition for the peak of the first sidelobe.

**Problem 11.4**

Let's design a 100-GHz communications link between Earth and the moon that can send a 10 kbps cell phone signal to an aperture antenna on the moon that points directly at the 1-watt isotropic cell-phone transmitter. The excellent lunar receiver has a receiver noise temperature  $T_R$  of 100K over the operating bandwidth of 5 kHz. Assume we need to receive ten times the receiver thermal noise power in order to decode the voice signal.

- (a) What is the thermal noise power  $P_{Th}$  [W] for this lunar receiver?
- (b) What receiver antenna effective area  $A_e$  is required on the moon?
- (c) Approximately what size spot on the ground within the first nulls is viewed by your lunar antenna if we assume the moon is  $3 \times 10^5$  m away?
- (d) Assuming this viewed spot is smaller than the earth, and that the earth looks like a blackbody at 250K, approximately how much larger should your lunar antenna now become to allow for the additional noise radiated by the earth itself? Discuss briefly.

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