6.01: Introduction to EECS I

Search Algorithms

April 26, 2011

Nano-Quiz Makeups

Wednesday, May 4, 6-11pm, 34-501.

– everyone can makeup/retake NQ 1
– everyone can makeup/retake two additional NQs
– you can makeup/retake other NQs excused by S^3

If you makeup/retake a NQ, the new score will replace the old score, even if the new score is lower!

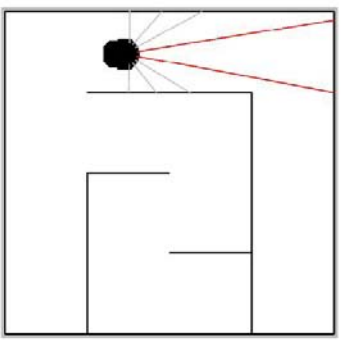
Module 4: Probability and Planning

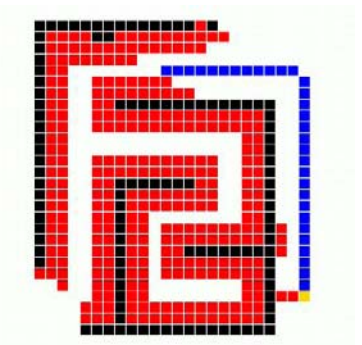
Modeling uncertainty and making robust plans.

Topics: Bayes' theorem, search strategies

Lab exercises:

- Mapping: drive robot around unknown space and make map.
- Localization: give robot map and ask it to find where it is.
- Planning: plot a route to a goal in a maze





Themes: Robust design in the face of uncertainty

Last Time: Probability

Modeling uncertainty and making robust plans.

Topics: **Bayes' theorem**, search strategies

Lab exercises:

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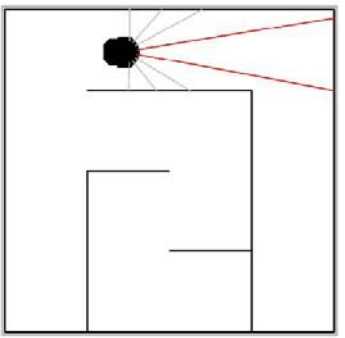
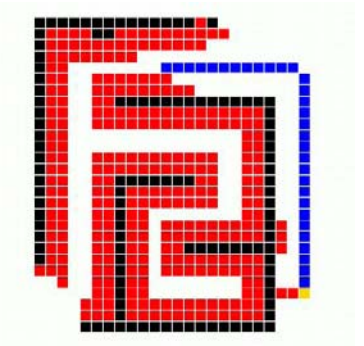
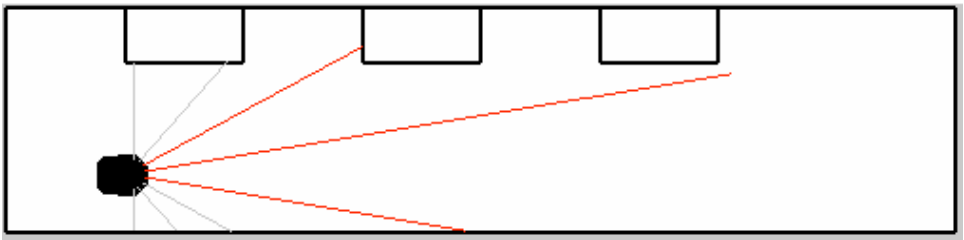




Themes: Robust design in the face of uncertainty

Design Lab 12: One-Dimensional Localizer

As robot drives along hallway with obstacles to its side, estimate its current position based on previous estimates and sonar information.



State S_t: discretized values of distance along the hallway (x).

Transition model $\Pr(S_{t+1} = s' \,|\, S_t = s)$: conditional distribution of next state given current state.

Observation model $\Pr(O_t = d \,|\, S_t = s)$: conditional distribution of left-facing sonar readings (y) given state.

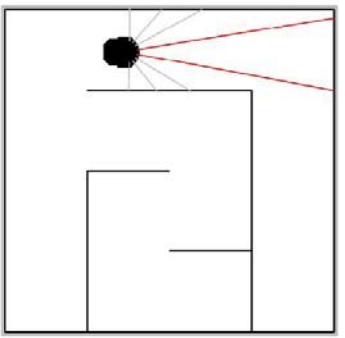
Today: Search Strategies

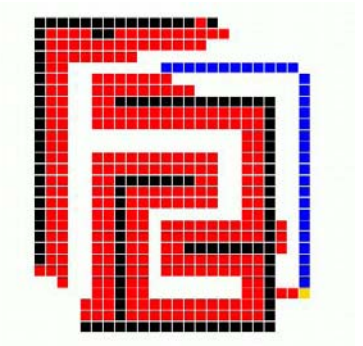
Modeling uncertainty and making robust plans.

Topics: Bayes' theorem, **search strategies**

Lab exercises:

- Mapping: drive robot around unknown space and make map.
- Localization: give robot map and ask it to find where it is.
- **Planning**: plot a route to a goal in a maze



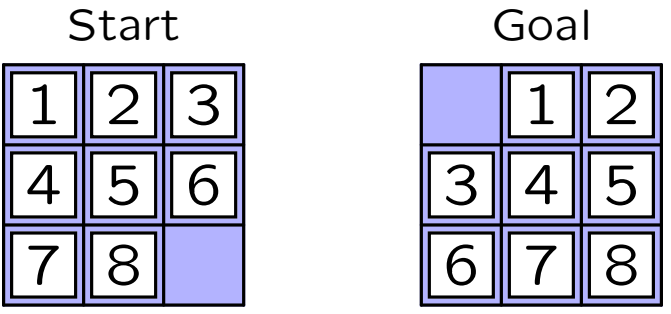


We will **plan** a route by **searching** through possible alternatives.

Planning

Make a plan by searching.

Example: Eight Puzzle



Rearrange board by sequentially sliding tiles into the free spot.

Eight Puzzle

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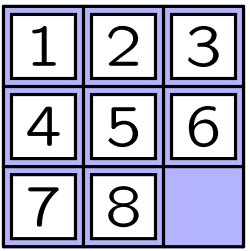

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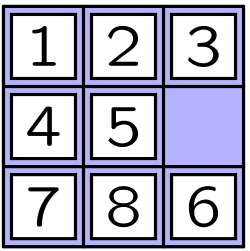

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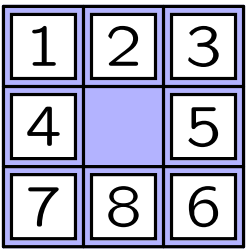

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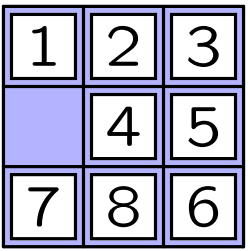

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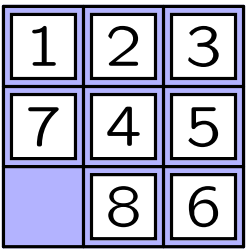

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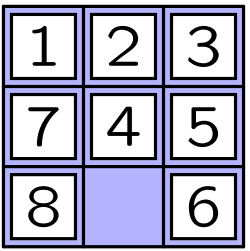

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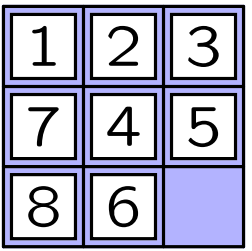

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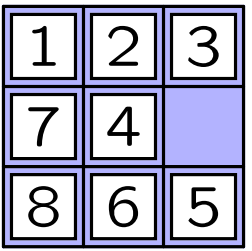

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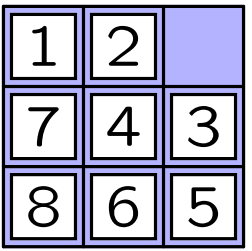

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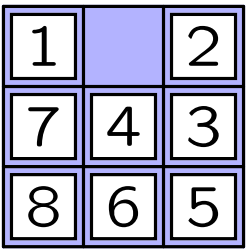

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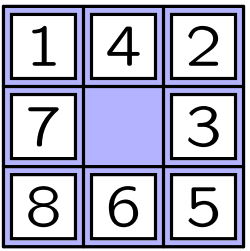

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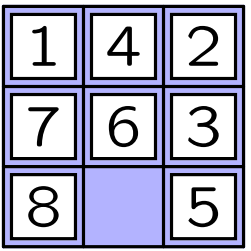

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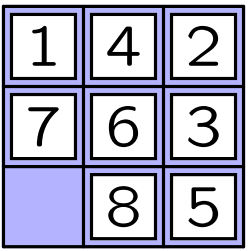

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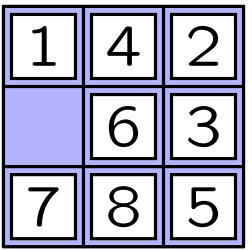

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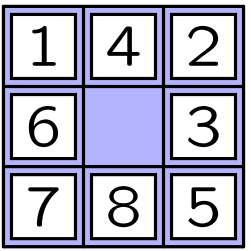

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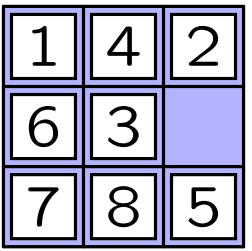

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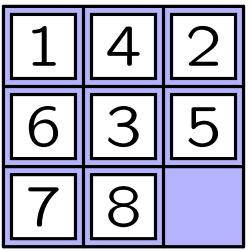

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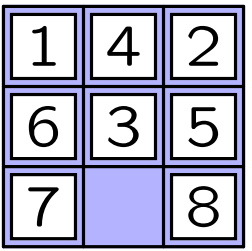

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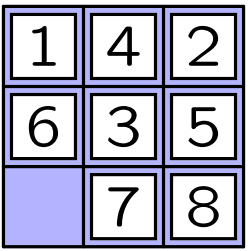

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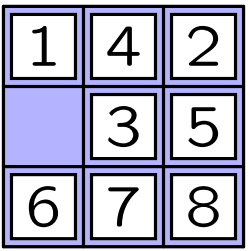

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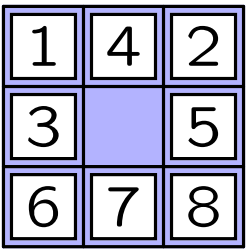

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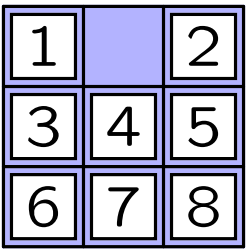

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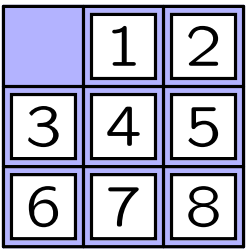

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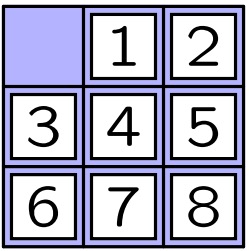
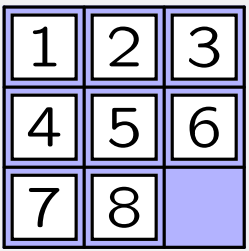


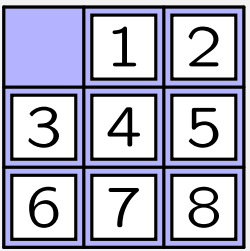
Twenty-two moves.

How difficult is this problem?

Check Yourself

How many different board configurations (states) exist?

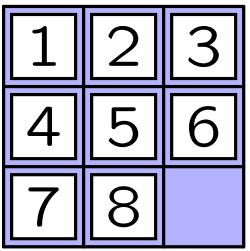


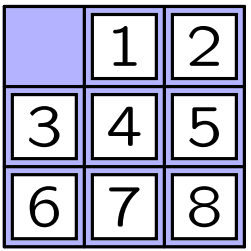


1. $8^2 = 64$
2. $9^2 = 81$
3. $8! = 40320$
4. $9! = 362880$
5. none of the above

Check Yourself

How many different board configurations (states) exist?





Nine possibilities for the first square.

Eight possibilities for the second square.

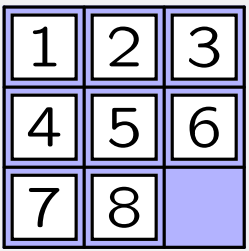
Seven possiblities for the third square.

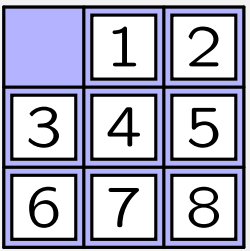
...

$9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 9!$

Check Yourself

How many different board configurations (states) exist? 4





1. $8^2 = 64$
2. $9^2 = 81$
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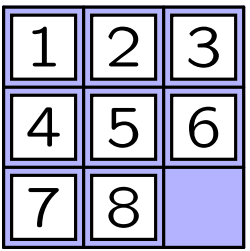
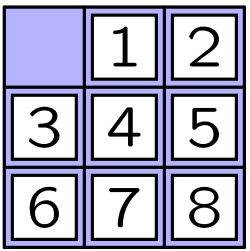
Eight Puzzle

We have to search through as many as $9! = 362,880$ configurations (more if we get confused and loose track of what we are doing)!





Is the solution with 22 moves optimal? Do shorter solutions exist?

Do we have to look at all 362,880 configurations to be sure?

Search Algorithm

Develop an algorithm to systematically conduct a search.

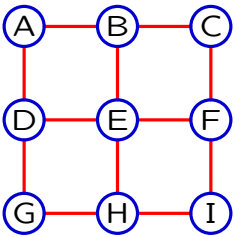
Analyze how well the algorithm performs.

Optimize the algorithm:

- find the "best" solution (i.e., minimum path length)
- by considering as few cases as possible.

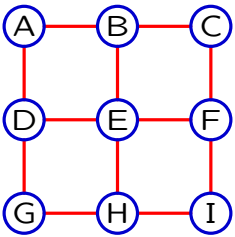
Algorithm Overview: Example

Find minimum distance path between 2 points on a rectangular grid.

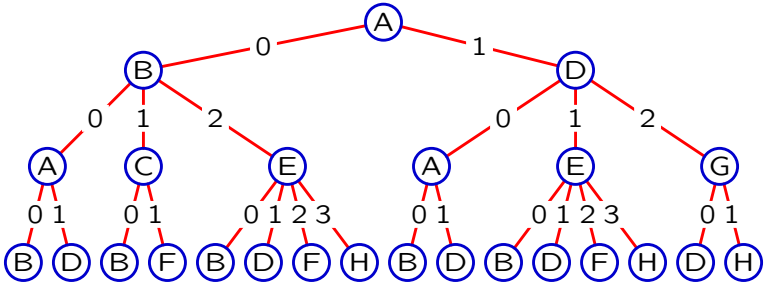


Algorithm Overview

Find minimum distance path between 2 points on a rectangular grid.



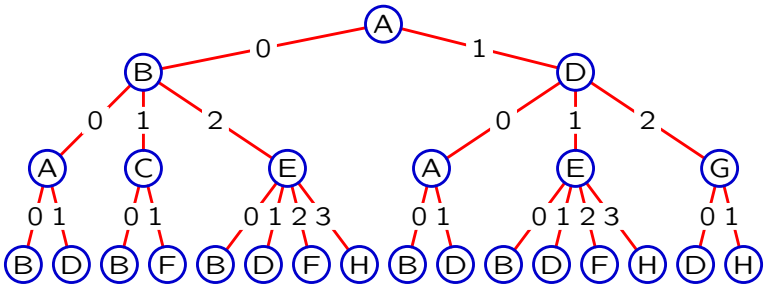
Represent **all possible paths** with a **tree** (shown to just length 3).



Find the shortest **path** from A to I.

Algorithm Overview

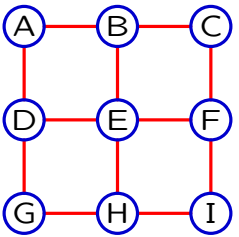
The tree could be infinite.



Therefore, we will construct the tree and search at the same time.

Python Representation

Represent possible locations by **states**: `'A','B','C','D',...'I'`



Represent possible transitions with **successor** procedure

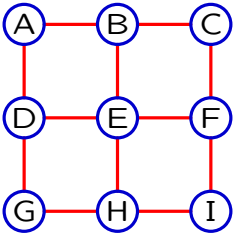
- inputs: current state (location) and action (e.g., up, right, ...)
- output: new state

Define **initialState** (starting location)

Determine if goal has been achieved with **goalTest** procedure

- input: state
- output: **`True`** if state achieves goal, **`False`** otherwise.

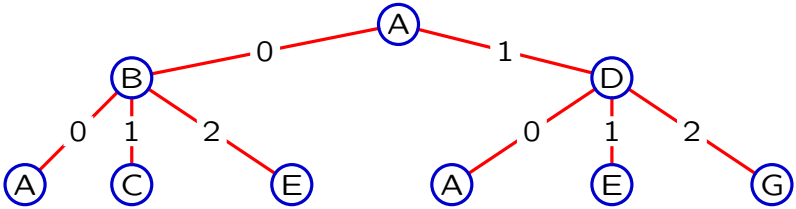
Python Representation




```
successors = { 'A': ['B','D'],          'B': ['A','C','E'],
               'C': ['B','F'],          'D': ['A','E','G'],
               'E': ['B','D','F','H'],  'F': ['C','E','I'],
               'G': ['D','H'],          'H': ['E','G','I'],
               'I': ['F','H'] }

actions = [0, 1, 2, 3]

def successor(s,a):
    if a<len(successors[s]): return successors[s][a]
    else: return s

initialState = 'A'

def goalTest(s):
    return s=='I'
```

Search Trees in Python

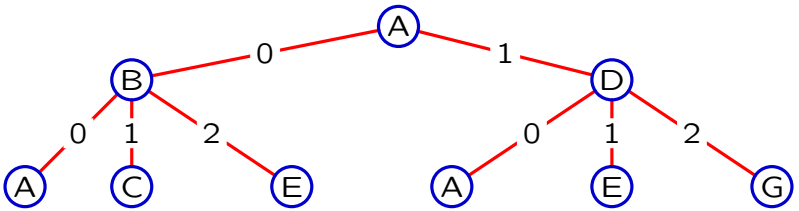
Represent each **node** in the tree as an instance of class **SearchNode**.




```
class SearchNode:
    def __init__(self, action, state, parent):
        self.action = action
        self.state = state
        self.parent = parent
    def path(self):
        if self.parent == None:
            return [(self.action, self.state)]
        else:
            return self.parent.path()+
                   [(self.action,self.state)]
```

Search Algorithm

Construct the tree and find the shortest path to the goal.



Algorithm:

- initialize **agenda** (list of nodes being considered) to contain starting node
- repeat the following steps:
  - remove one node from the agenda
  - add that node's children to the agenda

  until **goal is found** or **agenda is empty**
- return resulting path

Search Algorithm in Python

Define the **search** procedure.

```
def search(initialState, goalTest, actions, successor):
```

Search Algorithm in Python

Initialize the **agenda**.

```
def search(initialState, goalTest, actions, successor):
    if goalTest(initialState):
        return [(None, initialState)]
    agenda = [SearchNode(None, initialState, None)]
```

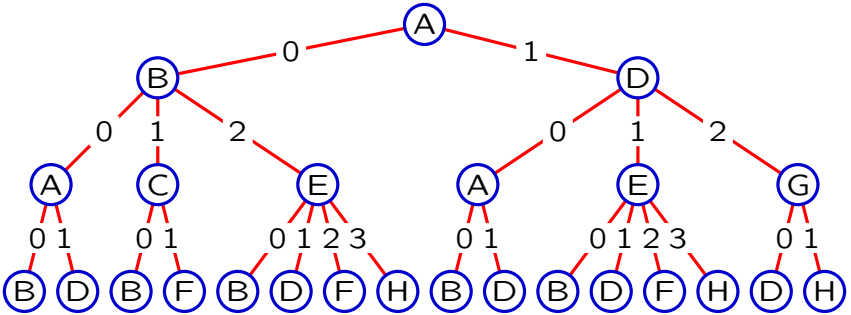
Search Algorithm in Python

Repeatedly (1) **remove** node (parent) from agenda and (2) **add** parent's children until goal is reached or agenda is empty.

```
def search(initialState, goalTest, actions, successor):
    if goalTest(initialState):
        return [(None, initialState)]
    agenda = [SearchNode(None, initialState, None)]
    while not empty(agenda):
        parent = getElement(agenda)
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            else:
                add(newN, agenda)
    return None
```

Order Matters

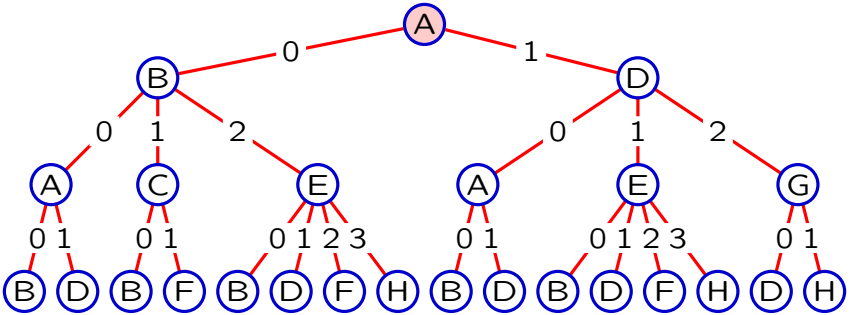
Replace first node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
2:	ABA ABC ABE AD
3:	ABAB ABAD ABC ABE AD

Order Matters

Replace first node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
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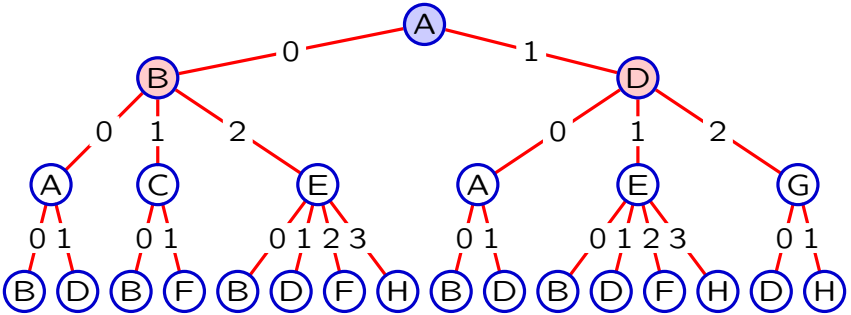
Order Matters

Replace first node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
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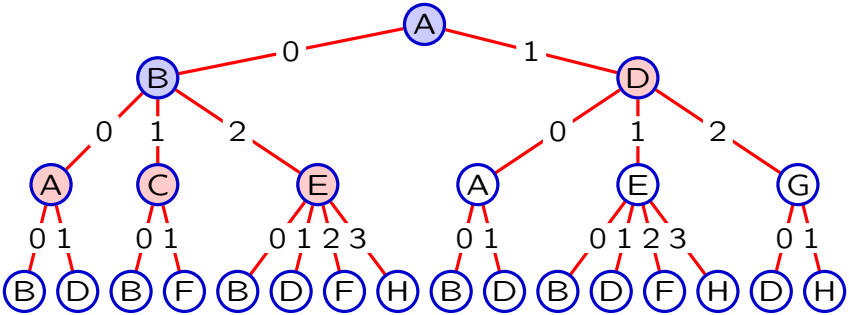
3:	ABAB ABAD ABC ABE AD

Order Matters

Replace first node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
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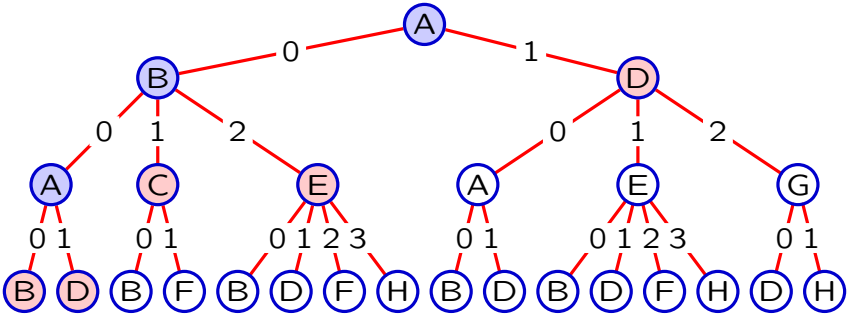
Order Matters

Replace first node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
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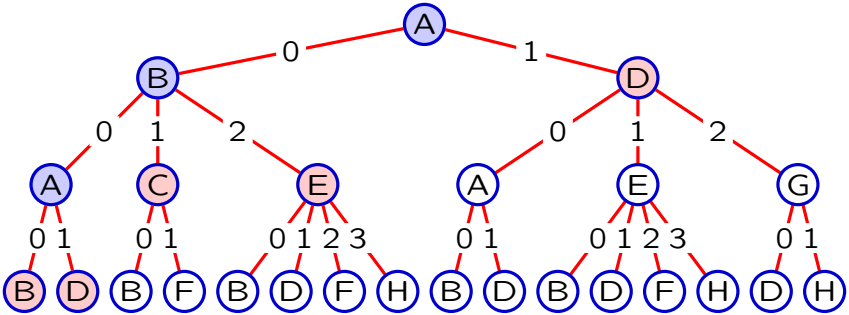
3:	ABAB ABAD ABC ABE AD

Order Matters

Replace first node in agenda by its children:

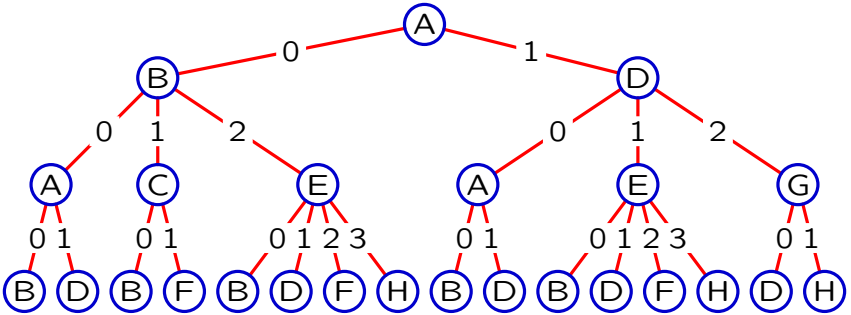


step	Agenda
0:	A
1:	AB AD
2:	ABA ABC ABE AD
3:	ABAB ABAD ABC ABE AD

Depth First Search

Order Matters

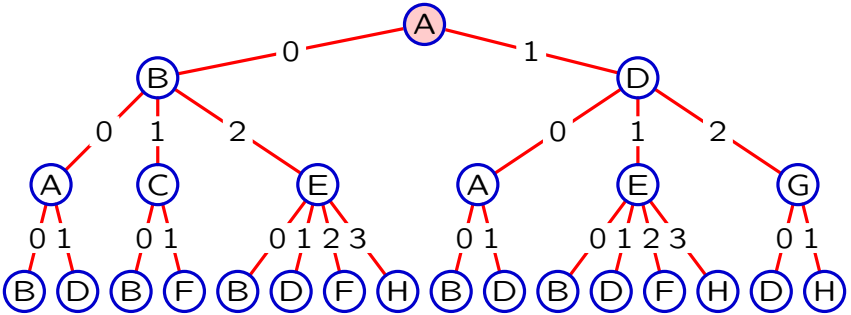
Replace last node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
2:	AB ADA ADE ADG
3:	AB ADA ADE ADGD ADGH

Order Matters

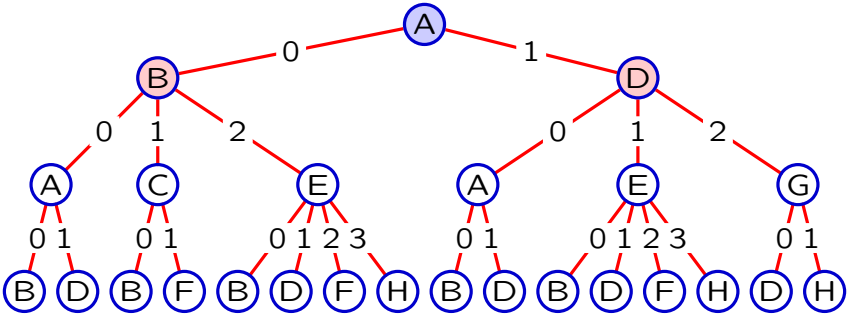
Replace last node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
2:	AB ADA ADE ADG
3:	AB ADA ADE ADGD ADGH

Order Matters

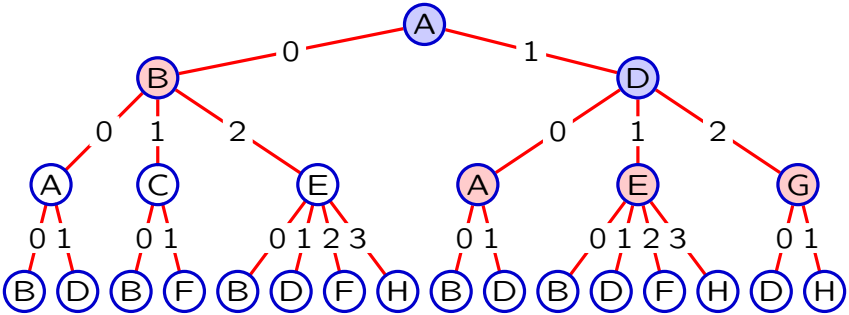
Replace last node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
2:	AB ADA ADE ADG
3:	AB ADA ADE ADGD ADGH

Order Matters

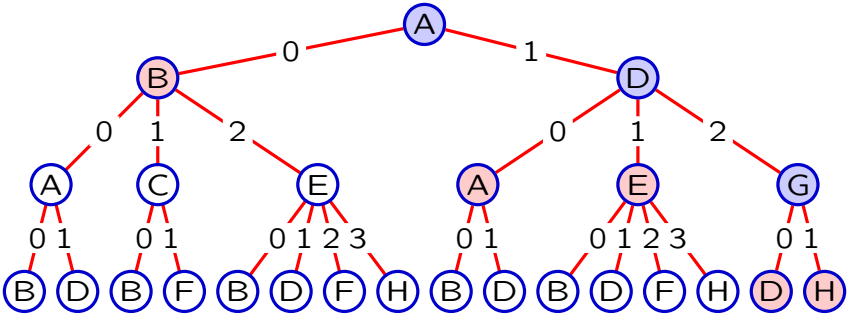
Replace last node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
2:	AB ADA ADE ADG
3:	AB ADA ADE ADGD ADGH

Order Matters

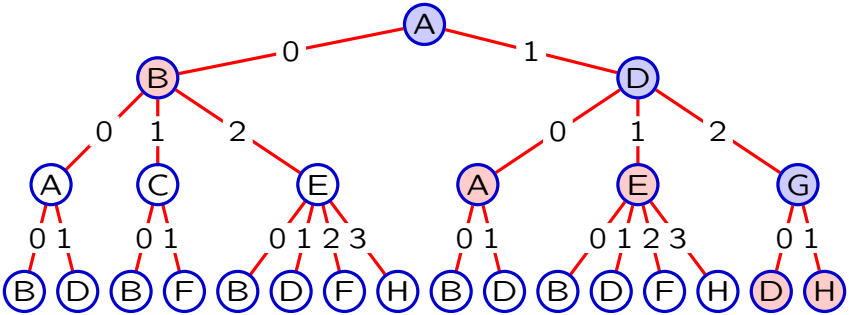
Replace last node in agenda by its children:



step	Agenda
0:	A
1:	AB AD
2:	AB ADA ADE ADG
3:	AB ADA ADE ADGD ADGH

Order Matters

Replace last node in agenda by its children:

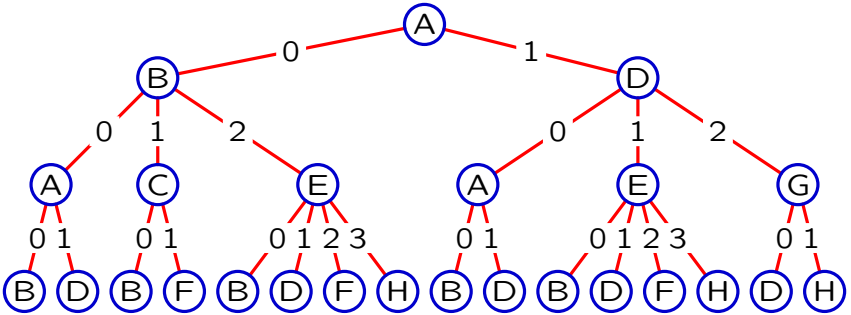


step	Agenda
0:	A
1:	AB AD
2:	AB ADA ADE ADG
3:	AB ADA ADE ADGD ADGH

also Depth First Search

Order Matters

Remove first node from agenda. Add its children to end of agenda.



step	Agenda
0:	A
1:	AB AD
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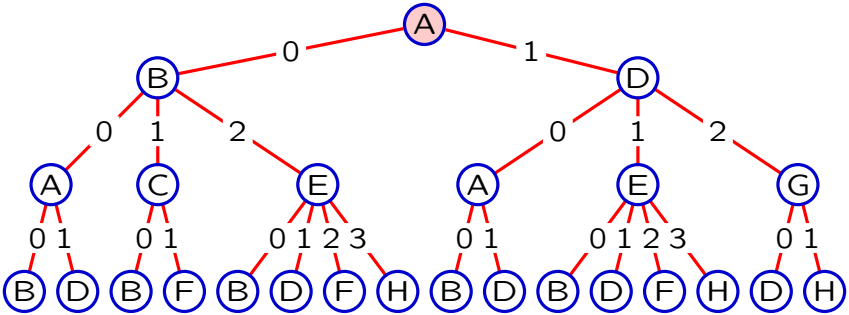
Order Matters

Remove first node from agenda. Add its children to end of agenda.



step	Agenda
0:	A
1:	AB AD
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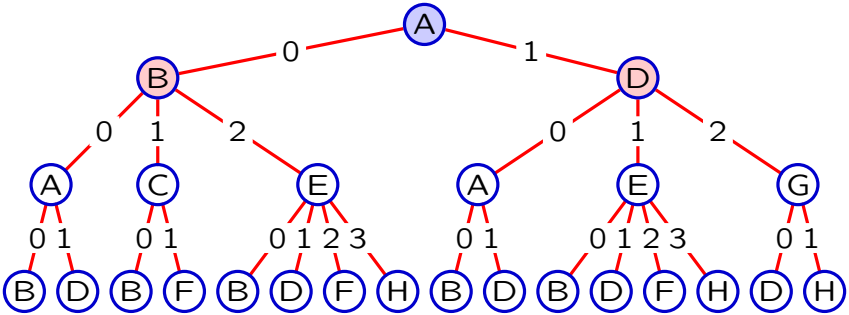
3:	ABA ABC ABE ADA ADE ADG

Order Matters

Remove first node from agenda. Add its children to end of agenda.



step	Agenda
0:	A
1:	AB AD
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3:	ABA ABC ABE ADA ADE ADG

Order Matters

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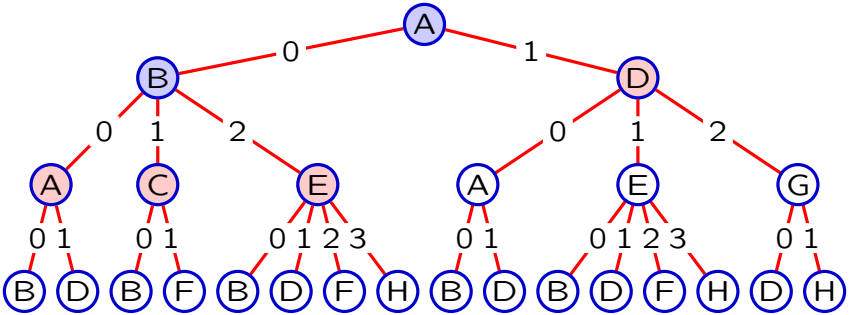
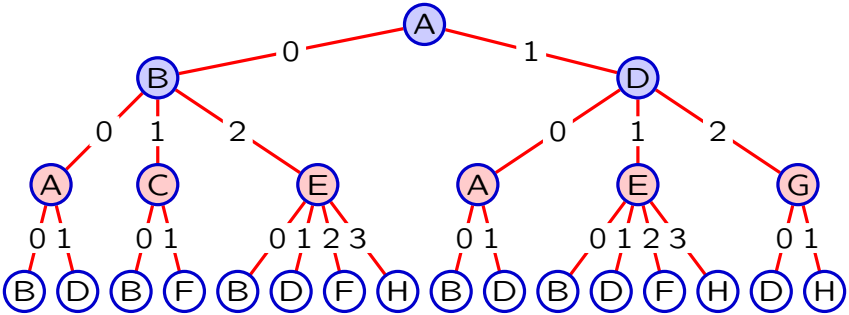


step	Agenda
0:	A
1:	AB AD
2:	AD ABA ABC ABE
3:	ABA ABC ABE ADA ADE ADG

Order Matters

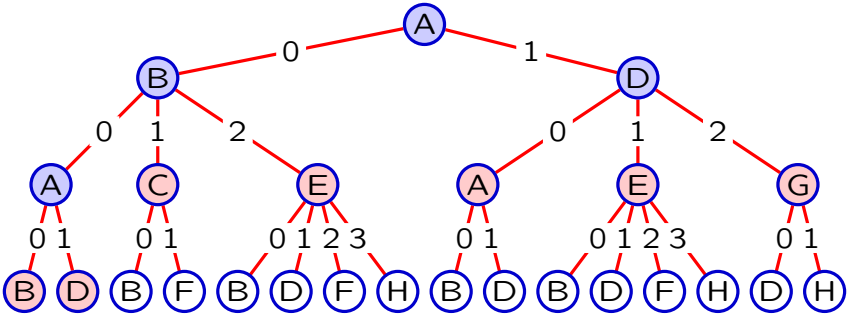
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
0:	A
1:	AB AD
2:	AD ABA ABC ABE
3:	ABA ABC ABE ADA ADE ADG

Order Matters

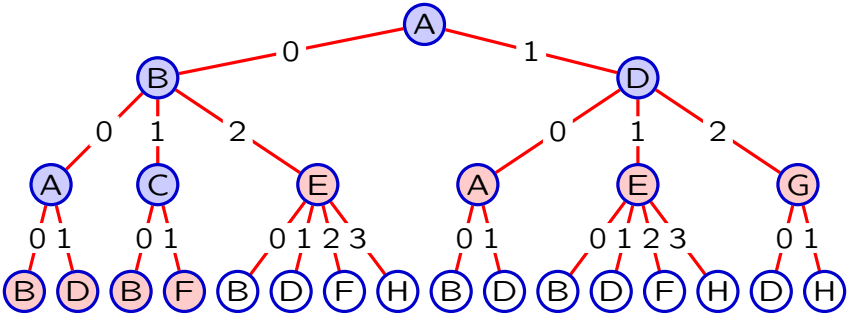
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
1:	AB AD
2:	AD ABA ABC ABE
3:	ABA ABC ABE ADA ADE ADG
4:	ABC ABE ADA ADE ADG ABAB ABAD

Order Matters

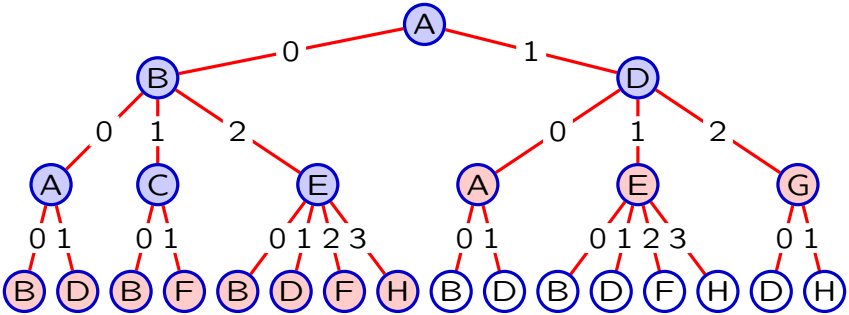
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
2:	AD ABA ABC ABE
3:	ABA ABC ABE ADA ADE ADG
4:	ABC ABE ADA ADE ADG ABAB ABAD
5:	ABE ADA ADE ADG ABAB ABAD ABCB ABCF

Order Matters

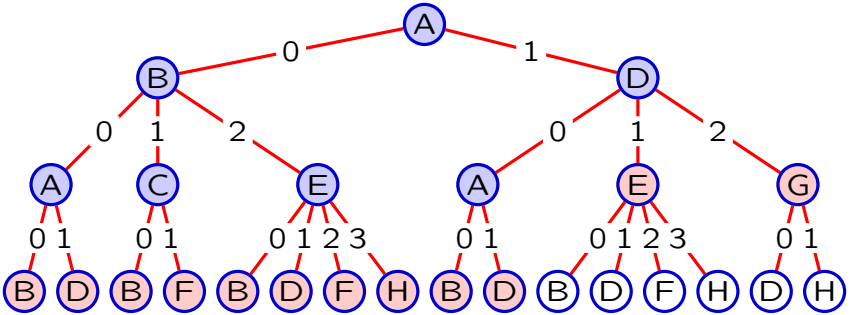
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
3:	ABA ABC ABE ADA ADE ADG
4:	ABC ABE ADA ADE ADG ABAB ABAD
5:	ABE ADA ADE ADG ABAB ABAD ABCB ABCF
6:	ADA ADE ADG ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH

Order Matters

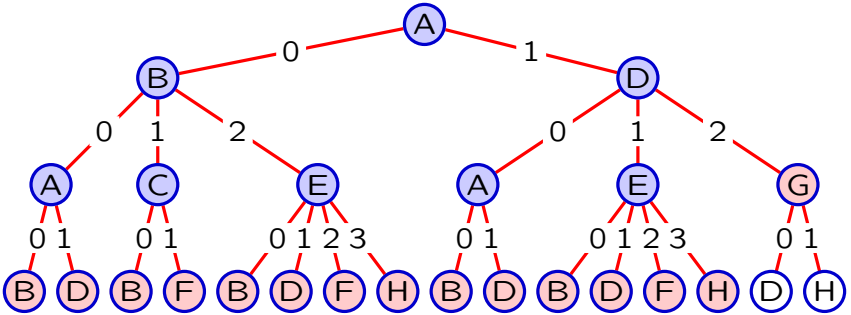
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
5:	ABE ADA ADE ADG ABAB ABAD ABCB ABCF
6:	ADA ADE ADG ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH
7:	ADE ADG ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH ADAB ADAD

Order Matters

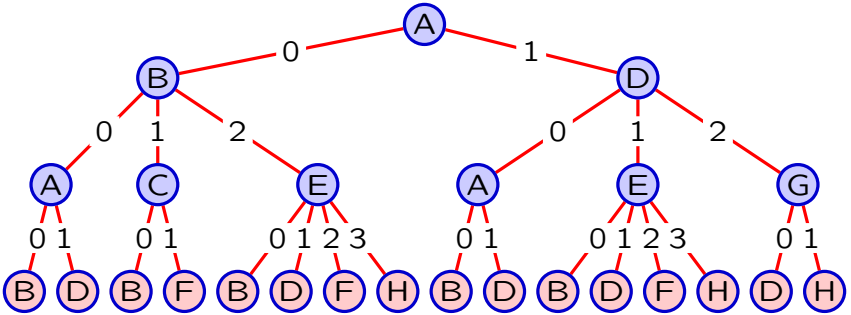
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
7:	ADE ADG ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH ADAB ADAD
8:	ADG ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH ADAB ADAD ADEB ADED ADEF ADEH

Order Matters

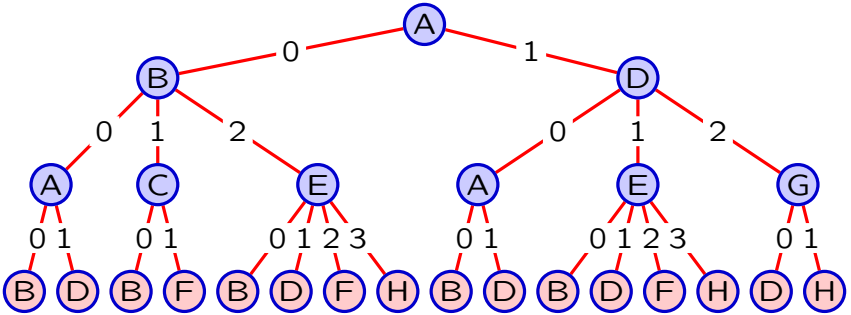
Remove first node from agenda. Add its children to end of agenda.



step	Agenda
8:	ADG ABAB ABAD ABCB ABCF ABEB ABED
	ABEF ABEH ADAB ADAD ADEB ADED ADEF ADEH
9:	ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH
	ADAB ADAD ADEB ADED ADEF ADEH ADGD ADGH

Order Matters

Remove first node from agenda. Add its children to end of agenda.



step	Agenda
8:	ADG ABAB ABAD ABCB ABCF ABEB ABED
	ABEF ABEH ADAB ADAD ADEB ADED ADEF ADEH
9:	ABAB ABAD ABCB ABCF ABEB ABED ABEF ABEH
	ADAB ADAD ADEB ADED ADEF ADEH ADGD ADGH

Breadth First Search

Order Matters

Replace last node by its children (depth-first search):
– implement with **stack** (last-in, first-out).

Remove first node from agenda. Add its children to the end of the agenda (breadth-first search):
– implement with **queue** (first-in, first-out).

Stack

Last in, first out.

```
>>> s = Stack()
>>> s.push(1)
>>> s.push(9)
>>> s.push(3)
>>> s.pop()
3
>>> s.pop()
9
>>> s.push(-2)
>>> s.pop()
-2
```

Stack Class

Last in, first out.

```
class Stack:
    def __init__(self):
        self.data = []
    def push(self, item):
        self.data.append(item)
    def pop(self):
        return self.data.pop()
    def empty(self):
        return self.data is []
```

Queue

First in, first out.

```
>>> q = Queue()
>>> q.push(1)
>>> q.push(9)
>>> q.push(3)
>>> q.pop()
1
>>> q.pop()
9
>>> q.push(-2)
>>> q.pop()
3
```

Queue Class

First in, first out.

```
class Queue:
    def __init__(self):
        self.data = []
    def push(self, item):
        self.data.append(item)
    def pop(self):
        return self.data.pop(0)   #NOTE: different argument
    def empty(self):
        return self.data is []
```

Depth-First Search

Replace **getElement**, **add**, and **empty** with **stack** commands.

```
def search(initialState, goalTest, actions, successor):
    agenda = Stack()
    if goalTest(initialState):
        return [(None, initialState)]
    agenda.push(SearchNode(None, initialState, None))
    while not agenda.empty():
        parent = agenda.pop()
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            else:
                agenda.push(newN)
    return None
```

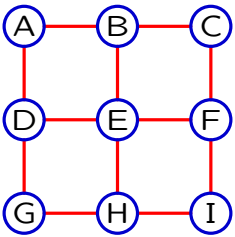
Breadth-First Search

Replace **getElement**, **add**, and **empty** with **queue** commands.

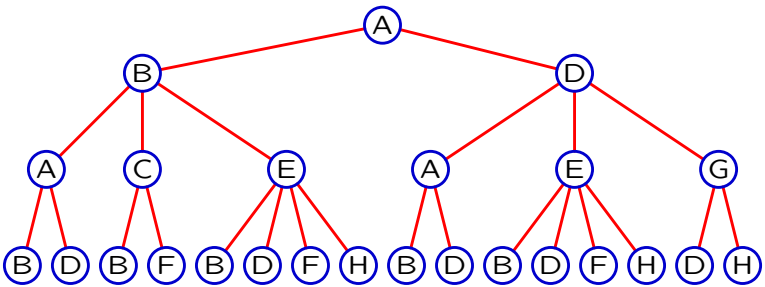
```
def search(initialState, goalTest, actions, successor):
    agenda = Queue()
    if goalTest(initialState):
        return [(None, initialState)]
    agenda.push(SearchNode(None, initialState, None))
    while not agenda.empty():
        parent = agenda.pop()
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            else:
                agenda.push(newN)
    return None
```

Too Much Searching

Find minimum distance path between 2 points on a rectangular grid.



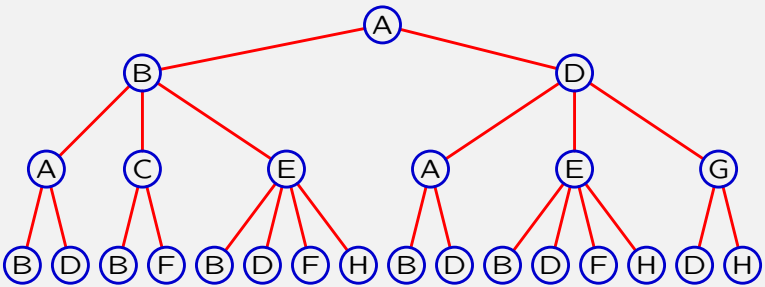
Represent **all possible paths** with a **tree** (shown to just length 3).



Not all of the nodes of this tree must be searched!

Check Yourself

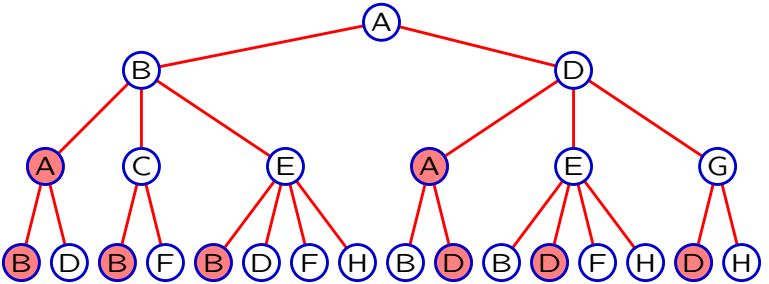
How many of these terminal nodes can be ignored?



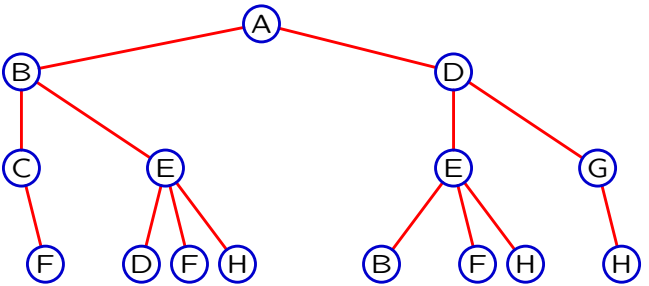
1. 0 2. 2 3. 4 4. 6 5. 8

Check Yourself

The red states represent returns to a previously visited state.

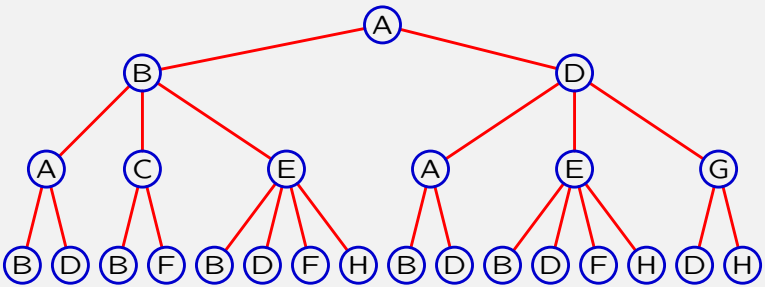


We only need to consider paths that do not revisit states.



Check Yourself

How many of these terminal nodes can be ignored? 5



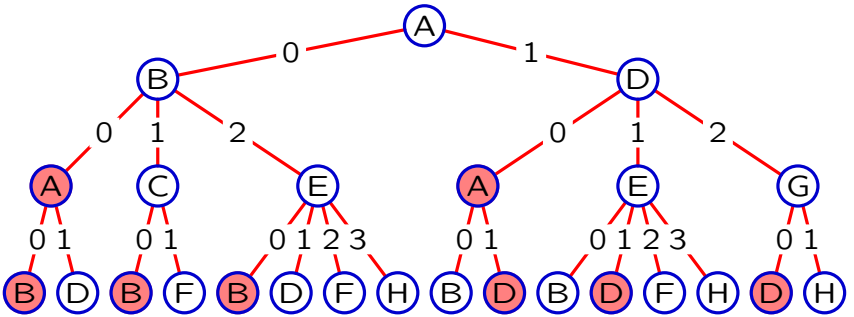
1. 0 2. 2 3. 4 4. 6 5. 8

Pruning

Prune the tree to reduce the amount of work.

Pruning Rule 1:

Don't consider any path that visits the same state twice.

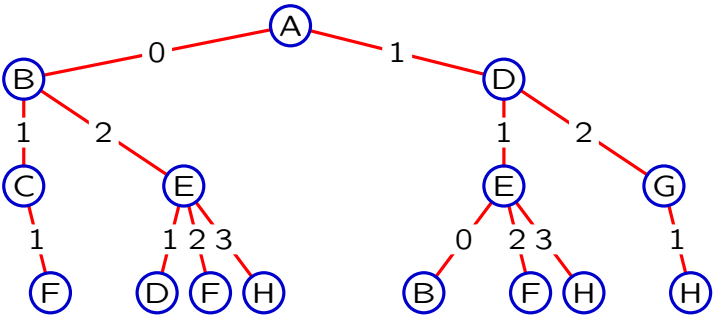


Pruning

Prune the tree to reduce the amount of work.

Pruning Rule 1:

Don't consider any path that visits the same state twice.



Pruning Rule 1

Implementation (depth first, switch to **Queue** for breadth first)

```
def search(initialState, goalTest, actions, successor):
    agenda = Stack()
    if goalTest(initialState):
        return [(None, initialState)]
    agenda.push(SearchNode(None, initialState, None))
    while not agenda.empty():
        parent = agenda.pop()
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            elif parent.inPath(newS):      # pruning rule 1
                pass
            else:
                agenda.push(newN)
    return None
```

Pruning Rule 1

Add **inPath** to SearchNode.

```
class SearchNode:
    def __init__(self, action, state, parent):
        self.action = action
 self.state = state
 self.parent = parent
    def path(self):
        if self.parent == None:
            return [(self.action, self.state)]
        else:
            return self.parent.path() + [(self.action, self.state)]
    def inPath(self, state):
        if self.state == state:
            return True
        elif self.parent == None:
            return False
        else:
            return self.parent.inPath(state)
```

Pruning

Prune the tree to reduce the amount of work.

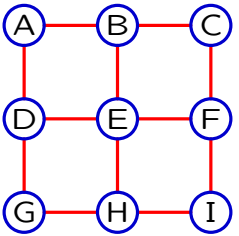
Pruning Rule 2:

If multiple actions lead to the same state, consider only one of them.

Pruning Rule 2

```
def search(initialState, goalTest, actions, successor):
    agenda = Stack()
    if goalTest(initialState):
        return [(None, initialState)]
    agenda.push(SearchNode(None, initialState, None))
    while not agenda.empty():
        parent = agenda.pop()
        newChildStates = []
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            elif newS in newChildStates:  # pruning rule 2
                pass
            elif parent.inPath(newS):     # pruning rule 1
                pass
            else:
                newChildStates.append(newS)
                agenda.push(newN)
    return None
```

Depth-First Search Example



Depth-First Search Transcript

```
agenda:  Stack([A])                                                    1
   expanding:  A -> AB, AD                                            +2
agenda:  Stack([AB, AD])
   expanding:  AD -> ABE, ADG                                         +2
agenda:  Stack([AB, ADE, ADG])
   expanding:  ADG -> ADGH                                            +1
agenda:  Stack([AB, ADE, ADGH])
   expanding:  ADGH -> ADGHE                                          +1
[(None, 'A'), (1, 'D'), (2, 'G'), (1, 'H'), (2, 'I')]
                                                                      ---
                                                   states visited = 7
```

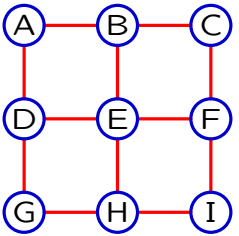
Depth-First Search Properties

- May run forever if we don't apply pruning rule 1.
- May run forever in an infinite domain.
- Doesn't necessarily find the shortest path.
- Efficient in the amount of space it requires to store the agenda.

Breadth-First Search

```
def search(initialState, goalTest, actions, successor):
    agenda = Queue()
    if goalTest(initialState):
        return [(None, initialState)]
    agenda.push(SearchNode(None, initialState, None))
    while not agenda.empty():
        parent = agenda.pop()
        newChildStates = []
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            elif newS in newChildStates:  # pruning rule 2
                pass
            elif parent.inPath(newS):     # pruning rule 1
                pass
            else:
                newChildStates.append(newS)
                agenda.push(newN)
    return None
```

Breadth-First Search Example



Breadth-First Search Transcript

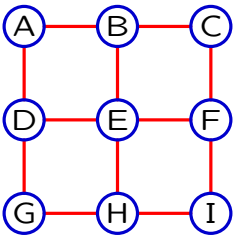
```
agenda:  Queue([A])                                                    1
   expanding:  A -> AB, AD                                            +2
agenda:  Queue([AB, AD])
   expanding:  AB -> ABC, ABE                                         +2
agenda:  Queue([AD, ABC, ABE])
   expanding:  AD -> ADE, ADG                                         +2
agenda:  Queue([ABC, ABE, ADE, ADG])
   expanding:  ABC -> ABCF                                            +1
agenda:  Queue([ABE, ADE, ADG, ABCF])
   expanding:  ABE -> ABED, ABEF, ABEH                                +3
agenda:  Queue([ADE, ADG, ABCF, ABED, ABEF, ABEH])
   expanding:  ADE -> ADEB, ADEF, ADEH                                +3
agenda:  Queue([ADG, ABCF, ABED, ABEF, ABEH, ADEB, ADEF, ADEH])
   expanding:  ADG -> ADGH                                            +1
agenda:  Queue([ABCF, ABED, ABEF, ABEH, ADEB, ADEF, ADEH, ADGH])
   expanding:  ABCF -> ABCFE                                          +1
[(None, 'A'), (0, 'B'), (1, 'C'), (1, 'F'), (2, 'I')]
                                                                      ---
                                                  states visited = 16
```

Breadth-First Search Properties

- Always returns a shortest path to a goal state, if a goal state exists in the set of states reachable from the start state.
- May run forever in an infinite domain if there is no solution.
- Requires more space than depth-first search.

Still Too Much Searching

Breadth-first search, visited 16 nodes: but there are only 9 states!



We should be able to reduce the search even further.

Dynamic Programming Principle

The *shortest* path from X to Z that goes through Y is made up of

- the *shortest* path from X to Y and
- the *shortest* path from Y to Z.

We only need to remember the *shortest* path from the start state to each other state!

Dynamic Programming in Breadth-First Search

The **first** path that BFS finds from start to X is the *shortest* path from start to X.

We only need to remember the *first* path we find from the start state to each other state.

Dynamic Programming as a Pruning Technique

Don't consider any path that visits a state that you have already visited via some other path.

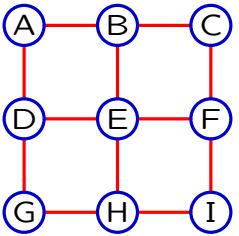
Need to remember the first path we find to each state.

Use dictionary called **visited**

Breadth-First Search with Dynamic Programming

```
def breadthFirstDP(initialState, goalTest, actions, successor):
    agenda = Queue()
    if goalTest(initialState):
        return [(None, initialState)]
    agenda.push(SearchNode(None, initialState, None))
    visited = {initialState: True}
    while not agenda.empty():
        parent = agenda.pop()
        for a in actions:
            newS = successor(parent.state, a)
            newN = SearchNode(a, newS, parent)
            if goalTest(newS):
                return newN.path()
            elif visited.has_key(newS): # rules 1, 2, 3
                pass
            else:
                visited[newS] = True
                agenda.push(newN)
    return None
```

Breadth-First with Dynamic Programming Example



Breadth-First with Dynamic Programming Transcript

```
agenda:  Queue([A])                 visited: A                          1
   expanding:  A -> AB, AD                                              +2
agenda:  Queue([AB, AD])            visited: A, B, D
   expanding:  AB -> ABC, ABE                                           +2
agenda:  Queue([AD, ABC, ABE])      visited: A, B, C, D, E
   expanding:  AD -> ADG                                                +1
agenda:  Queue([ABC, ABE, ADG])     visited A, B, C, D, E, G
   expanding:  ABC -> ABCF                                              +1
agenda:  Queue([ABE, ADG, ABCF])    visited A, B, C, D, E, F, G
   expanding:  ABE -> ABEH                                              +1
agenda:  Queue([ADG, ABCF, ABEH]) visited A, B, C, D, E, F, G, H
   expanding:  ADG -> x
agenda:  Queue([ABCF, ABEH])
   expanding:  ABCF -> x
[(None, 'A'), (0, 'B'), (1, 'C'), (1, 'F'), (2, 'I')]
                                                                        ---
                                                    states visited = 8
```

Summary

Developed two search algorithms
– depth-first search
– breadth-first search
Developed three pruning rules
– don't consider any path that visits the same state twice
– if multiple actions lead to same state, only consider one of them
– dynamic programming: only consider the first path to a given state

Nano-Quiz Makeup: Wednesday, May 4, 6-11pm, 34-501.
– everyone can makeup/retake NQ 1
– everyone can makeup/retake two additional NQs
– you can makeup/retake other NQs excused by S^3

If you makeup/retake a NQ, the new score will replace the old score, even if the new score is lower!

6.01SC Introduction to Electrical Engineering and Computer Science

Spring 2011