Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2010)

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1. Example 1.20, page 37 in the text.

   Consider two independent fair coin tosses, in which all four possible outcomes are equally likely. Let

$$\begin{aligned} H_1 &= \{\text{1st toss is a head}\}, \\ H_2 &= \{\text{2nd toss is a head}\}, \\ D &= \{\text{the two tosses produced different results}\}. \end{aligned}$$

   (a) Are the events H_1 and H_2 (unconditionally) independent?

   (b) Given event D has occurred, are the events H_1 and H_2 (conditionally) independent?

2. Imagine a drunk tightrope walker, in the middle of a really long tightrope, who manages to keep his balance, but takes a step forward with probability p and takes a step back with probability $(1 - p)$.

   (a) What is the probability that after two steps the tightrope walker will be at the same place on the rope?

   (b) What is the probability that after three steps, the tightrope walker will be one step forward from where he began?

   (c) Given that after three steps he has managed to move ahead one step, what is the probability that the first step he took was a step forward?

3. Problem 1.31, page 60 in the text.

   Communication through a noisy channel. A binary (0 or 1) message transmitted through a noisy communication channel is received incorrectly with probability ϵ_0 and ϵ_1, respectively (see the figure). Errors in different symbol transmissions are independent. The channel source transmits a 0 with probability p and transmits a 1 with probability $1 - p$.

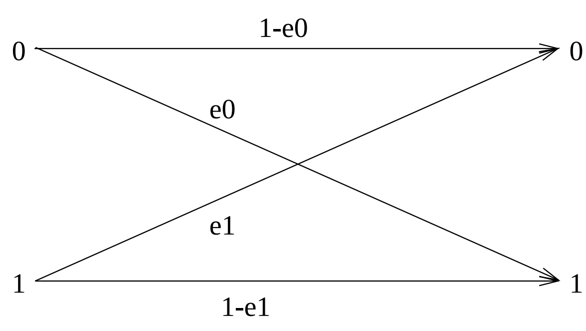

Figure 1: Error probabilities in a binary communication channel.

   (a) What is the probability that a randomly chosen symbol is received correctly?

   (b) Suppose that the string of symbols 1011 is transmitted. What is the probability that all the symbols in the string are received correctly?

(c) In an effort to improve reliability, each symbol is transmitted three times and the received symbol is decoded by majority rule. In other words, a 0 (or 1) is transmitted as 000 (or 111, respectively), and it is decoded at the receiver as a 0 (or 1) if and only if the received three-symbol string contains at least two 0s (or 1s, respectively). What is the probability that a transmitted 0 is correctly decoded?

(d) Suppose that the scheme of part (c) is used. What is the probability that a 0 was transmitted given that the received string is 101?

4. (a) Can an event A be independent of itself?

(b) Problem 1.43(a) on page 63 in text.

Let A and B be independent events. Use the definition of independence to prove that the events A and B^c are independent.

(c) Problem 1.44 on page 64 in text.

Let A, B, and C be independent events, with $\mathbf{P}(C) > 0$. Prove that A and B are conditionally independent of C.

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