MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

Tutorial
March 9-10, 2006

1. Consider the two-sided exponential PDF

$$f_X(x) = \begin{cases} p\lambda e^{-\lambda x}, & \text{if } x \geq 0 \\ (1-p)\lambda e^{\lambda x}, & \text{if } x < 0 \end{cases}$$

   where λ and p are scalars with $\lambda > 0$ and $p \in [0, 1]$. Find the mean and the variance of X.

2. A signal $s = 2$ is transmitted from a satellite but is corrupted by noise, so that the received signal is $X = s + W$. When the weather is good, which happens with probability 2/3, W is normal with zero mean and variance 1. When the weather is bad, W is normal with zero mean and variance 9. In the absence of any weather information, find the PDF of X and calculate the probability that X is between 1 and 3. (Express the probability using the standard normal CDF Φ.)

3. Beginning at time $t = 0$ we begin using bulbs, one at a time, to illuminate a room. Bulbs are replaced immediately upon failure. Each new bulb is selected independently by an equally likely choice between a Type-A bulb and a Type-B bulb.

   The lifetime, X, of any particular bulb of a particular type is an independent random variable with the following PDF:

$$\text{For Type-A Bulbs:} \quad f_X(x) = \begin{cases} e^{-x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$\text{For Type-B Bulbs:} \quad f_X(x) = \begin{cases} 3e^{-3x} & x \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

   (a) Find the expected time until the first failure.

   (b) Find $\mathbf{P}(D)$, the probability that there are no bulb failures during the first τ hours of this process.

   (c) Given that there are no failures during the first τ hours of this process, determine $\mathbf{P}(T_{1A}|D)$, the conditional probability that the first bulb used is a Type-A bulb.

   (d) Given that there are no failures during the first τ hours of this process, determine the total expected time until the first failure (i.e., the expected time elapsed from $t = 0$ until the first bulb fails).