# Massachusetts Institute of Technology 

Department of Electrical Engineering \& Computer Science

### 6.041/6.431: Probabilistic Systems Analysis

(Spring 2006)

## Tutorial 3: Answers

March 2-3, 2006

1. (a)

$$
\begin{gathered}
p_{S}(s)=\left\{\begin{array}{cl}
24 / 90, & s=1 ; \\
36 / 90, & s=2 ; \\
30 / 90, & s=3 ; \\
0, & \text { otherwise },
\end{array}\right. \\
p_{S \mid A}(s)=P(S=s \cap A) / P(A)=\left\{\begin{array}{cl}
24 / 60, & s=1 ; \\
36 / 60, & s=2 \\
0, & \text { otherwise } .
\end{array}\right.
\end{gathered}
$$

(The required sketches are omitted.)
(b) The joint PMF for $R$ and $Y$ is:

(c)

$$
p_{X \mid A}(x)\left\{\begin{array}{cl}
8 / 60, & x=2 \\
24 / 60, & x=3 \\
22 / 60, & x=4 \\
6 / 60, & x=5 \\
0, & \text { otherwise }
\end{array}\right.
$$

(The required sketch is omitted.)
2. (a)

$$
p_{N, K}(n, k)= \begin{cases}1 / 4 k & \text { if } k=1,2,3,4 \text { and } n=1, \ldots, k \\ 0 & \text { otherwise }\end{cases}
$$

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(b)

$$
p_{N}(n)= \begin{cases}1 / 4+1 / 8+1 / 12+1 / 16=25 / 48 & n=1 \\ 1 / 8+1 / 12+1 / 16=13 / 48 & n=2 \\ 1 / 12+1 / 16=7 / 48 & n=3 \\ 1 / 16=3 / 48 & n=4 \\ 0 & \text { otherwise }\end{cases}
$$

(c) The conditional PMF

$$
p_{K \mid N}(k \mid 2)=\frac{p_{N, K}(2, k)}{p_{N}(2)}= \begin{cases}6 / 13 & k=2 \\ 4 / 13 & k=3 \\ 3 / 13 & k=4 \\ 0 & \text { otherwise }\end{cases}
$$

(d) Let A be the event that Chuck bought at least 2 but no more than 3 books, $E[K \mid A]=3$ $\operatorname{var}(K \mid A)=\frac{3}{5}$
(e) $E[T]=\frac{21}{4}$
3. (a)
$p_{X, Y, Z}(x, y, z)= \begin{cases}\frac{1}{4} p(1-p)^{x-1}, & \text { if } x \text { is odd and }(y, z) \in\{(0,0),(0,2),(2,0),(2,2)\} \\ p(1-p)^{x-1}, & \text { if } x \text { is even and }(y, z)=(0,0) \\ 0, & \text { otherwise. }\end{cases}$
(b) (i) No.
(ii) Yes.
(iii) No.
(iv) Yes.
(c) The variance is

$$
(0-2)^{2} \frac{1}{4}+(4-2)^{2} \frac{1}{4}=2
$$

