# Massachusetts Institute of Technology <br> Department of Electrical Engineering \& Computer Science <br> 6.041/6.431: Probabilistic Systems Analysis <br> (Spring 2006) 

## Tutorial 05 <br> March 16-17, 2006

1. For $X$ a random variable uniformly distributed between -1 and 1 , find the density function of $Y$ where:
a) $Y=\sqrt{|X|}$.
b) $Y=-\ln |X|$.
2. a) Suppose a random variable $X$ is uniformly distributed between 0 and 1 . If $Y=\cos \pi X$ find the density function for $Y$.
b) Now suppose $X$ is uniformly distributed between $-1 / 2$ and $1 / 2$. Find the density function for $Y$, where $Y=\tan \pi X$.
3. Optional Suppose $X$ is a standard normal random variable, i.e. $X \sim N[0,1]$. Find the density for $Y$, where:
a) $Y=X^{2}$.
b) $Y=e^{X}$.
4. Let continuous random variables $X, Y$ and $Z$ be independent and identically distributed according to the uniform distribution in the unit interval $[0,1]$.
(a) Consider two new random variables defined by $V=X Y$ and $W=Z^{2}$. Derive the joint $\operatorname{PDF} f_{V, W}(v, w)$.
(b) Show that $\mathbf{P}\left(X Y<Z^{2}\right)=\frac{5}{9}$.
