MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

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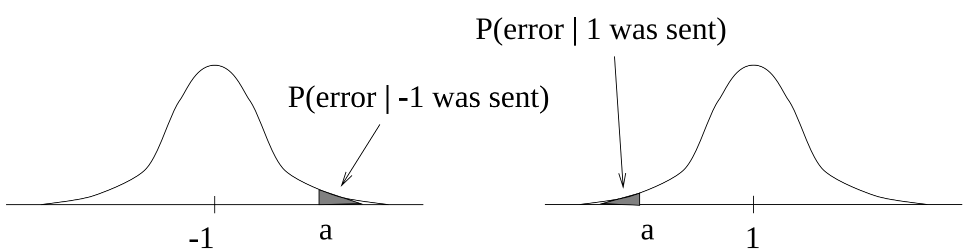
1. See solutions for supplementary problems, Problem 2, Section 3.1

2. See online solutions for supplementary problems (Problem 4, Section 3.2)

3. (a) Let Z be the random variable representing the additive zero-mean Gaussian noise; that is, $Z \sim N(0, \sigma^2)$. Let S_0 be the event that -1 is sent and S_1 be the event that $+1$ is sent. Let R_0 be the event that we conclude that an encoded signal of -1 was sent based on the received value being less than a. Let R_1 be the event that we conclude that an encoded signal of $+1$ was sent based on the received value being greater than a.

   There are two ways for errors to occur. The true encoded signal could be -1 but we could conclude that the encoded signal of $+1$ was sent. Conditioned on the true encoded signal being -1, the received signal is $Z - 1$; we would erroneously conclude that the encoded signal of $+1$ was sent if $Z - 1 > a$. Similarly, the true encoded signal could be $+1$ but we could conclude that the encoded signal of -1 was sent. In this case, conditioned on the true encoded signal being $+1$, the received signal is $Z + 1$ and we would erroneously conclude that the true signal was -1 if $Z + 1 < a$.

   The figure below illustrates the situations under which errors can occur.



   Let Φ such that

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-t^2/2} dt.$$

   Therefore

$$\begin{aligned}
\mathbf{P}(\text{error}) &= \mathbf{P}(R_1|S_0)\mathbf{P}(S_0) + \mathbf{P}(R_0|S_1)\mathbf{P}(S_1) \\
&= \mathbf{P}(Z - 1 > a)(p) + \mathbf{P}(Z + 1 < a)(1 - p) \\
&= p \cdot \left(1 - \Phi\left(\frac{a - (-1)}{\sigma}\right)\right) + (1 - p) \cdot \Phi\left(\frac{a - 1}{\sigma}\right) \\
&= p - p \cdot \Phi\left(\frac{a + 1}{\sigma}\right) + (1 - p) \cdot \left(1 - \Phi\left(\frac{1 - a}{\sigma}\right)\right) \\
&= 1 - p \cdot \Phi\left(\frac{a + 1}{\sigma}\right) - (1 - p) \cdot \Phi\left(\frac{1 - a}{\sigma}\right)
\end{aligned}$$

   (b) $\mathbf{P}(\text{error}) = 1 - 0.4 \cdot \Phi(\frac{3/2}{1/2}) - 0.6 \cdot \Phi(\frac{1/2}{1/2})$