# Massachusetts Institute of Technology 

Department of Electrical Engineering \& Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2006)

## Tutorial 06

March 23-24, 2006

1. The transform and the mean of a random variable $X$ are given by $M_{X}(s)=a e^{s}+b e^{13\left(e^{s}-1\right)}$ and $E[X]=5$ respectively. Determine the numerical values of:
(a) The constants $a$ and $b$.
(b) $E\left[e^{5 X}\right]$.
(c) $P(X=1)$.
(d) $E\left[X^{2}\right]$.
2. Let $X, Y$, and $Z$ be independent random variables. $X$ is Bernoulli with $p=1 / 4$. $Y$ is exponential with parameter 3. $Z$ is Poisson with parameter 5.
(a) Find the transform of $5 Z+1$.
(a) Find the transform of $X+Y$.
(b) Consider the new random variable $U=X Y+(1-X) Z$. Find the transform associated with $U$.
3. Let $X$ be uniform on $[0,2]$ and let $Y$ be uniform on $[3,4]$. Assume that $X$ and $Y$ independent. Find and sketch the PDF of $X+Y$, using convolutions.
