MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Electrical Engineering & Computer Science 6.041/6.431: Probabilistic Systems Analysis (Spring 2006)

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1. (a) The criminal will be caught if the first officer comes by in fewer than t seconds. Since the time until the first arrival is exponentially distributed, the desired probability is $1 - e^{-t\lambda}$.

(b) We are interested in the probability that the second arrival occurs before time t. The Erlang PDF of order 2 is $f_{Y_2}(y) = \lambda^2 y e^{-\lambda y}$. The desired probability is obtained by integrating by parts and is $1 - e^{-\lambda t}(\lambda t + 1)$.

- 2. (Problem 5.16) See textbook, page 307.
- 3. (a) The arrival time of each of the three calls is uniformly distributed in the interval of 90 minutes (see Problem 16 in Chapter 5 of the text). Furthermore, the three arrival times are independent of each other. This follows intuitively from the definition of the Poisson process: given that there was an arrival at some particular time, this gives us no information on what may have happened at other times. Therefore the probability that all three occur within the first 30 minutes is: $(1/3)^3 = 1/27$.

(b) The probability that at least one ocurs in the first 30 minutes is, by the same reasoning as above, 1 - (8/27) = 19/27.