# Massachusetts Institute of Technology <br> Department of Electrical Engineering \& Computer Science <br> 6.041/6.431: Probabilistic Systems Analysis <br> (Spring 2006) 

## Tutorial 11

May 4-5, 2006

1. Problem 5.14 in text, page 306
2. (a) The class $\{1,2\}$ is recurrent and aperiodic (i.e. the class has period 1 ). The class $\{4,5,6\}$ is recurrent with period 3 . The state $\{3\}$ is transient.
(b) $r_{33}(n)=(0.2)^{n}$
(c) Let $T_{33}$ be the number of trials up to and including the first trial on which the process leaves state 3, given that it starts in state 3. Then $\mathbf{E}\left[T_{33}\right]=\frac{5}{4}$.
(d) Let $X_{n}$ denote the state after $n$ trials. Then $\mathbf{P}\left(X_{n} \neq 1\right.$ for all $\left.n \mid X_{0}=3\right)=\frac{3}{8}$.
(e) $r_{34}(10)=(0.3)+(0.2)^{3}(0.3)+(0.2)^{6}(0.3)+(0.2)^{9}(0.3) \approx 0.3024$
(f) $\mathbf{P}\left(X_{1}=4 \mid X_{10}=4, X_{0}=3\right)=\frac{0.3}{r_{34}(10)}=0.992$
