MASSACHUSETTS INSTITUTE OF TECHNOLOGY Department of Electrical Engineering & Computer Science 6.041/6.431: Probabilistic Systems Analysis (Spring 2006)

Tutorial 11 May 4-5, 2006

- 1. Problem 5.14 in text, page 306
- 2. (a) The class $\{1,2\}$ is recurrent and aperiodic (i.e. the class has period 1). The class $\{4,5,6\}$ is recurrent with period 3. The state $\{3\}$ is transient.
 - (b) $r_{33}(n) = (0.2)^n$
 - (c) Let T_{33} be the number of trials up to and including the first trial on which the process leaves state 3, given that it starts in state 3. Then $\mathbf{E}[T_{33}] = \frac{5}{4}$.
 - (d) Let X_n denote the state after n trials. Then $\mathbf{P}(X_n \neq 1 \text{ for all } n \mid X_0 = 3) = \frac{3}{8}$.
 - (e) $r_{34}(10) = (0.3) + (0.2)^3(0.3) + (0.2)^6(0.3) + (0.2)^9(0.3) \approx 0.3024$
 - (f) $\mathbf{P}(X_1 = 4 \mid X_{10} = 4, X_0 = 3) = \frac{0.3}{r_{34}(10)} = 0.992$