# Massachusetts Institute of Technology <br> Department of Electrical Engineering \& Computer Science <br> 6.041/6.431: Probabilistic Systems Analysis <br> (Spring 2006) 

## Recitation 09

March 21, 2006

1. Al and Bo are in a race. Denote Al's and Bo's elapsed times with random variables $X$ and $Y$, respectively. These independent random variables are described by the following PDFs:

$$
\begin{aligned}
& f_{X}(x)= \begin{cases}0.5 & 1<x<3 \\
0 & \text { elsewhere }\end{cases} \\
& f_{Y}(y)= \begin{cases}0.5 & 2<y<4 \\
0 & \text { elsewhere }\end{cases}
\end{aligned}
$$

(a) Determine $P(A)$, the probability that Al wins the race.
(b) Determine the probability that Al wins a total of exactly 7 of the next 10 races. Assume all races are independent. You may use $P(A)$ symbolically in your answer. (As long as your answer is explicit, compact, and fully explained, it need not be simplified.)
(c) Determine, carefully sketch, and label the PDF for $W$, the elapsed time for the winner of the race. Fully explain each step of your work.
2. Random variables $X$ and $Y$ are independent and have PDFs as shown below.



Let $W=X+Y$, and find $f_{W}(w)$ using a graphical argument.
3. Alice and Bob flip bias coins independently. Alice's coin comes up heads with probability $1 / 4$, while Bob's coin comes up head with probability $3 / 4$. Each stop as soon as they get a head; that is, Alice stops when she gets a head while Bob stops when he gets a head. What is the PMF of the total amount of flips until both stop? (That is, what is the PMF of the combined total amount of flips for both Alice and Bob until they stop?)

