6.041SC Probabilistic Systems Analysis and Applied Probability, Fall 2013 Transcript – Tutorial: PMF of a Function of a Random Variable

Hey guys. Welcome back. Today, we're going to be working on a problem that asks you to find the PMF of a function of a random variable. So let's just jump right in. The problem statement gives you the PMF for a random variable called x. So we're told that there's this random variable x that takes on values minus 3, minus 2, minus 1, 1, 2, and 3.

And for each of those values, the probability mass lying over that value is given by this formula, x squared over a. Now I didn't write it here to save room, but we're also told that a is a real number that is greater than 0. And we're told that the probability of x taking on any value outside of the set is 0.

Now we're asked to do two things in the problem. First is to find the value of the parameter a. And that's sort of a natural question to ask, because if you think about it, the PMF isn't fully specified. And in fact, if you plug in the wrong number for a, you actually won't get a valid PMF. So we'll explore that idea in the first part.

And then the second part, you're given a new random variable called z. And z happens to be a function of x. In fact, it's equal to x squared. And then you're asked to compute that PMF. So this problem is a good practice problem. I think, at this point, you guys are sort of newly acquainted with the idea of a PMF, or probability mass function. So this problem will hopefully help you get more familiar with that concept and how to manipulate PMFs.

And by the way, just to make sure we're all on the same page, what does a PMF really tell you? So p sub X, where this is a capital X, because the convention in this class is to use capital letters for random variables. So p X of k, this is defined to be the probability that your random variable X takes on a value of k.

So essentially, this says-- and this is just some number. So in our particular case, this would be equal to k squared over a. And how you can interpret this is this px guy is sort of like a machine. He takes in some value that your random variable could take on, and then he spits out the amount of probability mass lying over that value. OK.

So now that we've done that quick recap, let's get back to the first part of the problem. So we have this formula for px of x, and we need to solve for a. So in order to do that, we're going to use one of our axioms of probability to set up an equation. And then we can solve precisely for a. So namely, we know that every PMF must sum to 1. And so essentially, if you sum this guy over all possible values of x, you should get a 1, and that equation will let us solve for a.

So let's do that. Summation over x of px of x. So here, essentially you're only summing over these six values. So this is equal to px of minus 3, plus px of minus 2, plus px of minus 1, et

cetera. Oops. px of 2 plus px of 3. OK. And again, like the interpretation as we said, this number here should be interpreted as the amount of probability mass lying over minus 3.

And to help you visualize this, actually, before we go further with the computation, let's actually plot this PMF. So the amount of probability mass lying over minus 3, the way we figure that out is we take minus 3 and we plug it into this formula up here. So you get 9/a. Now you can do this for minus 2. You've got 4/a, looking at the formula. For 1, you get 1/a. And of course, this graph, it's the mirror image over 0, because of the symmetry.

So hopefully this little visualization helps you understand what I'm talking about. And now we can just read these values off of the plot we just made. So we know px minus 3 is equal to px of 3. So we can go ahead and just take 2 times 9/a. Similarly, we get 2 times 4/a, and then plus 2 times 1/a. So now it's just a question of algebra.

So simplifying this, you're going to get 18 plus 8 plus 2, divided by a. And this gives you 28/a. And as I argued before, you know that if you sum a PMF over all possible values, you must get 1. So this is equal to 1, which of course implies that a is equal to 28.

So what we've shown here is that you actually don't have a choice for what value a can take on. It must take on 28. And in fact, if you plug in any other value than 28 in here, you actually are not going to have a valid PMF, because it's not going to sum to 1. OK. So I'm going to write my answer here, and then erase to give myself more room for part

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