# MASSACHUSETTS INSTITUTE OF TECHNOLOGY <br> 22.071/6.071 Introduction to Electronics, Signals and Measurement <br> Spring 2006 

Lab3. Fourier Transform and Fourier Series.
Experiment 1. (Listen to various signals)
Continuing with the material just presented in the class, lets investigate the frequency content of various signals by generating them and listening to them with the aid of our instrumentation.

We will use the Function generator of our instrument to generate and listen to three different signals characterized with the same "frequency."

Attach your speaker to your protoboard and connect it to the output of the function generator FUNC OUT. Connect the other lead of the speaker to ground.

Select a sine wave with a frequency of 1 kHz and listen to it carefully, Now switch to a triangular wave signal while maintaining the same frequency and listen to it again carefully.
Do the same for a square wave.
Which signal sounds more like the sine wave?
Triangle

Give a brief explanation for your answer.

The triangle wave has the least extraneous frequency components.
(any logical answer here is fine; this is just an intro question).

## Experiment 2.

Investigating the Fourier Series expansion of a square wave signal. We will perform a LabView based simulation for this experiment.

Download the programs called SquareWaveFS and SquareWaveFS_nterms from the materials section of the class web site.

Lets first start with the SquareWaveFS program.
Once started, the default screen of this program looks like


Run the instrument by clicking on the right going arrow on the upper left hand corner.
Construct a square wave varying between -1 and 1 and a frequency of 20 Hz .
On the left hand side of your virtual instrument you have the option of investigating the contributions of the various harmonics to the reconstruction of the square wave signal you just created.
Turn on the $1^{\text {st }}$ harmonic and observe the result. The plot of the harmonic appears on the left and its contribution to the signal reconstruction is shown by the blue trace on the right.
Continue in order with the other harmonics.
What is the frequency of the first 5 harmonics:

|  | $1^{\text {st }}$ harmonic | $2^{\text {nd }}$ harmonic | $3^{\text {rd }}$ harmonic | $4^{\text {th }}$ harmonic | $5^{\text {th }}$ harmonic |  |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: |
| Frequency | 20 Hz | 60 Hz | 100 Hz | 140 Hz | 180 Hz |  |
|  |  | Note; these are the 1st, 2nd, 3rd, 4th, and 5th NONZERO harmonics |  |  |  |  |  |

Consult the notes from today's lecture and extract the mathematical function corresponding to the signal for each harmonic.
Write the mathematical expression for each harmonic below:

| $1^{\text {st }}$ Harmonic | (4/pi) * $\sin \left(2^{*} \mathrm{pi}^{*} 20 \mathrm{t}\right.$ |
| :---: | :---: |
|  | (4/3pi) *sin(2*pi*60t) |
| $3^{\text {rd }}$ Harmonic | (4/5pi) * $\sin \left(2^{*} \mathrm{p}^{*} 100 \mathrm{t}\right)$ |
| $4^{\text {th }}$ Harmonic | (4/7pi) * $\sin \left(2^{*} \mathrm{p}^{*} * 140 \mathrm{t}\right)$ |
| $5^{\text {th }}$ Harmonic | (4/9pi) * $\sin \left(2^{*} \mathrm{p}^{*}{ }^{*} 80 \mathrm{t}\right)$ |

Plot the magnitude of the first five Fourier coefficients as a function of frequency in the space below:


From the information on the plot you just created and your observations from Experiment1 (listening to various signals) describe in more detail the reason why the sine wave and the square wave of the same frequency sounded so different.

[^0]Experiment 3.
This is another simulated experiment designed to further explore the Fourier Series expansion of a signal.

For this experiment we will use the program called SquareWaveFS_nterms that you downloaded earlier from the class web site. The interface with this virtual instrument looks like:


With this simulated experiment we will explore the error associated with the Fourier Series reconstruction of a square wave as a function of the number of terms in the series. Answer the following questions:

1. To what value does the Fourier series converges at the discontinuities of the square wave? (Give a general answer)

### 1.85 ; this is the Gibbs-constant

2. What happens to the observed ripple as you increase the number of terms in the expansion?
[^1]Experiment 4.
Next, let’s look at the Fourier Series expansion of a triangular wave signal and a sawtooth wave signal.

You may download the two LabView instruments called TriangularFS and SawtoothFS from the class web site.

Construct a triangular signal and a sawtooth signal with the same amplitude and frequency. Observe the Fourier Series reconstruction.

Plot the magnitude of the first 5 harmonics as a function of frequency for these two signals.



Compare these plot with the one you obtained for the square wave in Exercise 2.
What are the similarities and differences in these plots? Compare the spectral plot of the square wave and the sawtooth wave.

The triangle wave's harmonic amplitudes in the frequency domain scale with $1 / n^{\wedge} 2$ instead of $1 / n$ (contrasted to $1 / n$ for the square wave). The sawtooth wave has non-zero frequencies at every harmonic (contrasted to every odd harmonic for the square and triangle).


[^0]:    The sine wave only has one frequency component (fundamental frequency).
    The square wave with the same fundamental frequency has many other frequency components.

[^1]:    decreases

