6.071 Review Session
Quiz 1

3/1/2006

Signals:

RMS = Root mean squared = $\sqrt{\langle x^2(t) \rangle}$

Time domain representation $x(t) \overset{\mathcal{F}}{\longleftrightarrow}$ Frequency Domain $X(\omega)$

Frequency $f = \omega / 2\pi$

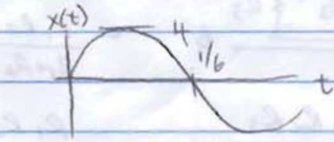
Period $T = 1/f$

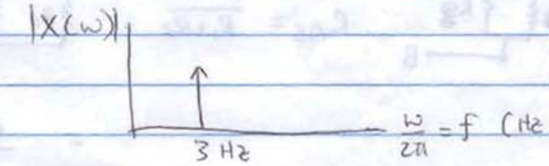
Amplitude $A = \frac{1}{2} \cdot$ (peak to peak value)

Signal Examples:

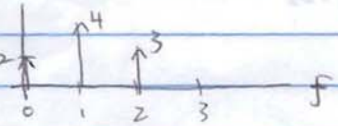
ex $x(t) = 4 \sin(6\pi t)$

$RMS = 4/\sqrt{2}$, $\omega = 6\pi$, $f = 3$ Hz, $A = 2+2 = 4$





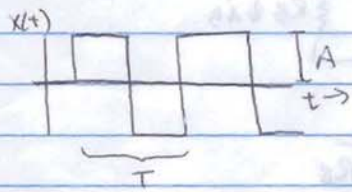
ex $|X(\omega)|$:

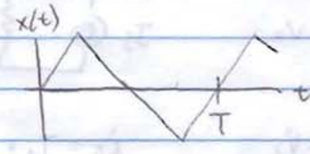


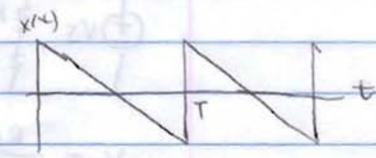
$x(t) = 2 + 4\cos(2\pi t) + 3\cos(4\pi t)$

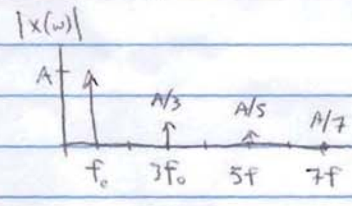
Nyquist rate = 2 × highest frequency
= 4 Hz

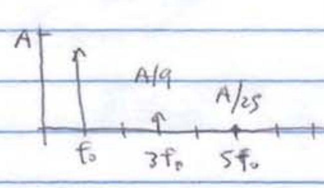
ex:

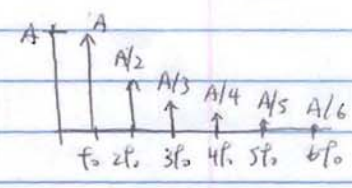




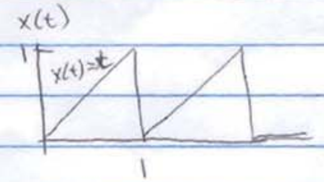








ex:



$$RMS = \sqrt{\text{mean}\{1 \text{ period of } x^2(t)\}}$$

$$= \sqrt{\frac{1}{1-0}\int_0^1 t^2\,dt}$$

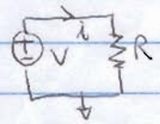
$$= \sqrt{1 \cdot \frac{1}{3}t^3 \Big]_0^1} = \frac{1}{\sqrt{3}}$$

Circuits:

$V=IR$ (Ohm's Law), parallel + series R's, voltage + current dividers, superposition

Ohm's Law, KVL, KCL, Mesh Method, Nodal Analysis, Thevenin, Norton, Power, ~~wheatstone bridge~~

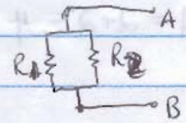
Ohm's Law: $V = IR$



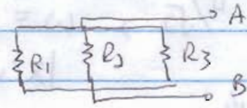
Parallel R's: $\frac{1}{R_{eq}} = \frac{1}{R_A} + \frac{1}{R_B}$

Series R's: $R_{eq} = R_A + R_B$

Parallel & Series:



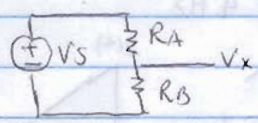
$R_{AB} = \frac{R_1 R_2}{R_1 + R_2}$

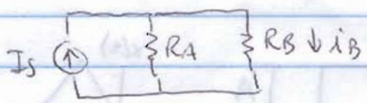


$R_{AB} = R_1 \parallel R_2 \parallel R_3 = \frac{R_1 R_2}{R_1+R_2} \parallel R_3$

$$= \frac{\frac{R_1 R_2 R_3}{R_1 + R_2}}{\frac{R_1 R_2}{R_1 + R_2} + R_3} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Voltage + Current Dividers:





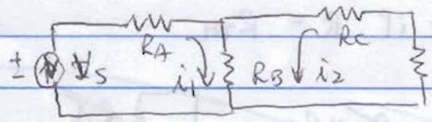
$V_x = \frac{R_B}{R_A + R_B} \cdot V_S$

$i_B = I_S \cdot \frac{R_A}{R_A + R_B}$

~~Mesh Method~~

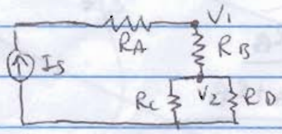
KVL: $\sum_{loop} V = 0$ KCL: $\sum_{node} i = 0$

Mesh Method:



$$V_S - i_1 R_A - (i_1 + i_2) R_B = 0$$
$$(i_1 + i_2) R_B + i_2 R_D + i_2 R_C = 0$$

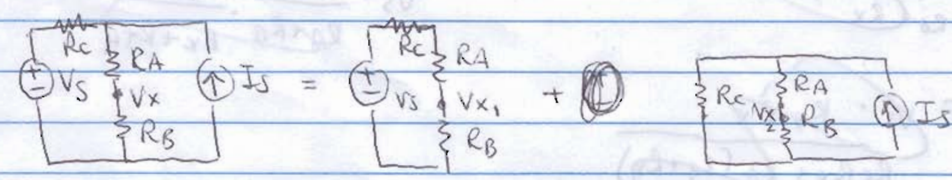
Nodal Analysis:



$$\left.\begin{aligned} \frac{V_1 - V_2}{R_B} - I_S &= 0 \\ \frac{V_2 - V_1}{R_B} + \frac{V_2}{R_C \| R_D} &= 0 \end{aligned}\right\} \quad \begin{aligned} V_1 - V_2 &= I_S R_B \\ V_1 (R_C \| R_D) + V_2 (R_B + R_C \| R_D) &= 0 \end{aligned}$$

$$\rightarrow \begin{bmatrix} 1 & -1 \\ -[R_C \| R_D] & R_B + R_C \| R_D \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} I_S R_B \\ 0 \end{bmatrix}$$

Superposition: Analyze Circuit w/ 1 independent source at a time.

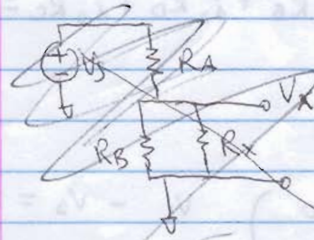


$$V_x = V_{x_1}' + V_{x_2}'$$

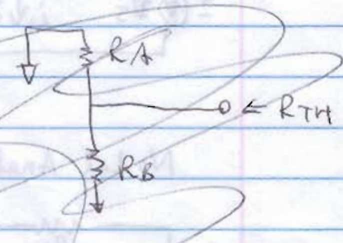
$$V_x = \frac{R_B}{R_A + R_B + R_C} V_S + \frac{R_C}{R_C + R_A + R_B} \cdot R_B \cdot I_S$$

Thevenin / Norton

- Take out the element between the measured nodes
- For R_{TH}, Short all Vs and open all Is
- $V_{oc} = I_N \cdot R_{TH}$
- Max power through a resistor if $R = R_{TH}$



$R_{TH}(R_x) = R_A \| R_B$



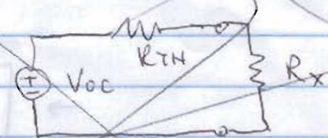
$V_{TH}(R_x) = V_S \cdot \frac{R_B}{R_A + R_B}$

$V_x = V_S \cdot \frac{R_B \| R_x}{R_B \| R_x + R_A}$

$= V_S \cdot \frac{\frac{R_B R_x}{R_B + R_x}}{\frac{R_B R_x}{R_B + R_x} + R_A}$

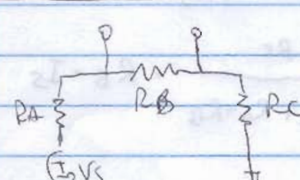
$= V_S \cdot \frac{R_B R_x}{R_B R_x + R_A (R_B + R_x)}$

$V_{oc} =$



$V_o = V_{oc} \cdot \frac{R_x}{R_x + R_{TH}}$

$= V_S \cdot \frac{R_B}{R_A + R_B} \cdot \frac{R_x}{R_x + R_{TH}}$



$V_{oc} = V_S$

$R_{TH} = R_A \| R_C \; R_A + R_C$

$V_o = \frac{R_B}{R_A + R_B + R_C}$