1) Ji Zhang 4/19/2006 6.071 Quiz 3 Review

I. RLC circuits
- transient behavior intuition
- be able to derive 1st or 2nd order differential equation
- solve differential equations
- Norton, Thevenin, KCL, KVL; stuff from 1st two quizzes

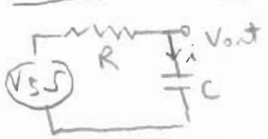
II. Diodes
- given a diode's forward voltage, reverse breakdown voltage, know its effect on an arbitrary circuit
- from its I-V characteristic derive its effect on any circuit

III. Transistors
- recognize regions of operation
- figure out bias for given quiescent conditions & vice versa
- sketch I/O

I: RLC Circuits:

* Always perform sanity check: $V(t=0)$ and $i(t=0)$, $V(t=\infty)$ and $i(t=\infty)$
- Energy is only dissapated through resistance; L and C just store energy
- Use your intuition; math errors are common!

1st order examples: ↙ WHAT DOES THIS CIRCUIT SERVE? (what purpose)



$V_s = u(t)$

$V_s - i_C R - V_C = 0$

$V_s - C\frac{dV_o}{dt}R - V_C = 0$

$\frac{1}{RC}V_s = \frac{dV_o(t)}{dt} + V_o(t)\cdot\frac{1}{RC}$ ← 1st order diff. eq.

($\frac{1}{RC}V_s$: response; $\frac{dV_o(t)}{dt} + V_o(t)\cdot\frac{1}{RC}$: system)

to find $V_o(t)$:
1. solve for $V_{oH}(t)$ (homogeneous)
2. guess $V_{oP}(t)$
3. fit to initial conditions

2]

① Homogeneous solution: set response to 0

$\rightarrow \frac{dV_o(t)}{dt} = -\frac{1}{RC}V_o(t)$

$\rightarrow \int \frac{dV}{V} = \int -\frac{1}{RC}dt$

$\rightarrow \ln V = -\frac{1}{RC}t + C$

$\rightarrow V_H = e^{-\frac{1}{RC}t + C} = K \cdot e^{-t/RC}$

② Guess particular $V_P = V_P(\infty) = V_S$

③ $V_o(t) = V_{oH} + V_{op} = Ke^{-t/RC} + V_S$

init conditions $V_o(t=0) = 0$

$\rightarrow K = -V_S$

$\rightarrow V_o(t) = V_S(1 - e^{-t/RC})$

* Do sanity check now!

E_{stored} in capacitor at $t = \infty$?

$E = \int_{-\infty}^{\infty} I \cdot V \cdot dt$

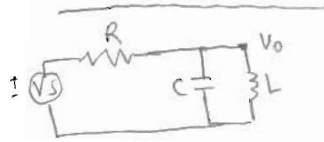
$i_c(t) = C\frac{dV}{dt} = C\frac{V_S}{RC}e^{-t/\tau} = \frac{V_S}{R}e^{-t/RC}$

$E = \int_0^\infty \frac{V_S^2}{R}(e^{-t/RC} - e^{-2t/RC})dt$

$= \frac{V_S^2}{R}\left[-RCe^{-t/RC} + \frac{1}{2}RCe^{-2t/RC}\right]_0^\infty = \frac{1}{2}CV_S^2$

* what about for other types of inputs? delta, ramp?

2nd order system



KCL: $\frac{V_o - V_S}{R} + C\frac{dV_o}{dt} + \int \frac{V_o}{L}dt = 0$

$\rightarrow \frac{1}{R}\frac{dV_o}{dt} + C\frac{d^2V_o}{dt^2} + \frac{V_o}{L} = 0$

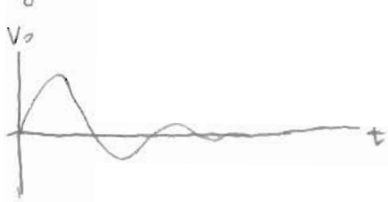
$\rightarrow \frac{d^2V_o}{dt} + \frac{1}{RC}\frac{dV_o}{dt} + \frac{V_o}{LC} = 0$

$\rightarrow x^2 + bx + c = 0, \quad b = \frac{1}{RC}, \quad c = \frac{1}{LC}$

$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2}$

underdamped? check if $\sqrt{b^2 - 4ac}$ = imaginary

graph underdamped case:

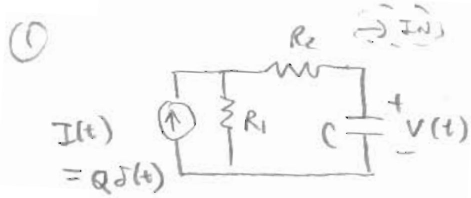


$V_o(t=0) = 0$

$V_o(t=\infty) = 0$

3)

(1)



1) Derive diff. eq that relates $V(t)$ to $I(t)$
hint: try Thevenin/Norton

→ 

$R_N = R_1 + R_2$

$$I_N = \frac{R_1}{R_1+R_2} I(t)$$

by KCL:

$$\frac{1}{R_N} V(t) + C\frac{dV(t)}{dt} - I_N(t) = 0$$

$$R_N C \frac{dV}{dt} + V(t) = I_N(t) R_N$$

b) find $V(t)$ for ~~before~~ $t=0^+$ & $t \to \infty$

$$\rightarrow \frac{dV}{dt} + \frac{1}{R_N C} V(t) = \frac{1}{C} I_N(t) \qquad V(0^-) = 0$$

$$\rightarrow \int_{0^-}^{0^+} dV + \frac{1}{R_N C}\int_{0^-}^{0^+} V(t)\,dt = \frac{1}{C}\int_{0^-}^{0^+} I_N(t)\,dt$$

$$\rightarrow V(t)\Big|_{0^-}^{0^+} + 0 = \frac{1}{C} \cdot \frac{R_1}{R_1+R_2} Q$$

$$\rightarrow V(0^+) = \frac{1}{C} \cdot \frac{R_1}{R_1+R_2} Q$$

$\int_{0^-}^{0^+} V(t)\,dt = 0$ since $V(t)$ is finite & integral over 0 time $= 0$. we know $V(t)$ finite since $I(t)$ delivers a finite amt of charge.

$$\boxed{V(\infty) = 0}$$

c) $\tau = ?$ $\quad \tau = R_N \cdot C$

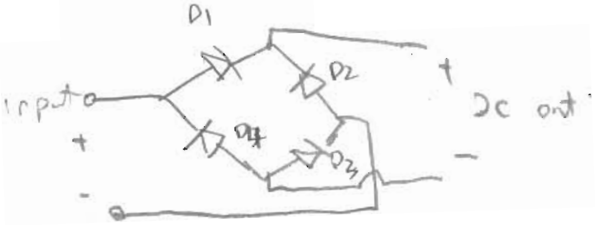
d) $v(t)$ for $t \geq 0$ w/o solving eq:

$$\boxed{v(t) = \frac{R_1}{R_1+R_2} \cdot \frac{Q}{C} \cdot e^{-t/\tau}}$$

Diode bridge:

Full wave rectifier.

$V_{Forward} = .7$ unless specified

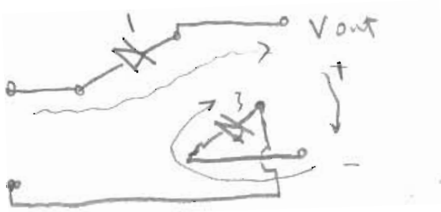


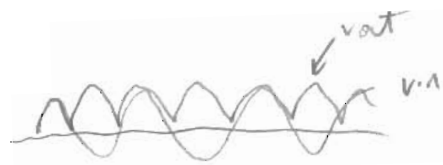
in:

Vout = ??

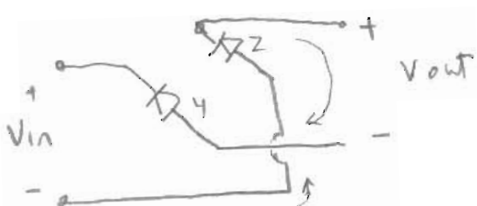
$V_{Forward} = 0$

$v_{in} > 0$:

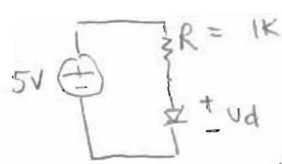


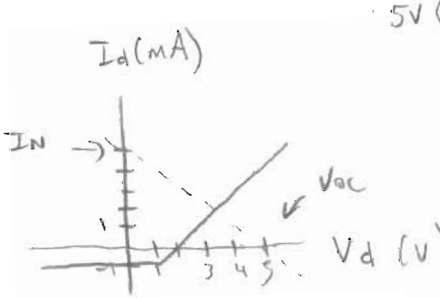


$v_{in} < 0$



Load Lines:

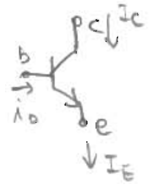




Zeners

What does the zener do?

4) Transistors:

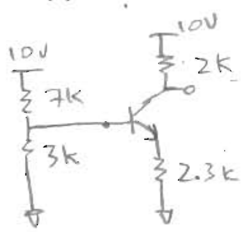


$V_{be} = .7$ in FAR

$I_c = \beta i_b$

$\beta \approx 200 \rightarrow I_c = I_E$, and we often disregard i_b

ex: $V_o = ?$



$V_b = 3V$

$V_e = 2.3V$

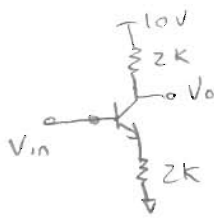
$I_e = 1mA$

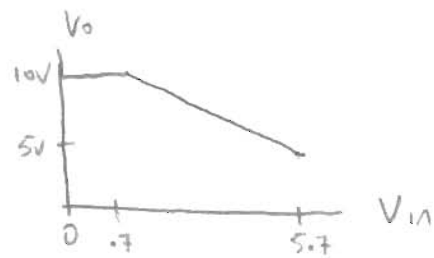
$V_o = 10V - I_c \cdot 2k$

$= 10V - I_E \cdot 2k$

$= 8V$

ex: Draw V_o for $0 < V_{in} < 5.7V$





$V_e = V_{in} - .7$

$I_E = \frac{V_{in} - .7}{2k}$

$V_{OUT} = 10V - 2k \cdot I_E$

$= 10V - V_{IN} + .7$

$= 10.7 - V_{in}$ for $V_{IN} \geq .7V$ (turn on BJT)

ex: Load Lines