#### 6.096 - Algorithms for Computational Biology - Lecture 9

#### 6.096 Lecture 10

#### Evolution

Gene correspondence - Rearrangements - Genome duplication



#### **Evolutionary change**

- Lecture 1 Introduction Lecture 2 - Hashing / BLAST Lecture 3 - Combinatorial Motif Finding Lecture 4 - Statistical Motif Finding Lecture 5 - Sequence alignment and Dynamic Programming Lecture 6 - RNA structure and Context Free Grammars Lecture 7 - Gene finding and Hidden Markov Models Lecture 8 - HMMs algorithms and Dynamic Programming Lecture 9 - Evolutionary change, phylogenetic trees
- Lecture 10 Genome rearrangements, genome duplication



#### **Overview**

#### Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements



#### Inferring orthologous gene relationships

- · BBH Best bi-directional hits
- COG Clusters of orthologous genes
- BUS Best unambiguous subgraphs











S.cerevisiae Chromosome VI 250-300kbp

#### Overview

Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements





# Specific mechanisms mediate rearrangements 10 translocations 8 across Ty elements 2 across neadwidentical genes 2 across neadwidentical genes 19 inversions All flanked by tRNA genes Multiple to the transformation of the transformation o





Transposons selectively kept in specific loci







Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements

#### Gene order rearrangement: overlapping inversions



$$1 2 3 4 5 6 7 8 9 10$$

$$1 2 -6 -5 -4 -3 7 8 9 10$$

$$1 2 -6 -8 -7 3 4 5 9 10$$

$$1 2 -6 -8 -7 3 4 5 9 10$$

$$1 2 -6 -8 -7 3 4 -10 -9 -5$$

Reversals: Example  

$$\pi = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$$
  
 $r(3,5)$   
 $1 \ 2 \ 5 \ 4 \ 3 \ 6 \ 7 \ 8$ 

Reversals: Example  

$$\pi = 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8$$
  
 $r(3,5)$ 
 $1 \ 2 \ 5 \ 4 \ 3 \ 6 \ 7 \ 8$   
 $r(5,6)$ 
 $1 \ 2 \ 5 \ 4 \ 6 \ 3 \ 7 \ 8$ 

#### **Reversals and Gene Orders**

Gene order is represented by a permutation pr

$$p = p_{1} \dots p_{i+1} \frac{p_{i} p_{i+1} \dots p_{j-1} p_{j}}{r(i,j)} p_{j+1} \dots p_{n}$$

 $p_1 \dots p_{i-1} p_j p_{j-1} \dots p_{i+1} p_i p_{j+1} \dots p_n$ 

• Reversal **r** (*i*, *j*) reverses (flips) the elements from *i* to *j* in **p** 

#### **Reversal Distance Problem**

- <u>Goal</u>: Given two permutations, find the shortest series of reversals that transforms one into another
- Input: Permutations p and s
- Output: A series of reversals  $r_1, \dots r_t$  transforming **p** into **s**, such that *t* is minimum
- *t* reversal distance between *p* and *s*
- $d(p, \sigma)$  smallest possible value of t, given p and  $\sigma$

#### Sorting By Reversals Problem

- Goal: Given a permutation, find a shortest series of reversals that transforms it into the identity permutation (1 2 ... n)
- Input: Permutation p
- Output: A series of reversals  $r_1, \ldots r_t$  transforming **p** into the identity permutation such that *t* is minimum

### Sorting By Reversals: Example • $t = d(\mathbf{p})$ - reversal distance of $\mathbf{p}$ • Example : $\mathbf{p} = \frac{3}{4} \stackrel{2}{2} \stackrel{1}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{10}{10} \stackrel{9}{8} \stackrel{4}{4} \stackrel{3}{3} \stackrel{2}{2} \stackrel{1}{1} \stackrel{5}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{2}{2} \stackrel{3}{3} \stackrel{4}{4} \stackrel{5}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{2}{2} \stackrel{3}{3} \stackrel{4}{4} \stackrel{5}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{2}{2} \stackrel{3}{3} \stackrel{4}{4} \stackrel{5}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{2}{2} \stackrel{3}{3} \stackrel{4}{4} \stackrel{5}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{2}{2} \stackrel{3}{3} \stackrel{4}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{1}{2} \stackrel{1}{3} \stackrel{1}{5} \stackrel{6}{6} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{1}{2} \stackrel{1}{3} \stackrel{1}{5} \stackrel{6}{5} \stackrel{7}{7} \stackrel{8}{8} \stackrel{9}{10} \stackrel{1}{1} \stackrel{1}{2} \stackrel{1}{3} \stackrel{1}{5} \stackrel{1}{5}$

#### Sorting by reversals: 5 steps

Step 0: p	2	-4	-3	5	-8	-7	-6	1
Step 1:	2	3	4	5	-8	-7	-6	1
Step 2:	2	3	4	5	6	7	8	1
Step 3:	2	3	4	5	6	7	8	-1
Step 4:	-8	-7	-6	-5	-4	-3	-2	-1
Step 5: g	1	2	3	4	5	6	7	8

#### Sorting by reversals: 4 steps

Step 0: p	2	-4	-3	5	-8	-7	-6	1
Step 1:	2	3	4	5	-8	-7	-6	1
Step 2:	-5	-4	-3	-2	-8	-7	-6	1
Step 3:	-5	-4	-3	-2	-1	6	7	8
Step 4: g	1	2	3	4	5	6	7	8

#### Sorting by reversals: 4 steps

Step 0: p	2	-4	-3	5	-8	-7	-6	1
Step 1:	2	3	4	5	-8	-7	-6	1
Step 2:	-5	-4	-3	-2	-8	-7	-6	1
Step 3:	-5	-4	-3	-2	-1	6	7	8
Step 4: g	1	2	3	4	5	6	7	8

What is the reversal distance for this permutation? Can it be sorted in 3 steps?

#### **Pancake Flipping Problem**

- The chef is sloppy; he prepares an unordered stack of pancakes of different sizes
- The waiter wants to rearrange them (so that the smallest winds up on top, and so on, down to the largest at the bottom)
- He does it by flipping over several from the top, repeating this as many times as necessary



Christos Papadimitrou and Bill Gates flip pancakes

#### **Pancake Flipping Problem: Formulation**

- <u>Goal</u>: Given a stack of *n* pancakes, what is the minimum number of flips to rearrange them into perfect stack?
- Input: Permutation p
- <u>Output</u>: A series of prefix reversals  $r_p \dots r_t$ transforming **p** into the identity permutation such that *t* is minimum

#### Pancake Flipping Problem: Greedy Algorithm

 Greedy approach: 2 prefix reversals at most to place a pancake in its right position, 2n-2 steps total

at most

• William Gates and Christos Papadimitriou showed in the mid-1970s that this problem can be solved by at most 5/3 (n + 1) prefix reversals

#### Sorting By Reversals: A Greedy Algorithm

- If sorting permutation p = 1 2 36 4 5, the first three elements are already in order so it does not make any sense to break them.
- The length of the already sorted prefix of *p* is denoted *prefix(p)*

- prefix(p) = 3

• This results in an idea for a greedy algorithm: increase *prefix(p)* at every step

#### Greedy Algorithm: An Example

• Doing so, p can be sorted

• Number of steps to sort permutation of length n is at most (n-1)

#### Greedy Algorithm: Pseudocode

#### SimpleReversalSort(p)

- 1 for  $i \leftarrow 1$  to n 1
- 2 j  $\leftarrow$  position of element i in p (i.e.,  $p_j = i$ )
- 3 if j?i
- 4  $p \leftarrow p * r(i, j)$
- 5 output p
- 6 if **p** is the identity permutation
- 7 return

#### Analyzing SimpleReversalSort

- SimpleReversalSort does not guarantee the smallest number of reversals and takes five steps on p = 6 1 2 3 4 5 :
  - Step 1: 1 6 2 3 4 5
  - Step 2: 1 2 6 3 4 5
  - Step 3: 1 2 3 6 4 5
  - Step 4: 1 2 3 4 6 5
  - Step 5: 1 2 3 4 5 6

#### Analyzing SimpleReversalSort (cont'd)

- But it can be sorted in two steps:
  - *p* = 6 1 2 3 4 5 - Step 1: 5 4 3 2 1 6
  - Step 2: 1 2 3 4 5 6
- So, SimpleReversalSort(p) is not optimal
- Optimal algorithms are unknown for many problems; approximation algorithms are used

#### **Approximation Algorithms**

- These algorithms find approximate solutions rather than optimal solutions
- The approximation ratio of an algorithm A on input p is:

A(**p**) / OPT(**p**)

where

A(p) -solution produced by algorithm A OPT(p) - optimal solution of the problem

#### **Approximation Ratio/Performance Guarantee**

- Approximation ratio (performance guarantee) of algorithm A: max approximation ratio of all inputs of size n
  - For algorithm A that minimizes objective function (minimization algorithm):

$$\cdot \max_{|p| = n} A(p) / OPT(p)$$

#### **Approximation Ratio/Performance Guarantee**

- Approximation ratio (performance guarantee) of algorithm A: max approximation ratio of all inputs of size n
  - For algorithm A that minimizes objective function (minimization algorithm):
     max<sub>|p| = n</sub> A(p) / OPT(p)
  - For maximization algorithm:
    - $\cdot \min_{|p| = n} A(p) / OPT(p)$

#### **Adjacencies and Breakpoints**

 $\boldsymbol{p} = \boldsymbol{p}_1 \boldsymbol{p}_2 \boldsymbol{p}_3 \dots \boldsymbol{p}_{n-1} \boldsymbol{p}_n$ 

- A pair of elements  $p_i$  and  $p_{i+1}$  are adjacent if
  - $p_{i+1} = p_i \pm 1$
- For example:
  - **p**=193478265
- (3, 4) or (7, 8) and (6,5) are adjacent pairs

**Breakpoints: An Example** There is a breakpoint between any pair of nonadjacent elements: *p*=193478265 • Pairs (1,9), (9,3), (4,7), (8,2) and (2,5) form breakpoints of permutation p

b(p) - # breakpoints in permutation p

Extending Permutations	Reversal Distance and Breakpoints	
<ul> <li>We put two elements p<sub>0</sub>=0 and p<sub>n+1</sub>=n+1 at the ends of p</li> <li>Example:</li> </ul>	<ul> <li>Each reversal eliminates at most 2 breakpoints.</li> </ul>	
$\pi = 1 \begin{vmatrix} 9 & 3 & 4 & 7 & 8 & 2 & 6 & 5 \\ Extending with 0 and 10 \\ \pi = 0 & 1 & 9 & 3 & 4 & 7 & 8 & 2 & 6 & 5 & 10 \\ \end{bmatrix}$ Note: A new breakpoint was created after extending	$p = 2 \ 3 \ 1 \ 4 \ 6 \ 5 \ 7 \qquad b(p) = 5$ $0 \ 2 \ 3 \ 1 \ 4 \ 6 \ 5 \ 7 \qquad b(p) = 4$ $0 \ 1 \ 2 \ 3 \ 4 \ 6 \ 5 \ 7 \qquad b(p) = 2$ $0 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \qquad b(p) = 0$	

#### **Reversal Distance and Breakpoints**

Each reversal eliminates at most 2 breakpoints.

This implies:

```
reversal distance = #breakpoints / 2
```

```
p = 231465
```

0	2	3	1	4	6	5	7	$b(\mathbf{p}) = 5$
0	1	<u>3</u>	2	4	6	5	7	$b(\mathbf{p}) = 4$
o	1	2	3	4	<u>6</u>	5	7	$b(\mathbf{p}) = 2$
0	1	2	3	4	5	6	7	$b(\boldsymbol{p})=0$

#### Sorting By Reversals: A Better Greedy Algorithm

BreakPointReversalSort(p)

- 1 while  $b(\mathbf{p}) > 0$
- 2 Among all possible reversals, choose reversal r minimizing  $b(p \cdot r)$
- 3  $p \leftarrow p \cdot r(i, j)$
- 4 output p
- 5 return

#### Sorting By Reversals: A Better Greedy Algorithm

#### BreakPointReversalSort(p)

- 1 while  $b(\mathbf{p}) > 0$
- 2 Among all possible reversals, choose reversal r minimizing  $b(p \cdot r)$
- 3  $p \leftarrow p \cdot r(i, j)$
- 4 output p
- 5 return

#### **Strips**

- Strip: an interval between two consecutive breakpoints in a permutation
  - Decreasing strip: strip of elements in decreasing order (e.g. 6 5 and 3 2 ).
  - <u>Increasing strip</u>: *strip* of elements in increasing order (e.g. 7 8)

#### 019437825610

A single-element strip can be declared either increasing or decreasing. We will choose to declare them as decreasing with exception of the strips with 0 and n+1

Problem: this algorithm may work forever

#### **Reducing the Number of Breakpoints**

#### Theorem 1:

If permutation p contains at least one decreasing strip, then there exists a reversal r which decreases the number of breakpoints (i.e.  $b(p \cdot r) < b(p)$ )

#### **Things To Consider**

#### • For p = 146578320146578329 $b(\mathbf{p}) = 5$

- Choose decreasing strip with the smallest element k in p(k = 2 in this case)

#### Things To Consider (cont'd)

#### • For p = 146578320146578329 b(p) = 5

- Choose decreasing strip with the smallest element k in p(k = 2 in this case)

#### Things To Consider (cont'd)

• For p = 146578320 1 4 6 5 7 8 3 2 9

- $b(\mathbf{p}) = 5$
- Choose decreasing strip with the smallest element k in p(k = 2 in this case)
- Find k 1 in the permutation

#### Things To Consider (cont'd)

- For p = 146578320 1 4 6 5 7 8 3 2 9 b(p) = 5- Choose decreasing strip with the smallest element k in p (k = 2 in this case)
  - Find k 1 in the permutation
  - Reverse the segment between k and k-1:

## b(p) = 5 $- 0 1 2 3 8 7 5 6 4 9 \qquad b(p) = 4$

#### Reducing the Number of Breakpoints Again

- If there is no decreasing strip, there may be no reversal r that reduces the number of breakpoints (i.e.  $b(p \bullet r) = b(p)$  for any reversal r).
- By reversing an increasing strip ( # of breakpoints stay unchanged ), we will create a decreasing strip at the next step. Then the number of breakpoints will be reduced in the next step (theorem 1).

#### Things To Consider (cont'd)

• There are no decreasing strips in p, for:

$$p = 0 \ 1 \ 2 \ 5 \ 6 \ 7 \ 3 \ 4 \ 8 \ b(p) = 3$$
$$p \bullet r(6,7) = 0 \ 1 \ 2 \ 5 \ 6 \ 7 \ 4 \ 3 \ 8 \ b(\pi) = 3$$

- $\checkmark$  r(6,7) does not change the # of breakpoints
- ✓ r(6,7) creates a decreasing strip thus guaranteeing that the next step will decrease the # of breakpoints.



#### ImprovedBreakpointReversalSort: Performance Guarantee

- *ImprovedBreakPointReversalSort* is an approximation algorithm with a performance guarantee of at most 4
  - It eliminates at least one breakpoint in every two steps; at most 2b(p) steps
  - Approximation ratio: 2b(p) / d(p)
  - Optimal algorithm eliminates at most 2 breakpoints in every step:  $d(p) \stackrel{a}{\rightarrow} b(p) / 2$
  - Performance guarantee:
    - $(2b(p) / d(p)) \approx [2b(p) / (b(p) / 2)] = 4$

#### **Signed Permutations**

- Up to this point, all permutations to sort were unsigned
- But genes have directions... so we should consider signed permutations



#### **GRIMM Web Server**

- Real genome architectures are represented by signed permutations
- Efficient algorithms to sort signed permutations have been developed
- GRIMM web server computes the reversal distances between signed permutations: http://nbcr.sdsc.edu/GRIMM/grimm.cgi

#### **GRIMM Web Server**



http://www-cse.ucsd.edu/groups/bioinformatics/GRIMM Courtesy of Glenn Tesler. Used with permission.

#### Sorting by reversals

Bader DA, Morei BM, Yan M. (2001) A linear-time algorithm for computing inversion distance between signed permutations with an experimental study. J Comput Biol 8:483-91.

Bergeron A. (in press) A very elementary presentation of the Hannenhalli-Pevzner theory. Discrete Applied Mathematics.

Hannenhalli, S. (1996). Polynomial-time algorithm for computing transforation distance between genomes. Discrete Applied Mathematics, 71:137–151.

Hannenhalli, S. and Pevzner, P. (1995). Transforming men into mice (polynomial algorithm for genomic distance problem). In Proceedings of the IEEE 36th Annual Symposium on Foundations of Computer Science, pages 581–592.

Hannenhalli, S. and Pevzner, P. A. (1999). Transforming cabbage into turnip (polynomial algorithm for sorting signed permutations by reversals). *Journal of the ACM*, 48:1–27.

Tesler G (2002) GRIMM: genome rearrangements web server. Bioinformatics, 18:492-3.

#### Overview

Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements





#### Whole genome duplication is controversial

Insufficient evidence

- Only 50% of genome in duplicate regions
- Only 8% of genes present in two copies
- Extensive redundancy outside duplicate regions

#### Evidence against WGD

- Divergence-based dating show multiple times
- Other species have similar level of redundancy
- Alternative evolutionary scenario proposed
  - Independent segmental duplications



	<ul> <li>"There was a whole-genome duplication." Wolfe, Nature '97</li> </ul>
5	<ul> <li>"There was no whole-genome duplication." Dujon, FEBS 2000</li> </ul>
	<ul> <li>"At least some chrom dup.</li> </ul>
	occurred independently" Langkjaer, JMB, 2000
	<ul> <li>"Dynamic equilibrium of duplications and loss" Llorente, FEBS, 2000</li> </ul>
	<ul> <li>"Recent evidence supports single event". Wong, PNAS '02</li> </ul>
	<ul> <li>"Continuous block duplications and deletions" Dujon, Yeast 2003</li> </ul>
	<ul> <li>"Dup. precedes divergence from Kluyveromyces." Piskur, Nature, 2003</li> </ul>
	<ul> <li>"Telomere-mediated duplication</li> </ul>

events" Coissac, Mol Bio Evo 1997 "Multiple closely spaced events" Friedman, Genome Res, 2003
 "Spontaneous duplication of large chromosomal segments" Koszul, EMBO '04











#### **Doubly Conserved Synteny Blocks (DCS)**

- 253 DCS blocks were identified containing 75% of K. waltii genes and 81% of S. cerevisae genes
- A typical DCS block has 27 genes (largest block has 81 genes).
- DCS blocks are separated by ~3 genes on the average.
- In a DCS block 90% of Kw genes have a match in at least 1 of the 2 Sc regions.
- 47 blocks have no duplicated gene.



Duplicate mapping tiles S. cerevisiae Orl ..... Or 1 1 million and the Or 4 0.3 cerevisiae Or OF 6 1888 18 118 cerevisiae Chr 3 047 Che 4 Over a second second Or 9 ...... Ś Or 7 Ś Or 10 . ..... Oc 8 Cir 11 .... Cir-11 the La e ti 50.04 Der 15 OV 16 145 blocks cover 88% of genome

Whole-genome duplication resolved



#### Overview

Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication **Duplicate gene evolution** Duplication and rearrangements

#### Accelerated gene divergence

- Ohno hypothesized that after duplication, one copy would preserve the original function, and the other copy would be free to diverge. Others argued that both copies would diverge.
- 76 of 457 duplicated gene pairs show accelerated evolution. In 95% of the cases, acceleration was limited to one of the 2 paralogs.
- Deletion of the ancestral paralog is lethal in 18% of the cases.
- Deletion of a derived paralog is never lethal.







	Ancestral function	Derived function
Gene deletion	Lethal (20%)	Never lethal

Gain new function and lose ancestral function

	Appartual function	Derived function
	Ancestrai function	Derived function
Gene deletion	Lethal (20%)	Never lethal
		Specific
Expression	Abundant	(Siless, Starvation)
Leadingtion	Concernal	Specific (mitochondrion_spores)
Localization	General	(mildenonanon, spores)

**Distinct functional properties** 

Gain new function and lose ancestral function





#### **Overview**

Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution **Duplication and rearrangements** 

#### Genome duplication in a vertebrate







#### How did the pre-duplicated ancestor look like?

- Can we derive the architecture of the current (human and tetraodon genomes) genomes in terms of the common ancestor?
- What was the sequence of rearrangement events after WGD?

#### **Genome rearrangements**













reversal distance = number of elements + 1 – number of cycles









#### Constructing Breakpoint Graph: Black Path

#### Constructing Breakpoint Graph: Superimposing Two Paths





#### Human-mouse breakpoint graph



#### **Constructing Rearrangement Scenarios**



#### **Reconstructing pre-duplicated Genome**

- WGD of genome *R* results in perfect duplicated genome *R*+*R*
- R+R becomes subject to rearrangements that shuffle genes in R+R and result in some rearranged duplicated genome P
- Problem: reconstruct pre-duplicated genome *R* from rearranged duplicated genome *P*.

#### **Genome Halving Problem**

- WGD of genome *R* results in perfect duplicated genome *R*+*R*
- R+R becomes subject to rearrangements that shuffle genes in R+R and result in some rearranged duplicated genome P
- Problem: reconstruct pre-duplicated genome *R* from rearranged duplicated genome *P*.
- Genome Halving Problem: Given a duplicated genome *P*, recover the ancestral pre-duplicated genome *R* minimizing the reversal distance from *R*+*R* to *P*











???	HP-theory: reminder
	Transforming signed gene order
	+a +b -c
	into unsigned gene order
+a +c +d +e +b	atah bibh chct
<u>+c +d +e +b +a +c +d +e</u> +b -e -d -c -a -b -e -d -c +b	<ul> <li>Elements x<sup>t</sup> and x<sup>h</sup> are called obverse pair</li> <li>t stands for tail and h stands for head</li> </ul>
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Breakpoint graph is formed by 3 matchings:
+d $+e$ $+d$ $+e$ $-c$ $-c$ $+b$ $+a$ $+b$	obverse matching
+d +e $-d$ +e $-c$ $-c$ +b +a +b	black matching (adjacent elements in 1st permutation)
-d +e -d +e -c -c +b +a +b	gray matching (adjacent elements in 2 <sup>nd</sup> permutation)

#### **HP-theory: reminder**

- · Breakpoint graph is formed by obverse, black and gray matchings.
- Every pair of matching forms a collection of alternating cycles:

black-gray cycles (#cycles in HP theory) single black-obverse cycle (1<sup>st</sup> permutation) single gray-obverse cycle (2<sup>nd</sup> permutation)

reversal distance between two circular permutations =

#elements - #black-gray cycles

#### Reversal distance between duplicated genomes

- While there exist fast algorithms for computing reversal distance between permutations (i.e., no duplicate genes), the problem of computing reversal distance between genomes with duplicated genese remains unsolved.
- Solution: label different copies of each gene (k! different labelings for a gene with k copies)
- One of these labelings is unavoidably an

#### optimal labeling

corresponding to the optimum rearrangement scenario

 Running time: (k!)<sup>n</sup>invocations of HP algorithms for a genome with n genes each present in k copies.

#### Labelings and breakpoint graphs

- Every labeling transforms genomes with duplicated genes into genomes without duplicated genes and enables applications of HP algorithm.
- Every labeling corresponds to a breakpoint graph
- Good labelings correspond to breakpoint graphs with large number of cycles.
- Can we construct a labeling corresponding to a large number of cycles?

#### Rearrangements in Duplicated Genomes: Challenges.

 Computing d(P, Q). Can we construct a labeling of duplicated genomes P and Q maximizing the number of cycles? NO

#### Rearrangements in Duplicated Genomes: Challenges.

- Computing d(P,Q). Can we construct a labeling of duplicated genomes P and Q maximizing the number of cycles? NO
- Computing d(P,R+R). Can we construct a labeling of duplicated genomes P and R+R maximizing the number of cycles? NO

#### Rearrangements in Duplicated Genomes: Challenges.

- Computing d(P,Q). Can we construct a labeling of duplicated genomes P and Q maximizing the number of cycles? NO
- Computing d(P,R+R). Can we construct a labeling of duplicated genomes P and R+R maximizing the number of cycles? NO
- Computing min<sub>R</sub> d(P,R+R).

#### Rearrangements in Duplicated Genomes: Challenges.

- Computing d(P,Q). Can we construct a labeling of duplicated genomes P and Q maximizing the number of cycles? NO
- Computing *d*(*P*,*R*+*R*). Can we construct a labeling of duplicated genomes *P* and *R*+*R* maximizing the number of cycles? NO
- Computing min<sub>R</sub> d(P,R+R). YES!

#### Rearrangements in Duplicated Genomes: Challenges.

- Breakpoint graphs are not defined for duplicated genomes.
- Can we generalize the notion of breakpoint graph for the case of duplicated genomes?
- Idea: Explore the connection between de Bruijn graphs and breakpoint graphs.

#### **De Bruijn Graphs**

• De Bruijn graph: Given a set of edgelabeled graphs, de Bruijn graph of this set is the result of "gluing" edges with the same label in all graphs in the set.

#### **De Bruijn Graphs**

- De Bruijn graph: Given a set of edgelabeled graphs, de Bruijn graph of this set is the result of "gluing" edges with the same label in all graphs in the set.
- Did we see de Bruijn graphs today?





#### De Bruijn Graphs

- De Bruijn graph: Given a set of edgelabeled graphs, de Bruijn graph of this set is the result of "gluing" edges with the same label in all graphs in the set.
- Did we see de Bruijn graphs today?

#### **De Bruijn Graphs**

- De Bruijn graph: Given a set of edgelabeled graphs, de Bruijn graph of this set is the result of "gluing" edges with the same label in all graphs in the set.
- Did we see de Bruijn graphs today?
- Breakpoint graph of permutations P and Q

de Bruijn graph of P-cycle and Q-cycle

#### **De Bruijn Graphs**

• Breakpoint graph of permutations *P* and Q

de Bruijn graph of *P*-cycle and *Q*-cycle

• Breakpoint graph of any genomes *P* and *Q* (with multiple gene copies)

#### **De Bruijn Graphs**

- Breakpoint graph of permutations *P* and *Q* ==
   de Bruijn graph of *P*-cycle and *Q*-cycle
- Breakpoint graph of any genomes *P* and *Q* (with multiple gene copies)

#### de Bruijn graph of P-cycle and Q-cycle

#### **Overview**

Genome correspondence Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements



#### Greedy Algorithms And Genome Rearrangements

#### Outline

- Transforming Cabbage into Turnip
- Genome Rearrangements
- Sorting By Reversals
- Pancake Flipping Problem
- Greedy Algorithm for Sorting by Reversals
- Approximation Algorithms
- Breakpoints: a Different Face of Greed

#### **Outline CHANGE**

- Genome Rearrangements give picture of splotch mouse
- Although Calbages and turnips share a recent common ancestor, they look and taste different





Turnip vs Cabbage: Almost Identical mtDNA gene sequences

- In 1980s Jeffrey Palmer studied evolution of plant organelles by comparing mitochondrial genomes of the cabbage and turnip
- 99% similarity between genes
- These surprisingly identical gene sequences differed in gene order
- This study helped pave the way to analyzing genome rearrangements in molecular evolution























Waardenburg's Syndrome: Mouse Provides Insight into Human Genetic Disorder

- Waardenburg's syndrome is characterized by pigmentary dysphasia
  Gene implicated in the disease was linked to human chromosome 2
- but it was not clear where exactly it is located on chromosome 2



#### Waardenburg's syndrome and splotch mice

- A breed of mice (with splotch gene) had similar symptoms caused by the same type of gene as in humans
- Scientists succeeded in identifying location of gene responsible for disorder in mice
- Finding the gene in mice gives clues to where the same gene is located in humans







#### Comparative Genomic Architecture of Human and Mouse Genomes