

## Evolution

Gene correspondence - Rearrangements - Genome duplication


Framework: graph of gene correspondence


- Weighted bipartite graph
- Graph represents gene correspondence
- Nodes: genes (w/ coordinates)
- Edges: sequence similarity (w/ weights)
- Two types of evolutionary relationships
- Orthologs (1-to-1 matches)
- Paralogs (1-to-many / many -to-many)
- Method
- Eliminate spurious edges (simplify graph)
- Select edges based on available information
- Blocks of conserved gene order
- Protein sequence similarity


## Evolutionary change

Lecture 1-Introduction
Lecture 2 - Hashing / BLAST
Lecture 3 - Combinatorial Motif Finding
Lecture 4-Statistical Motif Finding
Lecture 5 - Sequence alignment and Dynamic Programming
Lecture 6 - RNA structure and Context Free Grammars
Lecture 7-Gene finding and Hidden Markov Models
Lecture 8 - HMMs algorithms and Dynamic Programming
Lecture 9 - Evolutionary change, phylogenetic trees
Lecture 10-Genome rearrangements, genome duplication

## Overview Genome correspondence

Chromosome evolution Genome rearrangements Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements

Inferring orthologous gene relationships

- BBH - Best bi-directional hits
- COG - Clusters of orthologous genes
- BUS - Best unambiguous subgraphs

BUS: Best Unambiguous Subgraphs


Connected components of all best edges


Iterative refinement with increasing relative threshold

Conservation of gene order (synteny)


Preferentially select edges in synteny blocks

S.cerevisiae Chromosome VI 250-300kbp

## Overview

## Genome correspondence

Chromosome evolution
Genome rearrangements
Sorting by reversals
Genome duplication
Duplicate gene evolution Duplication and rearrangements

Regions of rapid change


## Specific mechanisms mediate rearrangements



Evolutionary advantage of transposons?


Specific regions of rapid evolution


Transposon locations are conserved


- Transposons are active
- Full--length Ty elements are recent
- Typically appear in only one genome
- Transposon locations are conserved
- Recent insertions reuse old loci
- LTR remnants found in other genomes



## Differences in gene content

- 8-10 genes unique to each genome
- Metabolism, regulation/silencing, stress
- Changes in gene dosage
- 10-20 tandem duplications (1-2 genes)
-2 segment duplications ( 56 genes)
- Protein family expansions
- 211 genes (3\%) with ambiguous correspondence
- Paralog duplication and/or loss

Different species, few novel genes


Reversals: Example

$$
\begin{gathered}
\pi=12345678 \\
\rho(3,5) \\
12543678
\end{gathered}
$$

Gene order rearrangement: overlapping inversions

|  |
| :---: |
| $12345 \bigvee 78910$ |
| $12-6-5-4-378910$ |
| $12-6-8-7345910$ |
| $12-6-8-734-10-9-5$ |

## Overview

Genome correspondence
Chromosome evolution Genome rearrangements

Sorting by reversals
Genome duplication
Duplicate gene evolution Duplication and rearrangements

Inference of inversion history: "sorting signed permutations by reversals"


Reversals: Example


## Reversals and Gene Orders

- Gene order is represented by a permutation $\pi$ :

- Reversal $\rho(i, j)$ reverses (flips) the elements from $i$ to $j$ in $\pi$


## Reversal Distance Problem

- Goal: Given two permutations, find the shortest series of reversals that transforms one into another
- Input: Permutations $\pi$ and $\sigma$
- Output: A series of reversals $\rho_{1}, \ldots \rho_{t}$ transforming $\pi$ into $\sigma$, such that $t$ is minimum
- $\boldsymbol{t}$ - reversal distance between $\pi$ and $\sigma$
- d( $\pi, \sigma$ ) - smallest possible value of $t$, given $\pi$ and $\sigma$


## Sorting By Reversals: Example

- $t=d(\pi)$ - reversal distance of $\pi$
- Example :

$$
\pi=\begin{array}{llllllllll}
3 & 4 & 2 & 1 & 5 & 6 & 7 & 10 & 9 & 8 \\
4 & 3 & 2 & 1 & 5 & 6 & 7 & 10 & 9 & 8 \\
4 & 3 & 2 & 1 & 5 & 6 & 7 & 8 & 9 & 10 \\
\hline 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10
\end{array}
$$

So $d(\pi)=3$

- Output: A series of reversals $\rho_{1}, \ldots \rho_{t}$ transforming $\pi$ into the identity permutation such that $t$ is minimum


## Sorting by reversals: 5 steps

Step 0: $\pi \quad 2$|  | -4 | -3 | 5 | -7 | -6 |
| :--- | :--- | :--- | :--- | :--- | :--- |

Step 1: $2 \overline{3} 45-8-7-61$
Step 2: $2345 \begin{array}{lllllll}678 & 1\end{array}$

Step 3: | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- |

$\begin{array}{llllllllll}\text { Step 4: } \\ \text { Step 5: } \gamma & \begin{array}{lllllllll}-8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 \\ & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8\end{array}\end{array}$

## Sorting by reversals: $\mathbf{4}$ steps

Step 0: $\pi \quad 2 \begin{array}{llllll}2 & -4 & -8 & -7 & -6 & 1\end{array}$

Step 1: | 2 | 3 | 4 | 5 | -8 | -7 | -6 |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |

| Step 2: | -5 | -4 | -3 | -2 | -8 | -7 | -6 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Step 4: $\gamma$|  | 12456 |
| :--- | :--- | :--- | :--- | :--- |

## Sorting by reversals: $\mathbf{4}$ steps

Step 0: $\pi \quad 2$-4 -3 5 5 -8 -7 -6 1
Step 1: $\begin{array}{r}2 \overline{3} 4 \\ 5\end{array}-8-7-6 \quad 1$
Step 2: $-5-4-3-2-8-7-6 \quad 1$
Step 3: $\quad-5-4-3-2-1678$
Step 4: $\gamma \begin{array}{llllllll} & 1 & 2 & 3 & 4 & 5 & 7 & 8\end{array}$

What is the reversal distance for this permutation? Can it be sorted in 3 steps?

## Pancake Flipping Problem

- The chef is sloppy; he prepares an unordered stack of pancakes of different sizes
- The waiter wants to rearrange them (so that the smallest winds up on top, and so on, down to the largest at the bottom)
- He does it by flipping over several from the top, repeating this as many times as necessary


Christos Papadimitrou and Bill Gates flip pancakes

## Pancake Flipping Problem: Formulation

- Goal: Given a stack of $n$ pancakes, what is the minimum number of flips to rearrange them into perfect stack?
- Input: Permutation $\pi$
- Output: A series of prefix reversals $\rho_{1}, \ldots \rho_{t}$ transforming $\pi$ into the identity permutation such that $t$ is minimum

Pancake Flipping Problem: Greedy Algorithm

- Greedy approach: 2 prefix reversals at most to place a pancake in its right position, $2 n-2$ steps total
at most
- William Gates and Christos Papadimitriou showed in the mid-1970s that this problem can be solved by at most $5 / 3(n+1)$ prefix reversals


## Sorting By Reversals: A Greedy Algorithm

- If sorting permutation $\pi=123645$, the first three elements are already in order so it does not make any sense to break them.
- The length of the already sorted prefix of $\pi$ is denoted prefix $(\pi)$
$-\operatorname{prefix}(\pi)=3$
- This results in an idea for a greedy algorithm: increase prefix $(\pi)$ at every step


## Greedy Algorithm: An Example

- Doing so, $\pi$ can be sorted

- Number of steps to sort permutation of length $n$ is at most ( $n-1$ )


## Greedy Algorithm: Pseudocode

```
SimpleReversalSort(\pi)
l for i
    j}\Leftarrow\mathrm{ position of element i in }\pi\mathrm{ (i.e., }\mp@subsup{\pi}{j}{}= i
    if j? i
        \pi\leftarrow\pi*\rho(i,j)
        output }
    if }\pi\mathrm{ is the identity permutation
        return
```


## Analyzing SimpleReversalSort (cont'd)

- But it can be sorted in two steps:

$$
\pi=612345
$$

- Step 1: 543216
- Step 2: 123456
- So, SimpleReversalSort $(\pi)$ is not optimal
- Optimal algorithms are unknown for many problems; approximation algorithms are used


## Analyzing SimpleReversalSort

- SimpleReversalSort does not guarantee the smallest number of reversals and takes five steps on $\pi=612345$ :
- Step 1: 162345
- Step 2: 126345
- Step 3: 123645
- Step 4: 123465
- Step 5: 123456


## Approximation Algorithms

- These algorithms find approximate solutions rather than optimal solutions
- The approximation ratio of an algorithm A on input $\pi$ is:

$$
\mathrm{A}(\pi) / \mathrm{OPT}(\pi)
$$

where
$A(\pi)$-solution produced by algorithm $A$
$\mathrm{OPT}(\pi)$ - optimal solution of the problem

## Approximation Ratio/Performance Guarantee

- Approximation ratio (performance guarantee) of algorithm A: max approximation ratio of all inputs of size $n$
-For algorithm A that minimizes objective function (minimization algorithm):
- $\max _{|\pi|={ }_{n} \mathrm{~A}(\pi) / \operatorname{OPT}(\pi)}$

Approximation Ratio/Performance Guarantee

- Approximation ratio (performance guarantee) of algorithm A: max approximation ratio of all inputs of size $n$
-For algorithm A that minimizes objective function (minimization algorithm):
- $\max _{|\pi|={ }_{n}} \mathrm{~A}(\pi) / \mathrm{OPT}(\pi)$
-For maximization algorithm:
- $\min _{|\pi|={ }_{n}} \mathrm{~A}(\pi) / \operatorname{OPT}(\pi)$


## Adjacencies and Breakpoints

$$
\pi=\pi_{1} \pi_{2} \pi_{3} \ldots \pi_{n-1} \pi_{n}
$$

- A pair of elements $\pi_{i}$ and $\pi_{i+1}$ are adjacent if

$$
\pi_{i+1}=\pi_{i} \pm 1
$$

- For example:

$$
\pi=193478265
$$

- $(3,4)$ or $(7,8)$ and $(6,5)$ are adjacent pairs


## Extending Permutations

- We put two elements $\pi_{0}=0$ and $\pi_{n+1}=n+1$ at the ends of $\pi$

Example:

$$
\begin{gathered}
\pi=\left.\left.\left.\left.1\right|_{9}\right|_{3} 4\right|_{7} ^{\left.\right|_{2}} 8\right|_{2} 65 \\
\pi=\left.\left.\left.01^{\mid} 9 l_{7} l_{7} 8_{2}\right|_{2}\right|_{6}\right|_{10}
\end{gathered}
$$

Note: A new breakpoint was created after extending

## Reversal Distance and Breakpoints

- Each reversal eliminates at most 2 breakpoints.
- This implies:

$$
\begin{aligned}
& \text { reversal distance }=\text { \#breakpoints } / 2 \\
& \pi=231465 \\
& 02314657 \quad b(\pi)=5 \\
& 011322465570 \\
& \begin{array}{llllllllll}
0 & 1 & 2 & 3 & 4 & 6 & 5 \\
0 & 1 & 2 & 3 & l_{4} & 5 & 6 & \left.\right|_{7} & b(\pi)=2 \\
b(\pi) & =0
\end{array} \\
& 01234 \left\lvert\, \begin{array}{lll}
0 & 1 & 7
\end{array}\right. \\
& b(\pi)=0
\end{aligned}
$$

## Breakpoints: An Example

There is a breakpoint between any pair of nonadjacent elements:

$$
\pi=19 \beta 4|78| 26|5|
$$

- Pairs $(1,9),(9,3),(4,7),(8,2)$ and $(2,5)$ form breakpoints of permutation $\pi$
- $b(\pi)$ - \# breakpoints in permutation $\pi$
- Each reversal eliminates at most 2 breakpoints.

$$
\pi=231465
$$

| 0 | 2 | 3 | 1 | 4 | 6 | 5 | 7 | $b(\pi)=5$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 1 | 3 | 2 | 4 | 6 | 5 | 7 | $b(\pi)=4$ |
| 0 | 1 | 2 | 3 | 4 | 6 | 5 | 7 | $b(\pi)=2$ |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | $b(\pi)=0$ |

## Sorting By Reversals: A Better Greedy Algorithm

## BreakPointReversalSort $(\pi)$

1 while $b(\pi)>0$
2 Among all possible reversals, choose reversal $\rho$ minimizing $b(\pi \cdot \rho)$
$3 \pi \leftarrow \pi \cdot \rho(\mathrm{i}, \mathrm{j})$
4 output $\pi$
5 return

Sorting By Reversals: A Better Greedy Algorithm
BreakPointReversalSort( $\pi$ )
1 while $b(\pi)>0$
2 Among all possible reversals, choose reversal $\rho$ minimizing $b(\pi \cdot \rho)$
$3 \pi \leftarrow \pi \cdot \rho(\mathrm{i}, \mathrm{j})$
4 output $\pi$
5 return

Problem: this algorithm may work forever

## Reducing the Number of Breakpoints

Theorem 1:
If permutation $\pi$ contains at least one decreasing strip, then there exists a reversal $\rho$ which decreases the number of breakpoints (i.e. $b(\pi \cdot \rho)<b(\pi)$ )

## Strips

- Strip: an interval between two consecutive breakpoints in a permutation
- Decreasing strip: strip of elements in decreasing order (e.g. 65 and 32 ).
- Increasing strip: strip of elements in increasing order (e.g. 78 )

019437825610

- A single-element strip can be declared either increasing or decreasing We strips with 0 and $n+1$


## Things To Consider

- For $\pi=14657832$

```
014 6 5 7 8 3 2|g | | | | | = 5
```

-Choose decreasing strip with the smallest element $k$ in $\pi$ ( $k=2$ in this case)

## Things To Consider (cont'd)

- For $\pi=14657832$
$0146578329 \quad b(\pi)=5$
-Choose decreasing strip with the smallest element $k$ in $\pi$ ( $k=2$ in this case)


## Things To Consider (cont'd)

- For $\pi=14657832$

$$
0146578329 \quad b(\pi)=5
$$

-Choose decreasing strip with the smallest element $k$ in $\pi$ ( $k=2$ in this case)
-Find $k-1$ in the permutation

## Things To Consider (cont'd)

- For $\pi=14657832$
- Choose decreasing strip with the smallest element $k$ in $\pi$ ( $k=2$ in this case)
- Find $k-1$ in the permutation
- Reverse the segment between $k$ and $k-1$ :



## Reducing the Number of Breakpoints Again

- If there is no decreasing strip, there may be no reversal $\rho$ that reduces the number of breakpoints (i.e. $b(\pi \cdot \rho)=b(\pi)$ for any reversal $\rho)$.
- By reversing an increasing strip (\# of breakpoints stay unchanged ), we will create a decreasing strip at the next step. Then the number of breakpoints will be reduced in the next step (theorem 1).


## Things To Consider (cont'd)

- There are no decreasing strips in $\pi$, for:

$\checkmark \rho(6,7)$ creates a decreasing strip thus guaranteeing that the next step will decrease the \# of breakpoints.


## ImprovedBreakpointReversalSort

```
mprovedBreakpointReversalSort(\pi)
1 while b}(\pi)>
2 if \pi}\mathrm{ has a decreasing strip
            Among all possible reversals, choose reversal \rho
                that minimizes b}(\pi\cdot\rho
    else
            Choose a reversal }\rho\mathrm{ that flips an increasing strip in }
    6 \pi}\leftarrow<\pi\cdot
    output 
8 return
```

Signed Permutations

- Up to this point, all permutations to sort were unsigned
- But genes have directions... so we should consider signed permutations



## GRIMM Web Server

- Real genome architectures are represented by signed permutations
- Efficient algorithms to sort signed permutations have been developed
- GRIMM web server computes the reversal distances between signed permutations:
http://nbcr.sdsc.edu/GRIMM/grimm.cgi


## GRIMM Web Server


http://www-cse.ucsd.edu/groups/bioinformatics/GRIMM
Courtesy of Glenn Tesler. Used with permission.

## Sorting by reversals

Boder DA. Moret BM, Yan M. [200]\} A linesr-time algorithm for computing inverion distabe buturen sipned permutations with an experimental study. I Comput Bod 8.483-91
Bergerun A. (in perse) A very elementary presentatice of the Hanmelails-Peveser thocry. Discrote Appolied Mathonatios
 pmomes. Diserte Appliod Mathowatice 71:137-151.
Hanumhall, S. and Pevaser, P. (1995). Transfoeming men into mice (polynomial algorithm for peowomic distance peoblem). In Procedings of the IEEE Mech Asvud Sgmposiaw on Foundstions of Computer Scerenes peas $581-502$
Hannenhalli, S. and Perxner, P. A. (1999). Trntsforming cabbugc into turnip (polynomial alpocithm for sarting nigned pectuntatione by revenalk), Jownen of the ACN, 48:1-27.
Tesker G (2002) GRDMAE genume rearrangemsenta weib never. Bivinformatics, 18:492-3

## Overview <br> Genome correspondence Chromosome evolution <br> Genome rearrangements <br> Sorting by reversals Genome duplication Duplicate gene evolution Duplication and rearrangements

Further back in evolutionary time


Ability to ask different set of questions

Whole Genome Duplication (WGD) in Yeast?


- However
- Interspersed with single-copy genes


## Interpretation: Genome duplication followed by gene loss

## Whole genome duplication is controversial




# Missing evidence supporting WGD 

Non-duplicated relative

Gene correspondence


Sister regions show gene interleaving


Few genes remain in 2 copies

Gene interleaving is evidence of complete duplication

Duplicate mapping tiles K. waltii


- 253 DCS blocks were identified containing $75 \%$ of K. waltii genes and $81 \%$ of S. cerevisae genes
- A typical DCS block has 27 genes (largest block has 81 genes).
- DCS blocks are separated by ~3 genes on the average.
- In a DCS block $90 \%$ of Kw genes have a match in at least 1 of the 2 Sc regions.
- 47 blocks have no duplicated gene.

Duplicate mapping tiles S. cerevisiae


## Overview

Genome correspondence
Chromosome evolution
Genome rearrangements
Sorting by reversals
Genome duplication

## Duplicate gene evolution

Duplication and rearrangements


Recognize sister regions solely based on gene order

Whole-genome duplication resolved


## Accelerated gene divergence

- Ohno hypothesized that after duplication, one copy would preserve the original function, and the other copy would be free to diverge. Others argued that both copies would diverge.
- 76 of 457 duplicated gene pairs show accelerated evolution. In 95\% of the cases, acceleration was limited to one of the 2 paralogs.
- Deletion of the ancestral paralog is lethal in $18 \%$ of the cases.
- Deletion of a derived paralog is never lethal.
- 457 genes kept in two copies, result of selection - Involved in sugar metabolism and fermentation

WGD


Evidence of accelerated protein divergence?

Emerging gene functions after duplication

- Origin of replication $\rightarrow$ silencing

- Translation initiation $\rightarrow$ anti-viral defense

Distinct functional properties

|  |  |  |
| :---: | :---: | :---: |
| Gene <br> deletion | Lethal (20\%) | Derived function |
| Expression | Abundant | Never lethal |
| Localization | General | Specific <br> (stress, starvation) |
| (mitochondrion, spores) |  |  |

Gain new function and lose ancestral function

Scenarios for rapid gene evolution


Ohno, 1970
Both copies faster


## 20\% of duplicated genes show acceleration

 $95 \%$ of cases: Only one copy fasterGain new function and lose ancestral function

Decelerated evolution


- 60 gene pairs ( $13 \%$ of 457 pairs)
- $98 \%$ protein identity (all pairs: $55 \%$ )
- $90 \%$ identity in 4 fold degenerate sites (all pairs: $41 \%$ )
- Not recent duplication
- Gene order argues ancestral WGD pairs

Evidence of gene conversion


- Tree root reveals time of duplication
- No acceleration in the K. waltii branch
- The two genes have recently replaced each other
- Branching order reveals gene conversion
- Paralogs are closer to each other than to their ortholog
- Both S. cerevisiae and S. bayanus show gene conversion

Periodic gene conversion


- Genome ancestry resolved
- Whole -genome duplication
- Massive gene loss

- Emergence of new functions
- Asymmetric acceleration
- Ancestral and derived functions - Repository for buffering mutations


Mammals: How many WGDs?


How did the pre-duplicated ancestor look like?

- Can we derive the architecture of the current (human and tetraodon genomes) genomes in terms of the common ancestor?
- What was the sequence of rearrangement events after WGD?


Rat Consortium, Nature, 2004

Genome rearrangements

-What is the architecture of the ancestral genome?

- What is the evolutionary scenario for transforming one genome into the other?
$1,2,3,-8,-7,-6,-5,-49,10$
- Blocks represent conserved genes.
- In the course of evolution, blocks $1, \ldots, 10$ could be misread as

$$
1,2,3,-8,-7,-6,-5,-49,10 .
$$

- Evolution: occurred one-two times every million years on the evolutionary path between human and mouse.
$1,2,3,-8,-7,-6,-5,-49,10$.


Reversals


- Blocks represent conserved genes.
- In the course of evolution, blocks $1,2, \ldots 9,10$ could be transformed into

1, 2, 3, -8, -7, $-6,-5,-4,9,10$


The inversion introduced two breakpoints (disruptions in order).

Sorting by reversals Most parsimonious scenarios

Step 0: $\pi \quad 2 \begin{array}{lllllll}2 & -4 & -3 & -8 & -7 & -6 & 1\end{array}$

| Step 1: | 2 | 3 | 4 | 5 | -8 | -7 | -6 | 1 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| Step 2: | -5 | -4 | -3 | -2 | -8 | -7 | -6 | 1 | $\begin{array}{llrrrrrrr}\text { Step 3: } \\ \text { Step 4: } \gamma & -5 & -4 & -3 & -2 & -1 & 6 & 7 & 8 \\ & 2 & 3 & 4 & 5 & 6 & 7 & 8\end{array}$

The reversal distance is the minimum number of reversals required to transform $\pi$ into $\gamma$.
Here, the reversal distance is $\mathrm{d}=4$.

## Breakpoint graph

$\mathbf{G}(\pi, \gamma)$


1. 6 breakpoints (consecutive elements in both genomes disagree)
$\wedge 3$ adjacencies (consecutive elements in both genomes agree)

## DualityTheorem:

```
reversal distance = #genes + 1 - #cycles + h
```

where h is rather complicated, but can be computed from breakpoint graph in polynomial time.

- Here, reversal distance $=8+1-5+0+0=4$


## Breakpoint graph


4. 6 breakpoints (consecutive elements in both genomes disagree) $\wedge 3$ adjacencies (consecutive elements in both genomes agree)

## - Duality Theorem for Sorting by Reversals - simple and imprecise version.

reversal distance $=$ number of elements +1 - number of cycles

Constructing Breakpoint Graph: Black Path


Constructing Breakpoint Graph: Dot Plot


Constructing Breakpoint Graph: Gray Path


Constructing Breakpoint Graph: Superimposing Two Paths



## Reconstructing pre-duplicated Genome

- WGD of genome $R$ results in perfect duplicated genome $R+R$
- $R+R$ becomes subject to rearrangements that shuffle genes in $R+R$ and result in some rearranged duplicated genome $P$
- Problem: reconstruct pre-duplicated genome $R$ from rearranged duplicated genome $P$.

Constructing Breakpoint Graph: Removing Dot-Plot


Constructing Rearrangement Scenarios


## Genome Halving Problem

- WGD of genome $R$ results in perfect duplicated genome $R+R$
- $R+R$ becomes subject to rearrangements that shuffle genes in $R+R$ and result in some rearranged duplicated genome $P$
- Problem: reconstruct pre-duplicated genome $\boldsymbol{R}$ from rearranged duplicated genome $P$.
- Genome Halving Problem: Given a duplicated genome $P$, recover the ancestral pre-duplicated genome $R$ minimizing the reversal distance from $R+R$ to $P$

| Illustration |
| :---: |
| $\boldsymbol{R}=+\mathrm{a}-\mathrm{d}+\mathrm{e}-\mathrm{c}+\mathrm{b}$ |
| $\boldsymbol{R}+\boldsymbol{R}=+\mathrm{a}-\mathrm{d}+\mathrm{e}-\mathrm{c}+\mathrm{b}+\mathrm{a}-\mathrm{d}+\mathrm{e}-\mathrm{c}+\mathrm{b}$ |
| $+\mathrm{a}-\mathrm{d}+\mathrm{d}-\mathrm{a}-\mathrm{b}+\mathrm{c}-\mathrm{e}+\mathrm{e}-\mathrm{c}+\mathrm{b}$ |
| $+a-d+d+c-e+e-c+b+a+b$ |
| $+\mathrm{a}-\mathrm{d}+\mathrm{d}-\mathrm{e}+\mathrm{e}-\mathrm{c}-\mathrm{c}+\mathrm{b}+\mathrm{a}+\mathrm{b}$ |

## Even worse!

$$
R=\text { ?? ?? ?? ?? ?? }
$$


$\boldsymbol{P}=+\mathrm{a}-\mathrm{d}+\mathrm{e}-\mathrm{d}+\mathrm{e}-\mathrm{c}-\mathrm{c}+\mathrm{b}+\mathrm{a}+\mathrm{b}$

## Recover it!

$$
R=\text { ?? ?? ?? ?? ?? }
$$


$\boldsymbol{P}=+\mathrm{a}-\mathrm{d}+\mathrm{e}-\mathrm{d}+\mathrm{e}-\mathrm{c}-\mathrm{c}+\mathrm{b}+\mathrm{a}+\mathrm{b}$

## Suppose we somehow figured out what is $R$ : would it help to find $d(P, R+R)$ ?

$$
\boldsymbol{R}=+\mathrm{a}+\mathrm{c}+\mathrm{d}+\mathrm{e}+\mathrm{b}
$$

$$
\boldsymbol{R}+\boldsymbol{R}=+\mathrm{a}+\mathrm{c}+\mathrm{d}+\mathrm{e}+\mathrm{b}+\mathrm{a}+\mathrm{c}+\mathrm{d}+\mathrm{e}+\mathrm{b}
$$

$$
\boldsymbol{P}=+\mathrm{a}-\mathrm{d}+\mathrm{e}-\mathrm{d}+\mathrm{e}-\mathrm{c}-\mathrm{c}+\mathrm{b}+\mathrm{a}+\mathrm{b}
$$

## HP-theory: reminder

- Transforming signed gene order

$$
+a+b-c
$$

into unsigned gene order

$$
a^{t} a^{h} \quad b b^{h} c^{h} c^{t}
$$

- Elements $x^{t}$ and $x^{h}$ are called obverse pair
- $t$ stands for tail and $h$ stands for head


## Breakpoint graph is formed by 3 matchings:

obverse matching
black matching (adjacent elements in $1^{\text {st }}$ permutation) gray matching (adjacent elements in $2^{\text {nd }}$ permutation)

## HP-theory: reminder

- Breakpoint graph is formed by obverse, black and gray matchings.
- Every pair of matching forms a collection of alternating cycles:
black-gray cycles (\#cycles in HP theory)
single black-obverse cycle ( $1^{\text {st }}$ permutation)
single gray-obverse cycle ( $2^{\text {nd }}$ permutation)
reversal distance between two circular permutations $=$
\#elements -\#black-gray cycles


## Labelings and breakpoint graphs

- Every labeling transforms genomes with duplicated genes into genomes without duplicated genes and enables applications of HP algorithm.
- Every labeling corresponds to a breakpoint graph
- Good labelings correspond to breakpoint graphs with large number of cycles.
- Can we construct a labeling corresponding to a large number of cycles?

Rearrangements in Duplicated Genomes: Challenges.

- Computing $d(P, Q)$. Can we construct a labeling of duplicated genomes $P$ and $Q$ maximizing the number of cycles? NO
- Computing $d(P, R+R)$. Can we construct a labeling of duplicated genomes $P$ and $R+R$ maximizing the number of cycles? NO
- While there exist fast algorithms for computing reversal distance between permutations (i.e., no duplicate genes), the problem of computing reversal distance between genomes with duplicated genes remains unsolved.
- Solution: label different copies of each gene ( $k$ ! different labelings for a gene with $\boldsymbol{k}$ copies)
- One of these labelings is unavoidably an
optimal labeling
corresponding to the optimum rearrangement scenario
- Running time: ( $k$ ! $)^{n}$ invocations of HP algorithms for a genome with $n$ genes each present in $k$ copies.

Rearrangements in Duplicated Genomes: Challenges.

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Rearrangements in Duplicated Genomes: Challenges.

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- Computing $d(P, R+R)$. Can we construct a labeling of duplicated genomes $P$ and $R+R$ maximizing the number of cycles? NO
- Computing $\min _{R} d(P, R+R)$.

Rearrangements in Duplicated Genomes: Challenges.

- Computing $d(P, Q)$. Can we construct a labeling of duplicated genomes $P$ and $Q$ maximizing the number of cycles? NO
- Computing $d(P, R+R)$. Can we construct a labeling of duplicated genomes $P$ and $R+R$ maximizing the number of cycles? NO
- Computing $\min _{R} d(P, R+R)$. YES!


## De Bruijn Graphs

- De Bruijn graph: Given a set of edgelabeled graphs, de Bruijn graph of this set is the result of "gluing" edges with the same label in all graphs in the set.

Rearrangements in Duplicated Genomes: Challenges.

- Breakpoint graphs are not defined for duplicated genomes.
- Can we generalize the notion of breakpoint graph for the case of duplicated genomes?
- Idea: Explore the connection between de Bruijn graphs and breakpoint graphs.


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- Did we see de Bruijn graphs today?

De Bruijn graph of a path

de Bruijn graph of another path


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## De Bruijn Graphs

- Breakpoint graph of permutations $P$ and $Q$ == de Bruijn graph of $P$-cycle and $Q$-cycle
- Breakpoint graph of any genomes $P$ and $Q$ (with multiple gene copies)


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de Bruijn graph of $P$-cycle and $Q$-cycle

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$=$
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## Overview

## Genome correspondence

Chromosome evolution
Genome rearrangements
Sorting by reversals
Genome duplication
Duplicate gene evolution Duplication and rearrangements

## Greedy Algorithms

And
Genome Rearrangements

## Outline

- Transforming Cabbage into Turnip
- Genome Rearrangements
- Sorting By Reversals
- Pancake Flipping Problem
- Greedy Algorithm for Sorting by Reversals
- Approximation Algorithms
- Breakpoints: a Different Face of Greed


## Outline CHANGE

- Genome Rearrangements - give picture of splotch mouse
 common ancestor, they look and taste different


Turnip vs Cabbage: Almost Identical mtDNA gene sequences

- In 1980s Jeffrey Palmer studied evolution of plant organelles by comparing mitochondrial genomes of the cabbage and turnip
- $99 \%$ similarity between genes
- These surprisingly identical gene sequences differed in gene order
- This study helped pave the way to analyzing genome rearrangements in molecular evolution
- Gene order comparison:


Turnip vs Cabbage: Different mtDNA Gene Order

- Gene order comparison:

- Gene order comparison:


Evolution is manifested as the divergence in gene order

- Gene order comparison:

- Gene order comparison:


Transforming Cabbage into Turnip


-What are the similarity blocks and how to find them?
-What is the architecture of the ancestral genome?

- What is the evolutionary scenario for transforming one genome into the other?

History of Chromosome X


Blocks represent conserved genes.

- In the course of evolution or in a clinical context, blocks $1, \ldots, 10$ could be misread as 1, 2, 3,

9, 10

Reversals and Breakpoints


The reversion introduced two (disruptions in order).

Reversals: Example


5' ATGCETGTAETA 3'
3' TACGGACATGAT 5'


5' ATGTACAGGCTA 3'
3' TACATGTCCGAT 5'

## Types of Rearrangements



Translocation


Waardenburg's Syndrome: Mouse Provides Insight into Human Genetic Disorder

- Waardenburg's syndrome is characterized by pigmentary dysphasia
- Gene implicated in the disease was linked to human chromosome 2 but it was not clear where exactly it is located on chromosome 2


Comparative Genomic Architecture of Human and Mouse Genomes
To locate where corresponding gene is in humans, we have to analyze the relative architecture of human and mouse genomes


Comparative Genomic Architertırac. Mnnıeave Hımяn

- Humans and mice hagenom similar genomes, but their genes are ordered differently
- ~245 rearrangements
- Reversals
- Fusions
- Fissions
- Translocation



## Waardenburg's syndrome and splotch mice

- A breed of mice (with splotch gene) had similar symptoms caused by the same type of gene as in humans
- Scientists succeeded in identifying location of gene responsible for disorder in mice
- Finding the gene in mice gives clues to where the same gene is located in humans

