Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.243j (Fall 2003): DYNAMICS OF NONLINEAR SYSTEMS

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Take-Home Test 1[1]

For each problem, give an answer and provide supporting arguments, not to exceed one page per problem. Return your test paper by 11.05 am on Friday October 17, in the classroom. Remember that collaboration is not allowed on test assignments.

Problem T1.1

Find all values of $\mu \in \mathbf{R}$ for which the function $V: \mathbf{R}^2 \mapsto \mathbf{R}$, defined by

$$V\left(\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}\right) = \max\{|\bar{x}_1|, |\bar{x}_2|\}$$

is monotonically non-increasing along solutions of the ODE

$$\begin{cases} \dot{x}_1(t) &= \mu x_1(t) + \sin(x_2(t)), \\ \dot{x}_2(t) &= \mu x_2(t) - \sin(x_1(t)). \end{cases}$$

Hint: $|\sin(y)| < |y|$ for all $y \neq 0$, and $\sin(y)/y \to 1$ as $y \to 0$.

[1] Posted October 16, 2003. Due at 11.05 am on October 17, 2003

Problem T1.2

Find all values of $r \in \mathbf{R}$ for which differential inclusion of the form

$$\dot{x}(t) \in \eta(x(t)), \quad x(0) = \bar{x}_0,$$

where $\eta: \ \mathbf{R}^2 \mapsto 2^{\mathbf{R}^2}$ is defined by

$$\eta(\bar{x}) = \{f(\bar{x}/|\bar{x}|)\} \ \text{ for } \ \bar{x} \neq 0,$$

$$\eta(0) = \{f(y) : \ y = [y_1; y_2] \in \mathbf{R}^2, \ |y_1| + |y_2| \leq r\},$$

has a solution $x : \ [0, \infty) \mapsto \mathbf{R}^2$ for every continuous function $f : \ \mathbf{R}^2 \mapsto \mathbf{R}^2$ and for every initial condition $\bar{x}_0 \in \mathbf{R}^2$.

Problem T1.3

Find all values $q, r \in \mathbf{R}$ for which $\bar{x}_0 = 0$ is *not* a (locally) stable equilibrium of the ODE

$$\dot{x}(t) = Ax(t) + B(Cx(t))^{1/3}$$

for every set of matrices A, B, C of dimensions n-by-n, n-by-1, and 1-by-n respectively, such that A is a Hurwitz matrix and

$$\mathrm{Re}[(1 + j\omega q)G(j\omega)] > r \ \ \forall \, \omega \in \mathbf{R}$$

for

$$G(s) = C(sI - A)^{-1}B.$$