MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Fall Term 2003 Issued : 1:30 pm
Final Exam Due : 4:30 pm

Final Exam
December 18, 2003
180 minutes

	Points	Grader
1		
2		
3		
4		
5		
6		

1. This examination consists of six problems. Work all problems.
2. This examination is closed book. Helpful equations, calculations, and the root-locus rules appear at the end of this packet.
3. You must summarize your solutions in the answer sheet booklet included with this examination. Draw all sketches neatly and clearly where requested. Remember to label ALL important features of any sketches.
4. Make sure that your name is on each answer sheet and on each examination booklet.

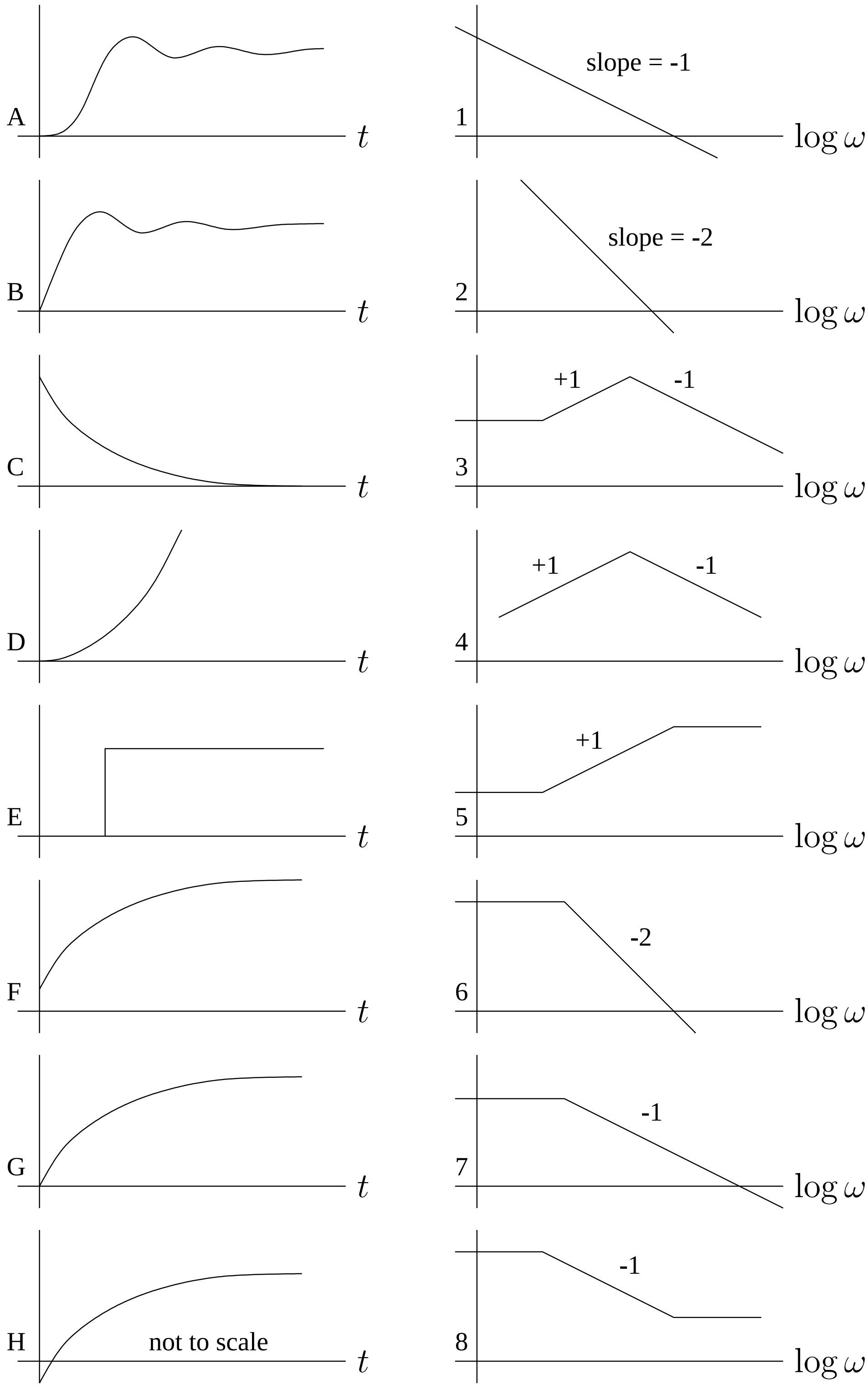
We encourage you to do the work for all of the problems in the answer sheet booklet as well. If you find that the answer sheets do not contain enough space for your scratch work, you may do additional work in the accompanying examination booklet. Make sure that you clearly denote which problem is on each page of the examination booklet. Your examination booklet will also be read by the graders, but only if your answers appear on the answer sheets.

Good luck.

Problem 1 (10%)

The figure on the next page shows eight linear-system step responses, A–H, and eight asymptotic magnitude Bode plots 1–8 (on standard log(magnitude)-versus-log(ω) axes). Note that the step responses and magnitude plots are the same ones included on Problem Set 3.

Match the step responses to the corresponding Bode plot(s). One or more step responses may not correspond to any of the Bode plots (in this case, enter "none" in the table on the answer sheet). Also, one or more of the step response may correspond to multiple Bode plots.



A
t
slope = -1
1
log ω
B
t
slope = -2
2
log ω
C
t
+1
-1
3
log ω
D
t
+1
-1
4
log ω
E
t
+1
5
log ω
F
t
-2
6
log ω
G
t
-1
7
log ω
H
not to scale
t
-1
8
log ω

Problem 2 (15%)

Consider the transfer function

$$G(s) = \frac{1}{25s}\left(\frac{1}{s^2 + 0.02s + 1}\right).$$

$G(s)$ is put into the forward path of a unity-gain feedback system.

(a) Draw the Nyquist plot for this system. Is it stable?

(b) Now suppose that a delay is added to the feedback path.

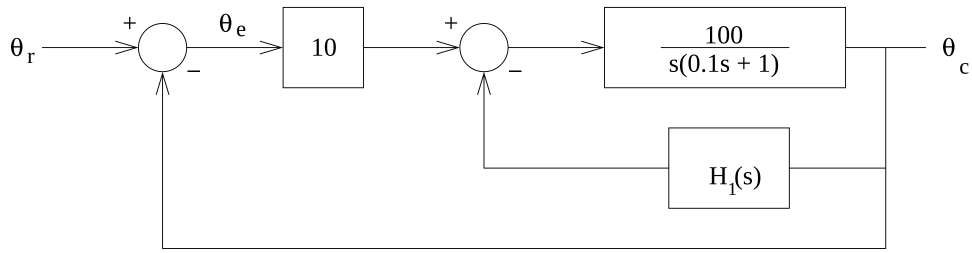
$$L(s) = G(s)e^{-s\pi}$$

Draw the Nyquist plot for the delayed system. Is it stable?

(c) The delay component has doubled from π to 2π. Draw the Nyquist plot for this system. Is it stable?

Problem 3 (15%)

A simplified representation of a positional servomechanism that includes a motor in its forward path is shown below.



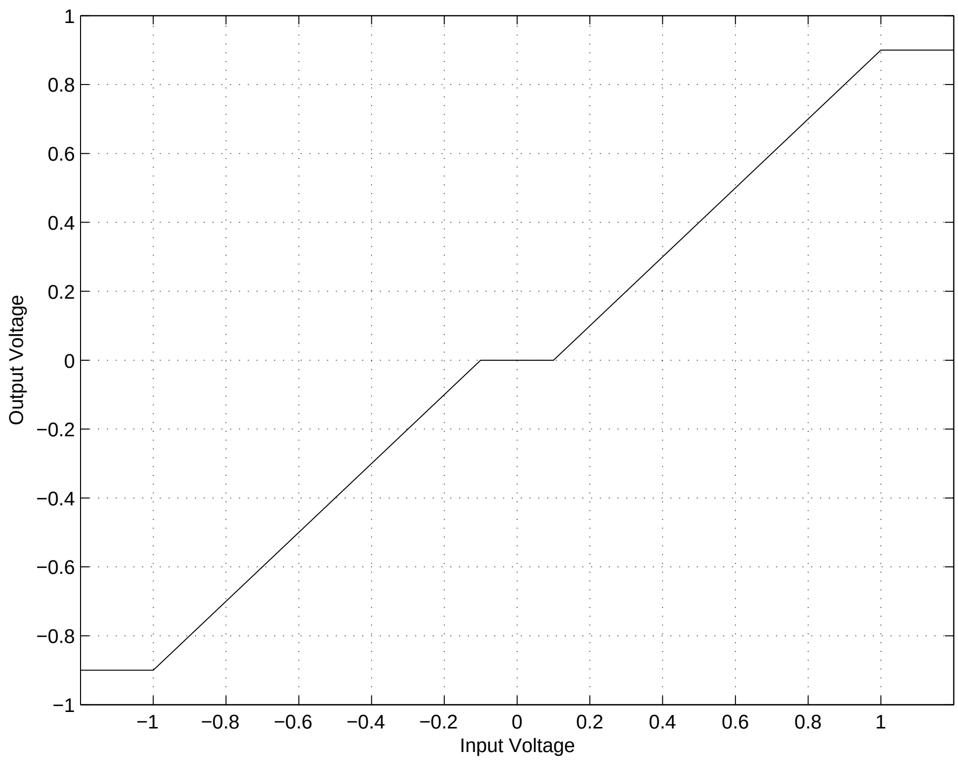
(a) Assume initially that the transfer function $H_1(s) = 0$. Find the natural frequency and the damping ratio of the closed-loop transfer function $\frac{\theta_c}{\theta_r}(s)$ in this case.

(b) One way to improve the stability of this system is to apply velocity feedback around the motor. This type of feedback can be modeled by making $H_1(s) = Ks$. Find the value of K that increases the phase margin of the major loop to 45°.

(c) Many design decisions for feedback systems are made based on the tradeoffs that exist between various performance metrics. For this system, the velocity error coefficient becomes larger as stability is improved. Illustrate this tradeoff by finding the steady state value of θ_e that results when a unit ramp is applied to the systems of part (a) and (b).

Problem 4 (25%)

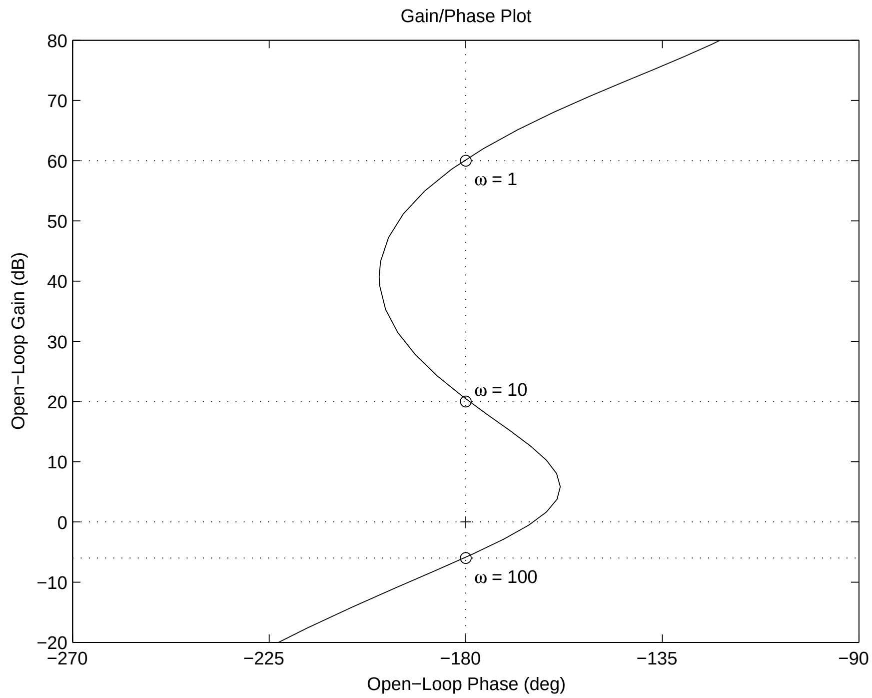
The input-output characteristics of a nonlinear element that has a dead zone and saturation is shown below.



(a) Sketch the important features of $G_D(E)$ for this element on the axes shown on the answer sheet. Extreme accuracy is not required, but your sketch should show the general shape of $G_D(E)$, and indicate values of E where important changes occur.

(b) Estimate the maximum value of $G_D(E)$ and the value of E that results in the maximum. 20% accuracy is adequate for these answers. Also determine an asymptotic expression for $G_D(E)$ for large E.

The nonlinear element is included in a negative feedback system with a linear element. The transfer function of the linear element is shown below in gain-phase form. The remaining parts of this question relate to the feedback system.



Hint:

$$-6 \text{ dB} \approx 0.5$$

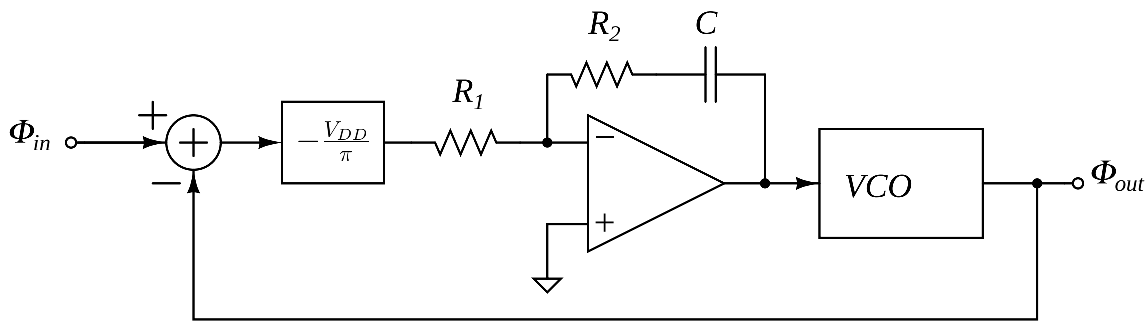
(c) Is it possible for the system to exhibit stable performance? Explain.

(d) For a certain set of initial conditions, it is found that the system is oscillating with the amplitude of the signal into the nonlinear element much greater than 1. What is the frequency of oscillation?

(e) For different initial conditions, it is found that the system is oscillating with the amplitude of the signal into the nonlinear element greater than 0.1 but less than 1. What is the frequency of oscillation in this case?

Problem 5 (15%)

Consider the PLL shown below. Assume that the relationship between control voltage and output frequency of the VCO is 10 MHz per volt and the op-amp is ideal.



(a) Find the loop transfer function $L(s)$.

(b) Assume V_{DD}=5 V, R_1=100 Ω and R_2=0. What value of C gives a loop crossover frequency of 100 kHz? What is the phase margin?

(c) With the value of C from part (b), find the value of R_2 that will provide a phase margin of 45° while preserving the crossover frequency.

Problem 6 (20%)

Designing a Dolphin Pool Heating System

Divya is a young engineer at the Franklin Park Zoo. He recently completed 6.302 and is eager to use his newfound skills as a feedback engineer. Divya walks around the zoo and eagerly notes room for improvement and opportunities to use his expertise.

Seeing the dolphin pool, Divya curiously dips his hand in the water and is shocked to find the water very cold. "Poor dolphins," Divya thinks and decides to help. Divya talks to his boss, who is somewhat apprehensive about the project. He says that dolphins are pretty content in their current environment, but decides to let Divya give it a try. "As long as the water does not go beyond $140^o F$, that would be very bad," Divya's boss comments.

Divya heads home and after an unslept night designs a temperature heating system as in Figure 1. The improved pool contains a heater at the bottom and a temperature sensor near the surface. In order to quickly bring the surface to a desired temperature, one can dynamically adjust the temperatue of the heater.

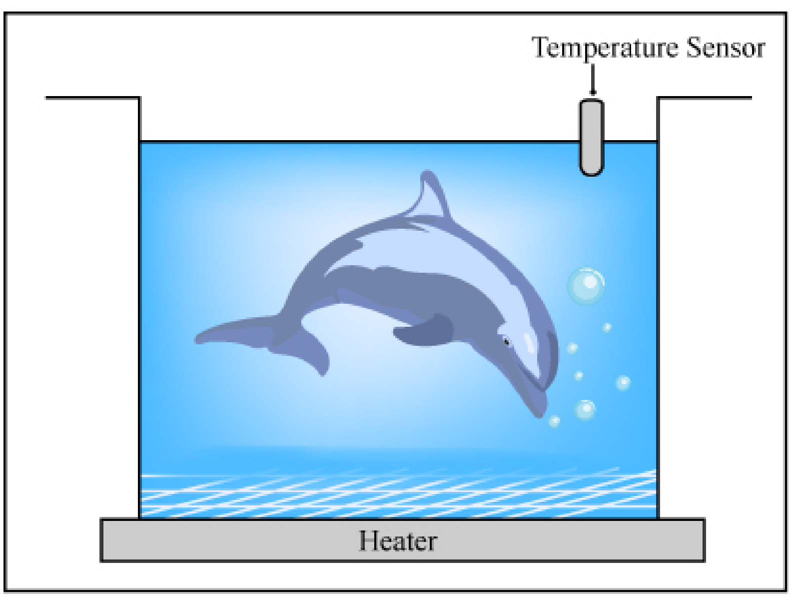


Figure by MIT OpenCourseWare.

Figure 1: Dolphin pool heating system.

After some measurements, transfer function relating the surface temperature $c(t)$ to the heater temperature $T_{heater}(t)$ turns out to be:

$$G_h(s) = \frac{C(s)}{T_{heater}(s)} = \frac{40}{s^2 + 15s + 50}$$

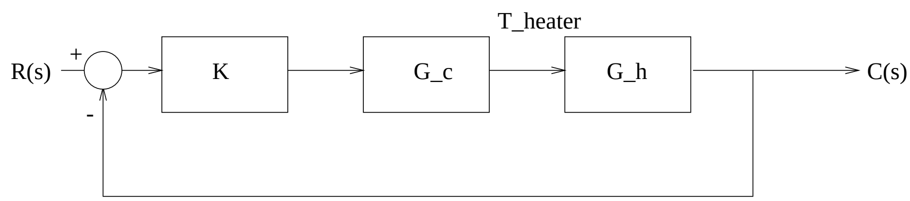


Figure 2: Water temperature control loop - desired temperature R, output C

(a) Sketch the shape of the open loop step response $c(t)$ (label initial/final value, initial slope) to $100^o F$ step from the heater. Why does the steady-state temperature at the water's surface differ from the steady-state temperature of the heater? Give a short physical/intuitive explanation.

(b) Divya now embarks upon using feedback, as in the block diagram in Figure 2. At first, Divya tries compensating the system with $G_c(s) = 1$ and just varying the proportional gain K.

1. Help Divya compute the smallest value of K required to keep the steady-state surface temperature within 2% of the desired temperature (you can make reasonable approximations to simplify math).
2. Sketch the shape of the response $c(t)$ (label initial/final value, initial slope) assuming $r(t) = 100u(t)$ for the value of K you just found (if you did not find K at least let Divya know what it looks like in terms of K). What happened to the response time (t_r) and the steady-state error? (just state whether it stayed the same, increased or decreased)
3. Divya is really excited about his success and decides to call Prof. Roberge to brag about how he has mastered feedback design in such a short time. Prof. Roberge examines his compensated system and fearfully exclaims "Thank goodness you called me." Why was Prof. Roberge terrified? Provide quantitative measure(s) if possible.

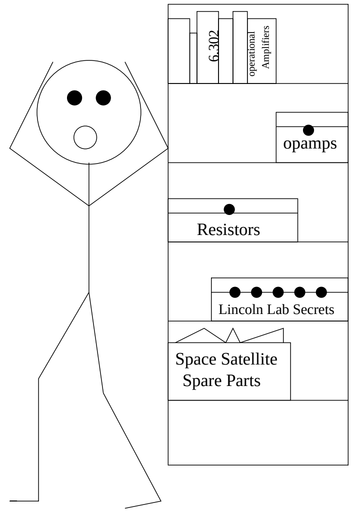


Figure 3: Terrified Professor Roberge

(c) Prof. Roberge suggests Divya tries a different compensator

$$G_c(s) = 1 + 0.05s$$

and starts with $K = 1$.

1. What kind of controller is the one Prof. Roberge suggested? Give a common name for it.
2. With $K = 1$, compute the closed loop system function. How does the closed loop function differ from the one in part (b)?
3. Now vary K so that the steady-state error value of $c(t)$ is within 2% of the desired input value. What's the minimum value of K to get us 2% steady-state error? (You can make reasonable approximations to simplify math.)
4. Is Prof. Roberge going to be okay with Divya implementing the compensator in part (c)? Why or why not?

(d) After Divya completes his work in part (c), Prof. Roberge teases Divya's ambition and suggests adding another term to Divya's compensator that would further improve the design. What is the new term? Do not compute anything, your answer to this part should just be the new improved $G_c(s)$ (you may normalize your suggested term to have a gain of 1).