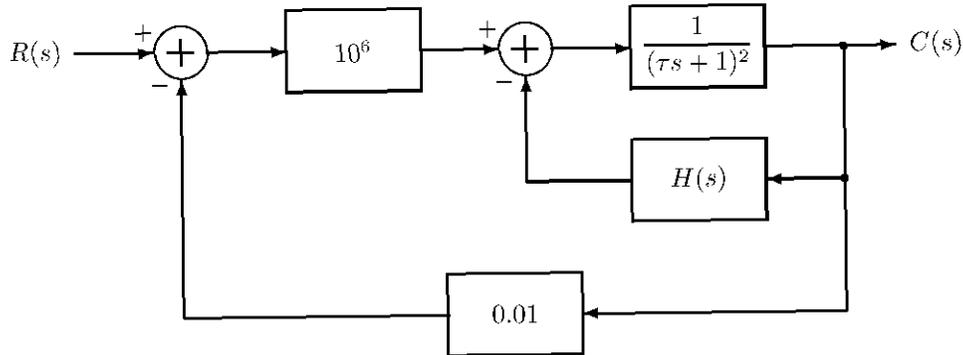


6.302 Feedback Systems

Spring Term 2007
Problem Set 8

Issued : April 11, 2007
Due : **Thursday, May 3, 2007**

Problem 1: A system with minor loop compensation is shown below.



where $\tau = 10^{-4}$.

- (a) Do we want $H(s) = Ks$ or $H(s) = \frac{K}{s}$?
- (b) Find K such that $\omega_c = 10^6$ rps and $\phi_M = 45^\circ$.
- (c) What is the dynamic tracking error $\left| \frac{E}{R} \right|$ at 10^3 rps?
- (d) Your customer wants $\omega_c = 10^6$ rps, $\phi_M = 45^\circ$ and $\left| \frac{E}{R} \right| < 2 \times 10^{-4}$ at 10^3 rps. Choose the $H(s)$ which satisfies all of these criteria.

$$H_1(s) = \frac{10^{-6}s^2}{10^{-4}s + 1}$$

$$H_2(s) = \frac{10^{-4}s + 1}{10^{-6}s^2}$$

$$H_3(s) = \frac{10^{-5}s^2}{10^{-3}s + 1}$$

$$H_4(s) = \frac{10^{-3}s + 1}{10^{-5}s^2}$$

$$H_5(s) = \frac{10^{-4}s^2}{10^{-2}s + 1}$$

$$H_6(s) = \frac{10^{-2}s + 1}{10^{-4}s^2}$$

- (e) What is $\left| \frac{E}{R} \right|$ evaluated at 10^3 rps now?

Problem 2: For each plant transfer function $G_p(s)$ in the left column (1–6), select the “best” compensator transfer function $G_c(s)$ from the right column (a–g). Additionally, specify the frequency or frequencies where the loop transfer function is unity, and give the phase margin at these frequencies. Some of the compensator transfer functions may be best for more than one plant, while others may not be used.

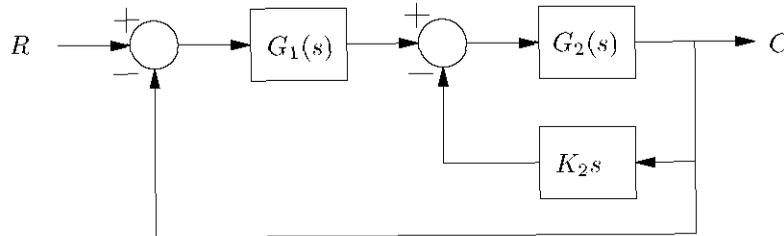
As with all design issues, what is “best” is often a compromise between competing considerations. You should consider the following three criteria: a phase margin requirement between 30° and 90° , the widest bandwidth consistent with this range of phase margin, and high open-loop gain at frequencies below crossover.

1.	$G_p(s) = \frac{10^2}{(s+1)(10^{-2}s+1)}$
2.	$G_p(s) = \frac{\sqrt{10} \times 10^6}{s^2(10^{-5}s+1)}$
3.	$G_p(s) = e^{-s}$
4.	$G_p(s) = \frac{1}{s(10^{-6}s+1)}$
5.	$G_p(s) = \frac{\sqrt{10} \times 10^4}{(0.1s+1)(0.1s-1)(10^{-5}s+1)}$
6.	$G_p(s) = 10^4 \frac{s^2}{(s+1)^3}$

a.	$G_c(s) = \frac{1}{s}$
b.	$G_c(s) = 7 \times 10^7 \left(\frac{10^{-7}s+1}{10^{-8}s+1} \right)$
c.	$G_c(s) = 10\sqrt{2} \left(\frac{0.1s+1}{s+1} \right)$
d.	$G_c(s) = 100$
e.	$G_c(s) = \frac{10^{-3}s+1}{10^{-4}s+1}$
f.	$G_c(s) = 10 \left(\frac{10s+1}{1000s+1} \right)$
g.	$G_c(s) = \frac{10}{s^2}$

Problem 3: For the system shown below, we are told that the plant is

$$G_2(s) = \frac{100}{(0.1s + 1)(0.01s + 1)^2}.$$



It is required that the major loop crossover occur close to $\omega_c = 100$ rps with a phase margin greater than or equal to 60 degrees.

- Assume for this first part the $G_1(s) = K_1$, a constant. Find K_2 to meet these specifications.
- Now assume $K_2 = 0$. Find a series compensator $G_1(s)$ to meet the specs.
- Suppose the DC gain of $G_2(s)$ is increased from 100 to 1000. What happens to the phase margin in the two cases considered in parts (a) and (b)?
- Using MATLAB, find the closed-loop step response for the system when minor loop compensation is used. Comment on the robustness of the two designs.