

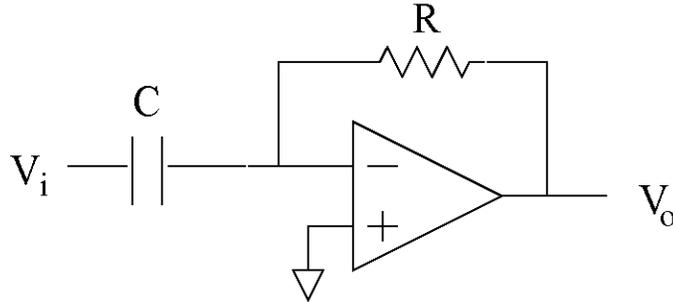
6.302 Feedback Systems

Spring Term 2007
Problem Set 7

Issued : April 3, 2007
Due : Tuesday, April 11, 2007

Problem 1: After dozing off in lecture, one of your classmates asks you the following question: “Lag compensation adds negative phase, thus *decreasing* the phase margin. Why would we ever want to do that?” Explain how you would answer him.

Problem 2: Consider the following operational amplifier circuit, where $R = 1\text{ k}\Omega$ and $C = 1000\text{ pF}$:



Assuming an ideal op amp:

- (a) What is $\frac{V_o(s)}{V_i(s)}$?
- (b) Sketch the output response for $V_i = \frac{1V}{\mu s}$ ramp.

Suppose the op amp has the μA 741 transfer function:

$$A(s) = \frac{200,000}{\frac{s}{2\pi \cdot 511z} + 1}$$

- (c) What is the phase margin, ϕ_m of $L(s)$?
- (d) Use Matlab to plot the output response for $V_i = \frac{1V}{\mu s}$ ramp.

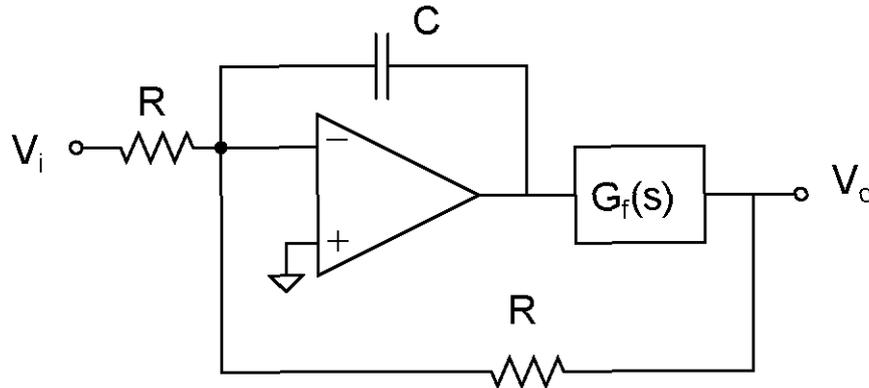
Now assume a more accurate μA 741 model:

$$A(s) = \frac{200,000}{\left(\frac{s}{2\pi \cdot 511z} + 1\right) \left(\frac{s}{6 \times 10^6} + 1\right)}$$

- (e) What is ϕ_m of $L(s)$?
- (f) Use Matlab to plot the output response for $V_i = \frac{1V}{\mu s}$ ramp.

Problem 3: A feedback-topology power amplifier is shown below. Assume that the op amp is ideal. Select a value for the capacitor C such that the system phase margin is 45° . Assume that $R = 10 \text{ k}\Omega$ and the transfer function of the power stage is

$$G_f(s) = \frac{1}{(10^{-6}s + 1)(10^{-7}s + 1)}$$



Problem 4: The fixed element of a unity-gain feedback system is

$$G(s) = \frac{K}{s(s^2 + 0.1s + 1.0025)}$$

To improve the low frequency behavior of this system, a lag network is added to $G(s)$ with $\alpha = 5$, so that the loop transfer function now becomes

$$L(s) = \frac{1 + Ts}{1 + \alpha Ts} G(s)$$

- Find T and K so that the system has at least 30° of phase margin.
- If we increased α without changing any other parameters of the system, what effect would that have? Support your argument with the appropriate root-locus plots.

Problem 5: The measured frequency response of a distributed system exhibits the behavior of the following transfer function:

$$G(\omega) = (-1 - j) \frac{e^{-0.001j\omega}}{\omega^{1.5}}$$

Compensate this system with a gain and a lead network with α no greater than 10. Maximize crossover frequency and maintain a phase margin of 45° . (Do not use MatLab).

Problem 6: Consider the following loop transfer function:

$$L(s) = G_C(s) \frac{100}{(s+1)(0.1s+1)(0.01s+1)}$$

What is the crossover frequency, phase margin, and steady state error to a step of the open loop system: $G_C(s) = 1$?

For each of the following compensation choices, compensate the system for 45 degrees of phase margin and maximize the crossover frequency. In each case, what are the necessary compensator parameters, what is the new crossover frequency, and what is the steady state error to a step?

(a) Reduced DC gain compensation:

$$G_C(s) = \frac{1}{K_R}$$

(b) Dominant pole compensation:

$$G_C(s) = \frac{K_D}{s}$$

(c) Lag compensation:

$$G_C(s) = \frac{\tau s + 1}{\alpha \tau s + 1}$$

(d) Lead compensation (use an α no larger than 10):

$$G_C(s) = K_L \frac{\alpha \tau s + 1}{\tau s + 1}$$