

6.302 Feedback Systems

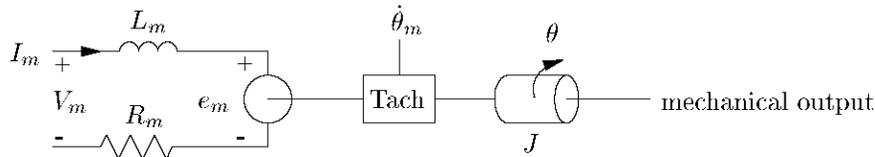
Spring Term 2007
 Lab 1A

Issued : February 13, 2007
 Due : Friday, February 23, 2007

Lab group size is restricted to no more than 2 people. You will hand in individual lab writeups. In your writeup, please indicate your lab station number as well as the name of your lab partner. Remember that labs will not be accepted for credit without a completed prelab, and everything must be turned in on time.

Background

The purpose of this lab is to specify a mathematical model for a physical system. The physical system explored in this lab is a simple servomechanism. The model developed in this lab will be used to design a variety of feedback control loops in subsequent labs (Labs 1B, 1C, and 1D). The electrical model employed to describe the motor is the following:



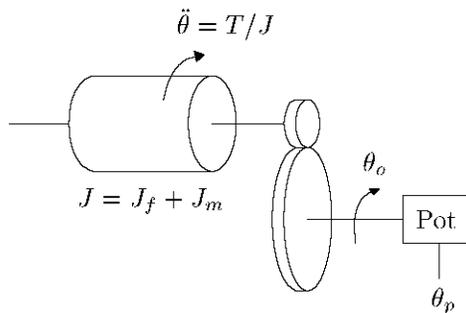
where the back voltage (or back emf) of the spinning motor is

$$e_m = K_e \dot{\theta}$$

and the output voltage of the tachometer is

$$\dot{\theta}_m = K_{tach} \dot{\theta}$$

The mechanical output of the servomechanism is modeled by:



where the output torque of the motor is

$$T = K_t I_m$$

the position of the output shaft is

$$\theta_o = \theta/n$$

and the output voltage of the position sensor is

$$\theta_p = K_p \theta_o$$

Note that the motor shaft angle is θ , and the output shaft angle is θ_o . The flywheel, with inertia J_f , is mounted directly on the motor shaft. The output shaft is geared to the motor shaft with a gear ratio n . The potentiometer is connected to the output shaft, and the voltage across the potentiometer is θ_p .

This lab consists of determining values for the model parameters R_m , L_m , K_e , K_{tach} , J_m , J_f , n , K_t , and K_p .

Equipment

Pick up 4 BNC connectors at the desk. Note that these are 1x connectors, unlike the 10x scope probes; be sure to check the scope input setting.

Setup

For this lab we will only be concerned with the **Power Amp** and **Monitor** sections of the control box. When you arrive at the lab area, check to see that the control box is connected to the ± 15 volt power supply and that the motor cable is connected to the control box output. Make sure that you record the number of the motor station which you are using, as you will be using this station for all four of the servomechanism laboratories.

Connect a signal generator output to the input of the **Power Amp** section. For the voltage-drive measurements, set the **Power Amp** gain select switch to the “ -2 V/V ” setting so that you can drive the motor with a voltage. For the current-drive measurements, be sure to change the switch to the “ -0.4 A/V ” setting.

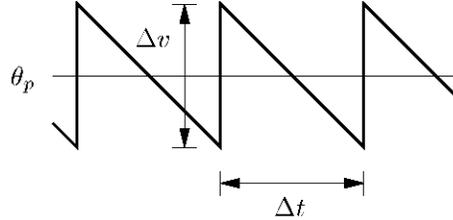
Triggering

For DC input waveforms (from the signal generator), you should trigger off of the channel you are recording. Use the “auto level” trigger mode for the DC outputs, and the “normal” trigger mode for time-varying outputs. For square wave and sine wave inputs from the waveform generator, it may be useful to connect the “TTL OUT” signal on the waveform generator to channel 3 or 4 on the scope, and trigger off of that channel. Again, use the “normal” trigger mode. If your display is blank or intermittent, you may need adjust the trigger level.

Measurements: Part I

Part I of the lab is done at one of the standard motor lab stations that have a flywheel. These stations will be the subject of all subsequent labs. Make sure that you record the number of the motor station which you are using, as you should use the same station for all four of the servomechanism laboratories.

1. Drive the motor with a constant (DC) voltage. Make sure the **Power Amp** gain select switch is in the “-2 V/V” setting. A DC voltage can be obtained from the output of the signal generator by making sure that none of the wave shape buttons are pushed in. To vary this DC voltage, twiddle the “DC Offset” knob. Make sure that the 0 dB button is pushed in. The θ_p should look like this:



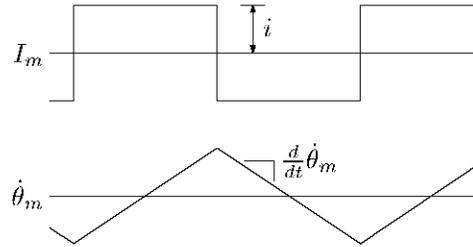
Note that the θ_p waveform is discontinuous — the monitor voltage “wraps around” when the output shaft passes through an angle of $\pm\pi$ radians (the measured voltage θ_p is continuous for output shaft angles in the range $-\pi < \theta_o < +\pi$). Measure the size of the discontinuity.

2. Drive the motor with a constant (DC) voltage. Measure and record the monitor values V_m , I_m , and $\dot{\theta}_m$. Also measure the output shaft speed by recording Δt , the time between wraps, in the θ_p waveform. Repeat this measurement for five different voltages in the range $0 \text{ V} \leq V_m \leq 10 \text{ V}$. To what does the time between the wraps correspond?

Note that all of the monitor outputs are voltages. A reading of 0.5 V on the I_m output corresponds to a motor current of 0.5 A, since the scale factor for the I_m monitor is 1 V/A.

Recall that the monitor voltage $\dot{\theta}_m$ is related to the motor shaft speed $\dot{\theta}$ by the scale factor K_{tach} . As part of this lab you will determine the value of K_{tach} ; note that it has units of V-sec/rad.

3. Drive the motor with a square wave of current (with no DC offset). Remember to switch the **Power Amp** gain setting to the “-0.4 A/V” setting. The motor current and angular velocity should look something like the following picture:

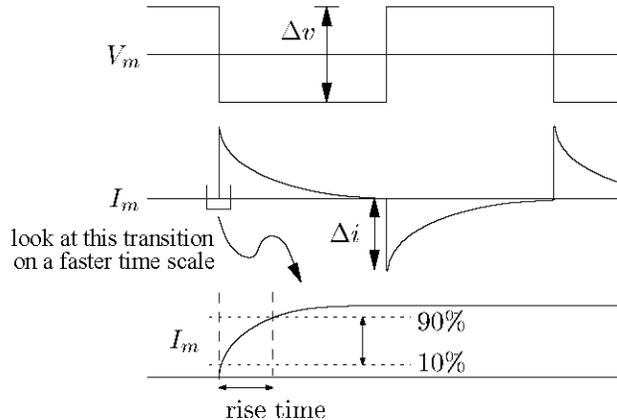


Measure and record the values of I_m and $\frac{d}{dt}\dot{\theta}_m$. Be sure that you are not saturating the amplifier when making these measurements (I_m should be a clean square wave). Drive the system fast enough that the $\dot{\theta}_m$ waveform is made up of relatively straight line segments. Repeat this for five different values in the range $0 \text{ A} \leq i \leq 1.5 \text{ A}$.

Measurements: Part II

Part II requires that you use a motor setup which does not include the flywheel. Go over to a setup in the 6.302 area that is labeled “NO FLYWHEEL” and take the following measurements at that station.

1. Drive the motor with a relatively slow square wave of voltage (with no DC offset). Make sure the **Power Amp** gain select switch is in the “ -2 V/V ” setting. The motor voltage and current waveforms should look something like the following artist’s conception:



Measure and record the values of Δi and Δv . Also measure the 10%–90% risetime of the motor current waveform. Note that it is easier to measure this risetime by switching to a much faster time-base setting on your oscilloscope. Since this waveform is quite noisy, you will probably want to select the *average* display mode by pressing the **DISPLAY** button on the scope. Make sure that you are measuring the rise time, as shown in the figure, and not the fall time.

Repeat this measurement for five different Δv values in the range $0 \text{ V} \leq \Delta v \leq 10 \text{ V}$.

2. Drive the motor with a square wave of voltage and measure the 10%–90% risetime of the motor velocity monitor, $\dot{\theta}_m$. Repeat this for five different voltages.
3. Drive the motor with a square wave of current (with no DC offset). Remember to switch the **Power Amp** gain setting to the “ -0.4 A/V ” setting. Repeat the measurement from Part I-3.

Data Reduction

We will use a linear-least-squared error approximation technique to reduce the measurement data and obtain the desired model parameter values. In a perfect world (like maybe on an exam) this technique wouldn't be necessary; a single set of measurements in the lab would sufficiently and uniquely determine the parameter values. However, measurement errors and modeling deficiencies limit the accuracy of parameter calculation in the real world. To improve our accuracy, several measurements are "averaged" to yield the desired model parameters.

K_{tach} calculation: Ideally, $\dot{\theta}K_{tach} = \dot{\theta}_m$. We can obtain several measurements of θ_o from the time between wraparounds in the θ_p waveform (corresponding to an output shaft rotation θ_o equal to 2π). Be sure to convert $\dot{\theta}_o$ to $\dot{\theta}$ by taking the gear ratio into account. We also have the corresponding tachometer monitor voltage measurements of $\dot{\theta}_m$. Thus, in vector form, we can write:

$$\begin{pmatrix} \dot{\theta}_1 \\ \vdots \\ \dot{\theta}_5 \end{pmatrix} K_{tach} = \begin{pmatrix} \dot{\theta}_{m1} \\ \vdots \\ \dot{\theta}_{m5} \end{pmatrix}$$
$$\vec{\dot{\theta}}K_{tach} = \vec{\dot{\theta}}_m$$

Determine the best-fit value of K_{tach} by minimization of the squared error, a la

$$K_{tach} = \frac{\vec{\dot{\theta}}^T \vec{\dot{\theta}}_m}{\vec{\dot{\theta}}^T \vec{\dot{\theta}}}$$

R_m calculation: Once again in an ideal world, $R_m \Delta i = \Delta v$. From Part II-1 we have several measurements of Δi , along with the corresponding Δv values; calculate the best-fit value for R_m using the least squares estimation technique as was used for K_{tach} .

L_m calculation: The time constant of an exponential response is equal to the 10%–90% risetime divided by a factor of 2.2. Average the five risetime measurements and divide by 2.2 to obtain the average time constant, which corresponds to the L_m/R_m time constant in the motor model. The value of L_m is calculated by multiplying the average time constant by the best-fit R_m value.

K_e calculation: $\dot{\theta}K_e$ should equal $(V_m - I_m R_m)$. We have obtained several measurements of $\dot{\theta}$ for the K_{tach} calculation above. We also have the corresponding V_m and I_m measurements, and the best-fit value of R_m . Once again, use the LLSE to determine the best-fit value for K_e .

Gear Ratio (n) calculation: Counting gear teeth isn't terribly interesting or educational, so we have done this for you. The gear on the motor shaft has 32 teeth, and the output shaft gear has 216 teeth. Thus the value of n is 6.75.

K_p calculation: The discontinuity at an output shaft angle of $\pm\pi$ radians corresponds to a jump of 2π radians. Thus the value of K_p is $\Delta v/2\pi$.

J_f calculation: The flywheel inertia J_f is calculated from basic physics (you may need to find your 8.01 book). The flywheel is a cylinder made of aluminum, which has a density of 2.7 g/cm³. The thickness of the flywheel is 6.35 mm, and its diameter is 63.5 mm.

K_t and J_m calculation: Consider first the data set taken in Part II-3 without the extra flywheel inertia. Note that the inertias of the motor armature, the gears, the potentiometer, etc., are all lumped together into a single effective inertia which we call the 'motor inertia', J_m . Once again considering the ideal scenario, $K_t i = J_m \frac{d}{dt} \dot{\theta}$. We have several measurements of $\frac{d}{dt} \dot{\theta}_m$ (which can be converted to $\frac{d}{dt} \dot{\theta}$ values through division by the model parameter K_{tach}), along with the corresponding i values. Calculate the best-fit value for the ratio K_t/J_m using the LLSE technique.

Repeat the above procedure for the data set taken in Part I-3 with the flywheel inertia J_f on the motor shaft. Note that now you are solving for the ratio $K_t/(J_m + J_f)$. We now have an expression for K_t/J_m and an expression for $K_t/(J_m + J_f)$. Since the value for J_f is known, we have two equations in two unknowns. Solve these equations for K_t and J_m .

τ_m **calculation:** Average the five risetime measurements for $\dot{\theta}_m$ from Part II-2 and divide by 2.2 to determine the motor time constant τ_m . Explain why this calculation gives you the motor time constant.

Questions:

1. In the prelab, you found an expression for the motor time constant. How well does the measured value agree with a calculated value based upon your model parameters? (Remember that the flywheel was *removed* for these measurements.)
2. Two of your motor parameters should be the same. Which two? Why? Are they?
3. How does the inertia of the flywheel compare to that of the big gear? Why?

Just to give you an idea...

Here are some order of magnitude approximations for the parameters you will be concerned with:

$$\begin{aligned}R_m &\sim 10^0 \\L_m &\sim 10^{-3} \\K_e &\sim 10^{-2} \\K_{latch} &\sim 10^{-2} \\J_f &\sim 10^{-5} \\J_m &\sim 10^{-6} \\K_t &\sim 10^{-2} \\K_p &\sim 10^0\end{aligned}$$

These are listed in MKS units (which you determined in the prelab...)

Write Up

The write up for the lab should be short, simple, and informal. All we require is your raw measured data, your reduced parameter values (in a neat table on the first page), and any brief insights or conclusions which you feel are relevant. Individual write ups are required from each student. Please document which motor setup you used, who your lab partner was, and who did what.

Be sure to make a copy of your motor parameters from this lab to use in calculations for the subsequent motor labs. You'll thank yourself later.