

# 6.302 Feedback Systems

Recitation 26: Final Recitation and Review

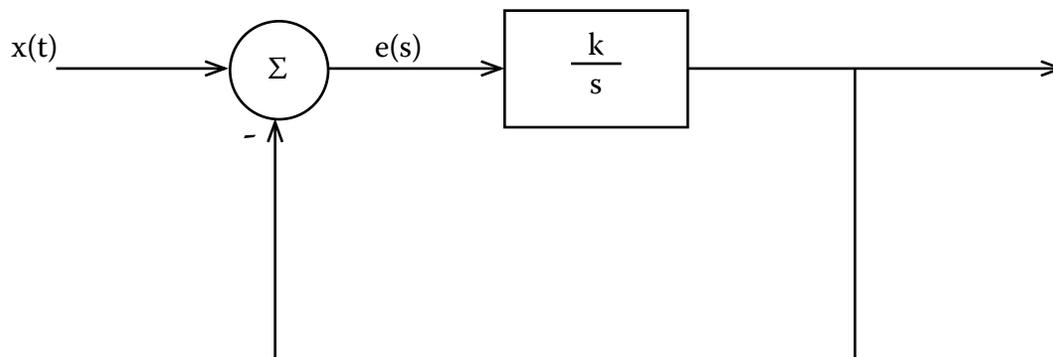
Prof. Joel L. Dawson

We're going to spend some time in this recitation revisiting some of the very early examples of recitation #1.

To aid us in this review, the final class exercise involves exploring one more aspect of the behavior of feedback systems.

## CLASS EXERCISE

Consider the following simple feedback system:



Suppose we drive this system with a unit step. By now, we all know (or knew earlier in the course) at the end of time,  $e(t) \rightarrow 0$ . But we typically can't wait that long...

Derive an expression that tells us how long we need to wait for the error to get below a certain value. In response to a unit step, how long must we wait until the error gets down to  $e_0$ ? Express your answer in terms of  $k$  and  $e_0$ .

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This problem illustrates a general truth: the more precision that we want out of our feedback system, the longer we'll have to wait for it.

We saw an example of this type of behavior in the very first class. Remember filling the water jar?



Figure by MIT OpenCourseWare.

Open-Loop: Thoroughly characterize the relevant physical characteristics such as flow rate, detailed geometry of jar and pitcher, etc. Then, looking only at a stopwatch, you pour according to your calculations.

We decided this was fraught with accuracy problems. But because we don't have to rely on our reflexes to tell us when to stop pouring, this type of operation could be very fast. Imagine a fire hose plus control valve setup...

So here, as in control systems, open-loop operation lets us be faster in general.

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What about closed-loop pouring?

Closed-Loop: What we always do. We pour water, constantly monitoring our progress, until we reach our goal, then we stop.

This type of filling has close parallels to feedback systems. Among them:

- 1) Feedback is provided by your eyes.
- 2) Dynamics introduced by your reflexes and hand-eye coordination.
- 3) You start off pouring fast, and slow as you get near the desired level.

↳ Step response of class exercise is  $\frac{k}{s(s+k)}$ . If we look at the time derivative of this response at  $t = 0$  and  $t = \infty$ :

$$\underline{t = 0^+}: \lim_{s \rightarrow \infty} s \left[ \cancel{s} \cdot \frac{k}{\cancel{s}(s+k)} \right] = k$$

$$\underline{t = \infty}: \lim_{s \rightarrow 0} s \left[ \cancel{s} \cdot \frac{k}{\cancel{s}(s+k)} \right] = 0$$

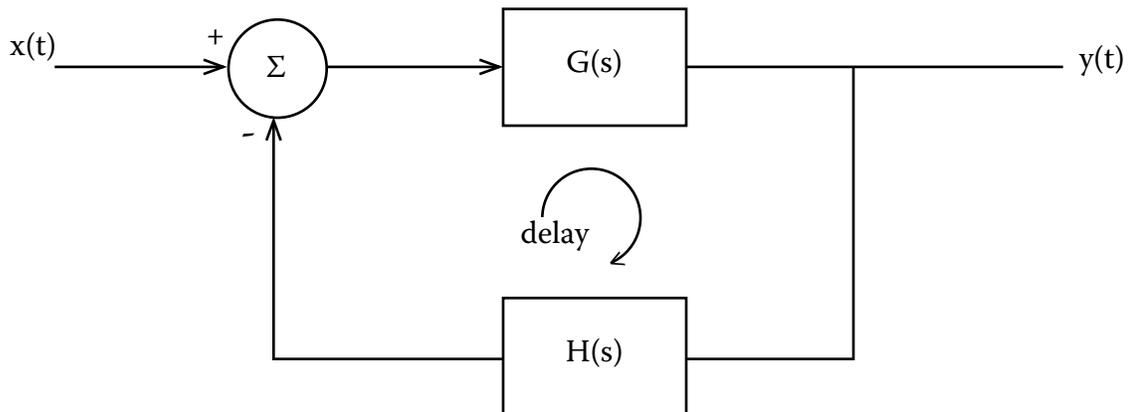
- 4) The more accurate you want to be, the longer it takes. (Drip, drip as you get close.)

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We also talked in that first recitation about delay in feedback systems.



Since there is always delay around the loop, we aren't really comparing  $x$  and  $y$  at truly corresponding times.

Question: Doesn't this complicate things? And if so, how?

Answer: Yes. See 6.302.

Almost everything we've been doing is related to this question of characterization delay. To see this, recall that delay of a time-domain function is expressed:

$$h(t) \rightarrow h(t - T_D)$$

Now look at how the loop transmission effects an incoming sinusoid:

$$\begin{aligned} L(j\omega)e^{j\omega t} &= |L(j\omega)|e^{-j\phi(\omega)}e^{j\omega t} \\ &= |L(j\omega)|e^{j\omega t - j\phi(\omega)} \\ &= |L(j\omega)|e^{j\omega(t - \frac{\phi(\omega)}{\omega})} \end{aligned}$$

↑  
Frequency dependent delay.

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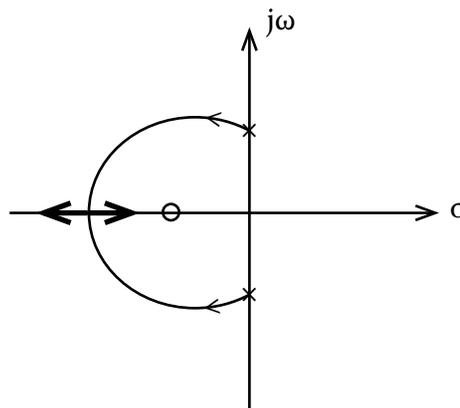
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Over the course of this semester, we have looked at this frequency-dependent delay from many angles.

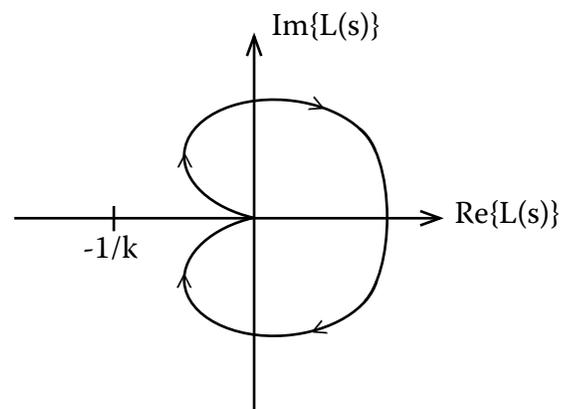
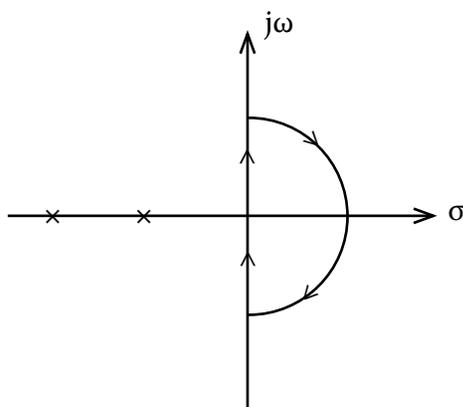
## 1) Root Locus Rules

In satisfying the angle condition,  $\angle L(s) = -180^\circ$ , we used the phase response to show us where the closed-loop poles were going to move.



## 2) Nyquist Plots

It was the phase response that made all those encirclements possible.



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3) Nichols Chart

Plotted  $L(s)$  on a gain-phase plane. Can't get anywhere near the top of Mt. Nichols without a phase shift.

4) Bode Plots

Gain and phase margin have clear connection to phase response.

FINAL EXAMPLE:

Car with delay built into steering column. How would you drive a car like this? Answer: You would drive so slowly that the delay of the steering would become insignificant. This, conceptually, is how we stabilize feedback systems.