

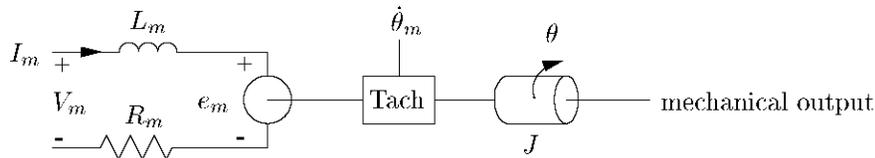
6.302 Feedback Systems

Spring Term 2007
 Prelab 1A

Issued : February 6, 2007
 Due : Thursday, February 15, 2007

Background

The purpose of this lab is to specify a mathematical model for a physical system. The physical system explored in this lab is a simple servo-mechanical apparatus. The model developed in this lab will be used to design a variety of feedback control loops in subsequent labs (Labs 1B, 1C, and 1D). The electrical model employed to describe the motor is the following:



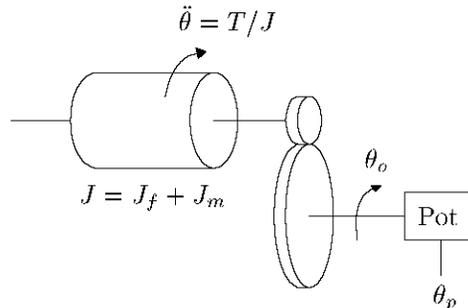
where the back voltage (or back emf) of the spinning motor is

$$e_m = K_e \dot{\theta}$$

and the output voltage of the tachometer is

$$\dot{\theta}_m = K_{tach} \dot{\theta}$$

The mechanical output of the servo-mechanism is modeled by:



where the output torque of the motor is

$$T = K_t I_m$$

the position of the output shaft is

$$\theta_o = \theta/n$$

and the output voltage of the position sensor is

$$\theta_p = K_p \theta_o$$

Note that the motor shaft angle is θ , and the output shaft angle is θ_o . The flywheel, with inertia J_f , is mounted directly on the motor shaft. The output shaft is geared to the motor shaft with a gear ratio n . The potentiometer is connected to the output shaft, and the voltage across the potentiometer is θ_p .

Lab 1A will consist of determining values for the model parameters R_m , L_m , K_e , K_{tach} , J_m , J_f , n , K_t , and K_p .

Prelab Exercises

The following questions concern the modeling of the motor which will be used as part of this servo-mechanism lab.

1. Draw a block diagram for the motor, with V_m as the input and $\dot{\theta}_m$ and θ as the outputs of the system. Clearly label the blocks as well as the following intermediate variables: I_m , T , e_m . (Hint: K_e should appear in the feedback path.)
2. Derive the following transfer functions: $\frac{\dot{\Theta}}{I_m}$, $\frac{\dot{\Theta}}{V_m}$, and $\frac{\Theta}{V_m}$.
3. Consider the addition of a damping term B , which represents the viscous damping component of the load (proportional to velocity). Draw a new block diagram for the motor which appropriately includes the damping term, and give the corresponding expressions for $\dot{\Theta}/I_m$ and $\dot{\Theta}/V_m$.
4. What change does damping cause in the $\dot{\Theta}/I_m$ transfer function? What is the significance of this change?
5. What is the electrical time constant for a circuit with an inductor L_m and a resistor R_m ?
6. Assuming that the electrical time constant is much faster (smaller) than the mechanical time constant, simplify the transfer function $\dot{\Theta}/V_m$ (without damping) to that of a first order system. What is the time constant of this transfer function?
7. Draw a block diagram for the mechanical output to the motor. Label clearly θ , θ_o , and θ_p . (Ignore the viscous damping that was modelled in Question 3.)
8. $\dot{\theta}_m$ and θ_p are electrical quantities (in volts) which you will be able to measure on the oscilloscope. Given this, please indicate the units (using standard SI units) for all of the following system parameters. (Hint: K_t , K_e , and K_{tach} are not unitless!)
 $R_m, L_m, I_m, V_m, \theta_p, \dot{\theta}_m, K_{tach}, K_e, K_t, K_p, T, J_m, J_F, B, n, \theta, \theta_o$.