

# 6.302 Feedback Systems

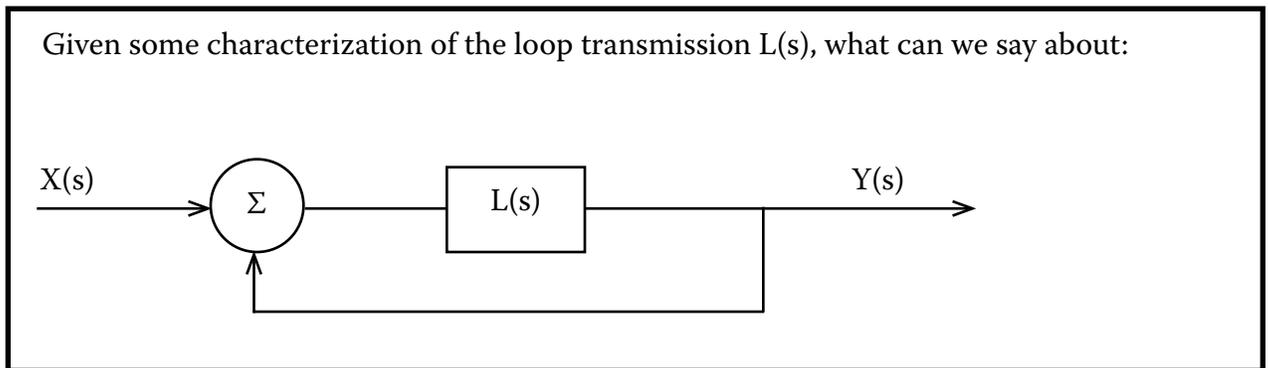
Recitation 13: Our Analytical Toolbox

Prof. Joel L. Dawson

We've covered a number of different analytical techniques in this class. What does it all mean? What are we doing all this for? Let's review the BIG PICTURE.

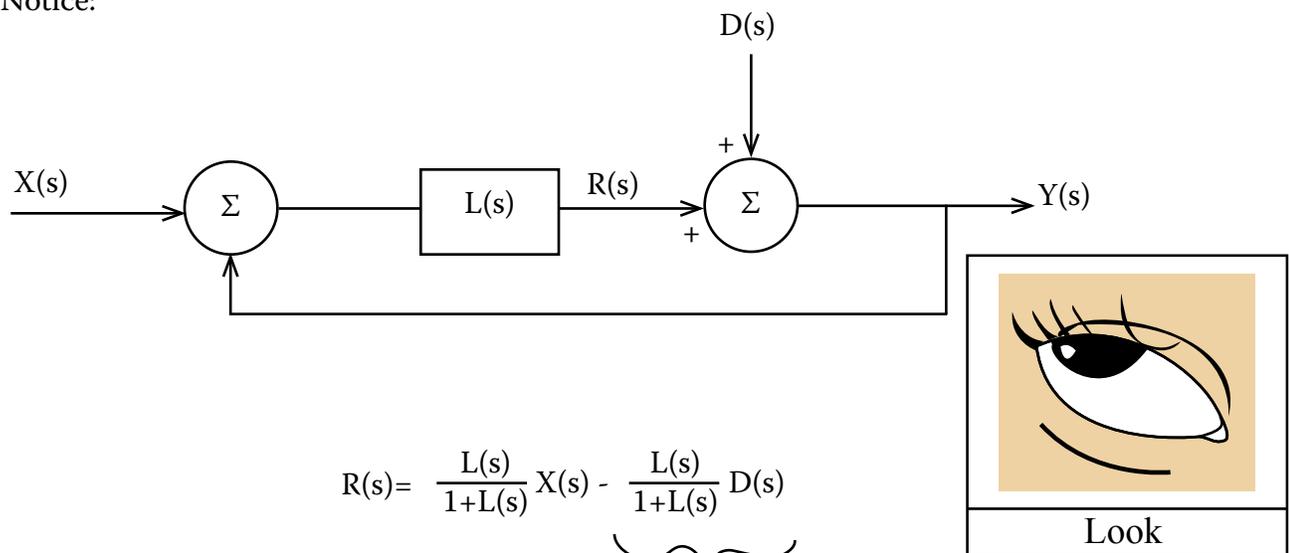
In some ways, they all exist to answer the following question:

Given some characterization of the loop transmission  $L(s)$ , what can we say about:



Remember, the reason this simple block diagram is of interest is because feedback is a very powerful design strategy. Because we “look” at the output, and “adjust” until we get as close as possible, the system is robust to variations in the forward path.

Notice:



$$R(s) = \frac{L(s)}{1+L(s)} X(s) - \underbrace{\frac{L(s)}{1+L(s)} D(s)}_{\text{“cancellation” signal!!}}$$

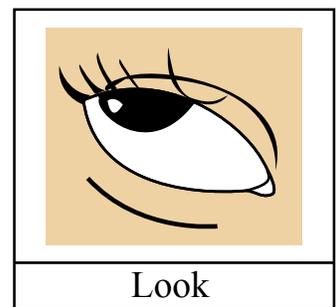


Figure by MIT OpenCourseWare.

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So what are our tools? Let's review:

Black's Formula:

$$\frac{Y(s)}{X(s)} = \frac{L(s)}{1 + L(s)}$$

Our first result, and in theory all we need. In practice, though:

- Sometimes exact transfer function  $L(s)$  is not known, or only available as measured frequency domain data.

Routh Criterion

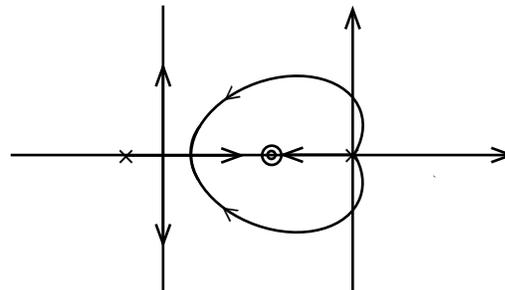
$1 + L(s) = P(s)$ , a polynomial in  $s$ .

Based on the coefficients of the various powers of  $s$ , we can determine if all of the roots of  $P(s)$  are in the left half plane.

- (see above)
- Also, this doesn't tell us where the poles of  $\frac{L(s)}{1 + L(s)}$  are, except for which half plane they are in.

Root Locus

$$L(s) = \frac{(s + 1)^2}{s^3(s + 10)}$$



Knowing the open-loop poles and zeros, we can rapidly sketch how the closed-loop poles will move as we vary the gain.

Conditional stability is immediately apparent.

- Numerical results awkward to obtain
- Can't handle delay,  $L(s)$ s that are not ratios of polynomials.

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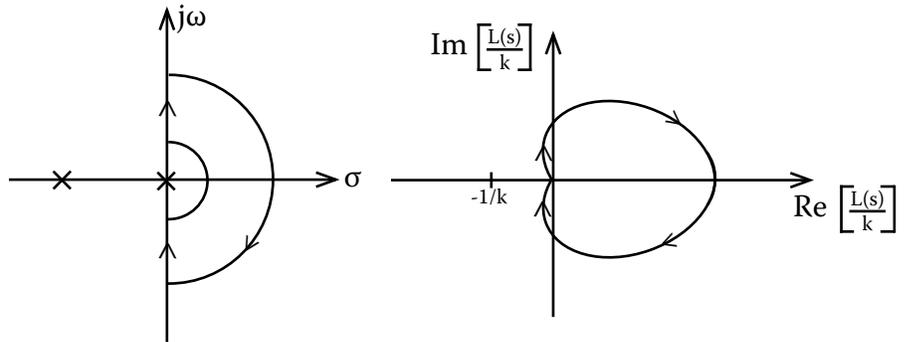
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## Nyquist Criterion

$$L(s) = \frac{1}{s(s+1)}$$

$$Z = N + P$$



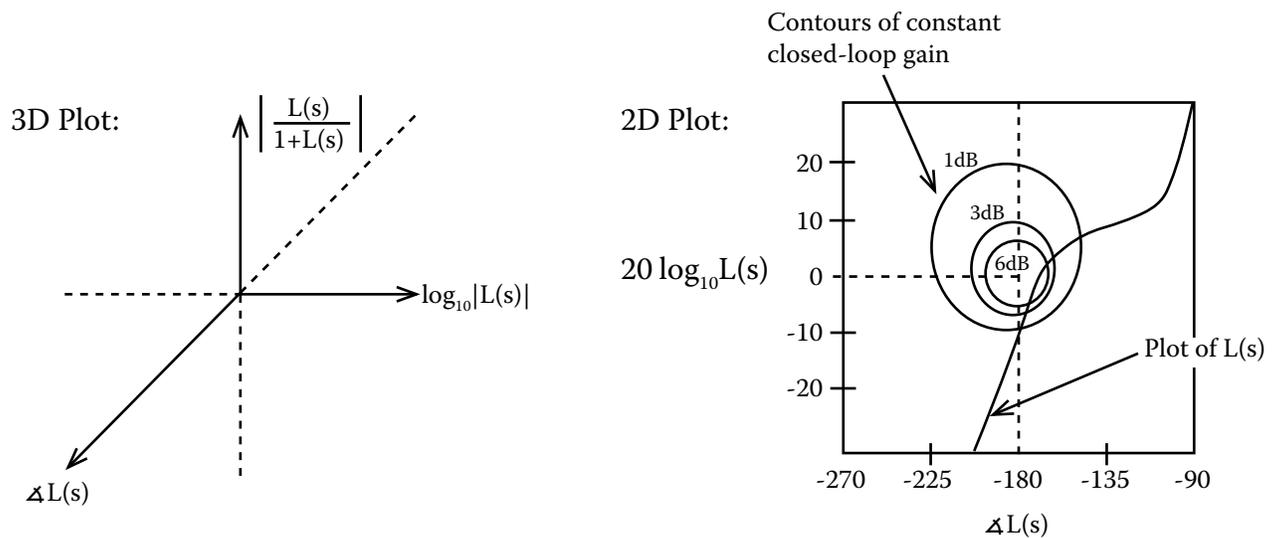
General. Definitive. Not always easy to plot, but we can use a Bode Plot as a guide to help us.

## Hall Chart

(We kind of skip this one. See Nichols Chart.)

## Nichols Chart

Graphical technique to help us determine  $M_p$ .



Notice that as we get near  $|L(s)| = 1$ ,  $\Delta L(s) = -180^\circ$ , we find ourselves on the peak of “Mount Nichols.”

## Gain and Phase Margin

Easy. We hope we can always use these. Actually, we hope we need only one (phase margin).