MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.302 Feedback Systems

Fall Term 2005 — Issued : `9:00 am`
Final Exam — Due : `12:00 pm`

Final Exam
December 21, 2005
180 minutes

1. This examination consists of four problems. Work all problems.
2. This examination is closed book. Calculators are allowed. Helpful equations and the root-locus rules appear at the end of this packet.
3. You MUST summarize your solutions in the answer sheets included in this packet. Draw all sketches neatly and clearly where requested. Remember to label ALL important features of any sketches.
4. Make sure that your name is on each answer sheet and on each examination booklet.
5. All problems have equal weight.

We encourage you to do the work for all of the problems in the answer sheets as well. If you find that the answer sheets do not contain enough space for your scratch work, you may do additional work in the accompanying examination booklet. Make sure that you clearly denote which problem is on each page of the examination booklet. Your examination booklet will also be read by the graders, but only if your answers appear on the answer sheets.

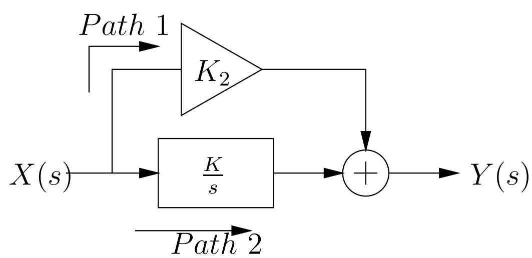
Good luck.

Problem 1

Feedforward Compensation

We've talked about a number of compensation techniques for this class: lead, lag, minor loop, etc. But there is at least one other intriguing possibility, which is called feedforward compensation.

(a) Consider the following block diagram:

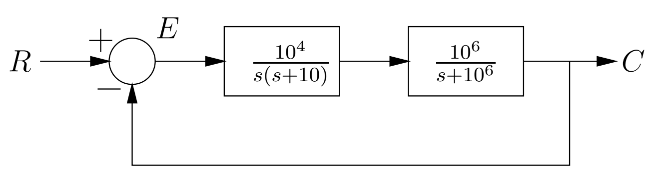


(i) Derive an expression for a new type of crossover frequency, ω_{fc}, for which the magnitude of the Path 1 gain equals the magnitude of the Path 2 gain.

(ii) Derive the transfer function $\frac{Y(s)}{X(s)}$. How does the location of the zero relate to ω_{fc}?

(iii) Write down approximations for $\frac{Y(s)}{X(s)}$ for the regions: $\omega >> \omega_{fc}$ and $\omega << \omega_{fc}$.

Now suppose we have a feedback system that can be modeled as follows:

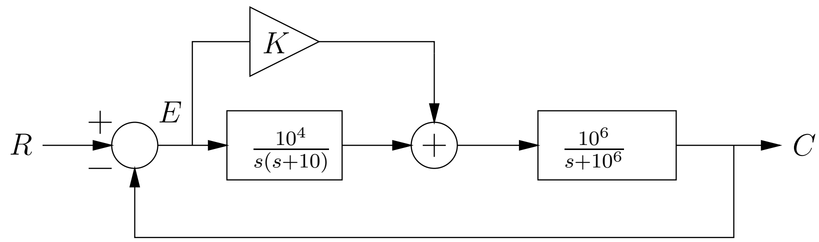


(b) Show, by whatever method you choose, that

(i) The system is very nearly unstable.

(ii) The steady state error in response to a step is zero.

(c) Now, add



(i) Choose a K such that the PM of this feedback loop is 90°.

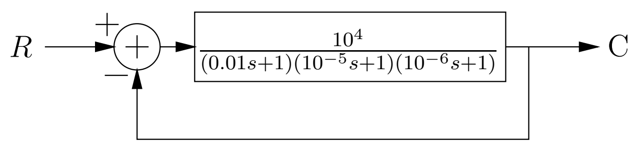
(ii) Show that the steady state error in response to a step is still zero.

Problem 2

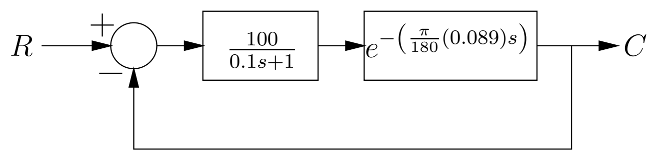
Bode Variety Pack

Important: Using <u>straight-line</u> approximations to construct Bode plots, find the crossover frequency(rad/s), phase margin(degrees), and gain margin of the following systems and indicate whether or not the overall closed-loop system $\frac{C}{R}(s)$ is stable(yes/no).

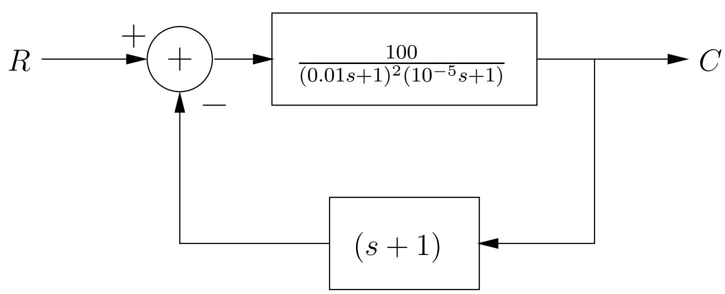
(a)



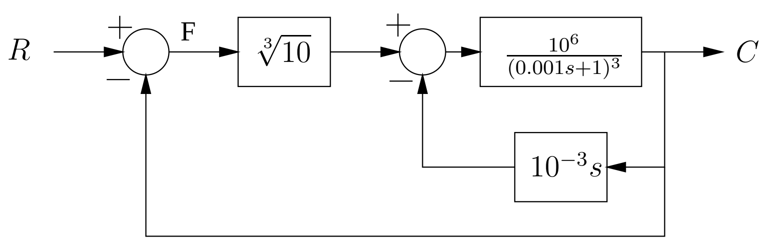
(b)



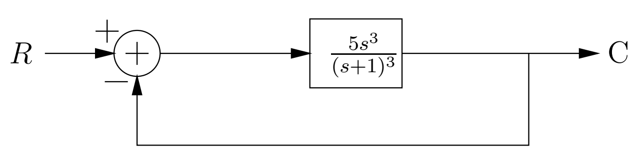
(c)



(d)



(e)



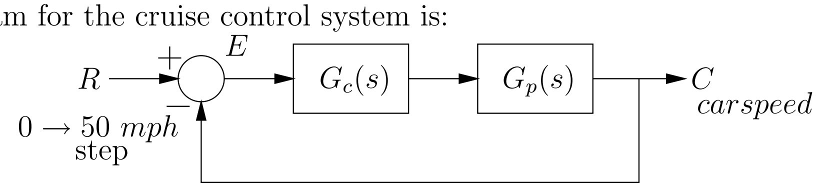
Does the stability of this system change if the feedback path gain is changed from unity to 2? Explain.

Problem 3

Speed Control

You are designing a feedback controller for the cruise control for the cruise control for the cruise control in your car. The speed limit is 50 mph, but you can only tolerate going 1mph under as you have an interview to catch. The police will give a leeway of about 10 mph over the speed lmit as long as its for a short while.

A block diagram for the cruise control system is:



The plant for the cruise control is:

$$G_p(s) = \frac{10}{(10s+1)^2(0.1s+1)} \tag{1}$$

(a) What specifications for the steady state error and peak-overshoot are required for the system?

(b) Does the system meet specifications with $G_c = 1$? Do you make it to the interview on time? Will you get pulled over? Explain.

(c) Design a lag compensator of the form:

$$G_c(s) = \frac{K_1(\tau_1 s+1)}{(10\tau_1 s+1)} \tag{2}$$

Choose K and τ_1 so that you will get to your interview on time and the crossover frequency remains as in part b, with approximately 5° less phase margin.

(d) Now add a lead compensator to your compensator of part c, so that the final compensator is:

$$G_c'(s) = \frac{K_1(\tau_1 s+1)}{(10\tau_1 s+1)} \frac{K_2(10\tau_2 s+1)}{(\tau_2 s+1)} \tag{3}$$

Find values for K_2 and τ_2 such that the system crossover frequency remains the same as in part c and phase margin is increased by approximately 40°.

Do not use an asymptotic approximation to the Bode plot for this part of the problem.

(e) Does the compensation of part d save you from being pulled over? Do you still get to your interview on time? Explain.

Problem 4

Describing Functions

(a) The input-output relationship for six frequency-independent nonlinearities are shown below, along with eight possible describing functions. Select the describing function that is associated with each of the nonlinearities. Also determine the values of the quantities C_1 and C_2 indicated on each selected describing function. It may help to recall that the fundamental component of a $\pm V$ square wave is $\frac{4V}{\pi}$.

(b) A negative feedback loop includes a linear portion combined with a nonlinearity. The angle of the linear transfer function is shown. This transfer function is minimum phase with a monotnoic decreasing gain vs. frequency and has a DC gain of K. If the nonlinearity is A, can the system exhibit a stable amplitude oscillation (for some range of values of K) at ω_1? At ω_2? At ω_3?

The non linearity is changed to C. Can this system exhibit stable amplitude oscillations at these frequencies?