

Speech Signal Representation

- Fourier Analysis
 - Discrete-time Fourier transform
 - Short-time Fourier transform
 - Discrete Fourier transform
- Cepstral Analysis
 - The complex cepstrum and the cepstrum
 - Computational considerations
 - Cepstral analysis of speech
 - Applications to speech recognition
 - Mel-Frequency cepstral representation
- Performance Comparison of Various Representations

Discrete-Time Fourier Transform

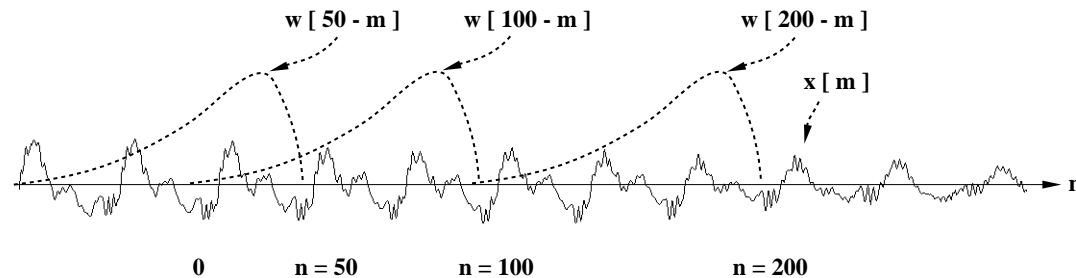
$$\begin{cases} X(e^{j\omega}) &= \sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n} \\ x[n] &= \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})e^{j\omega n} d\omega \end{cases}$$

- Sufficient condition for convergence: $\sum_{n=-\infty}^{+\infty} |x[n]| < +\infty$
- Although $x[n]$ is discrete, $X(e^{j\omega})$ is continuous and periodic with period 2π .
- Convolution/multiplication duality:

$$\begin{cases} y[n] &= x[n] * h[n] \\ Y(e^{j\omega}) &= X(e^{j\omega})H(e^{j\omega}) \end{cases}$$

$$\begin{cases} y[n] &= x[n]w[n] \\ Y(e^{j\omega}) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} W(e^{j\theta})X(e^{j(\omega-\theta)})d\theta \end{cases}$$

Short-Time Fourier Analysis (Time-Dependent Fourier Transform)



$$X_n(e^{j\omega}) = \sum_{m=-\infty}^{+\infty} w[n-m]x[m]e^{-j\omega m}$$

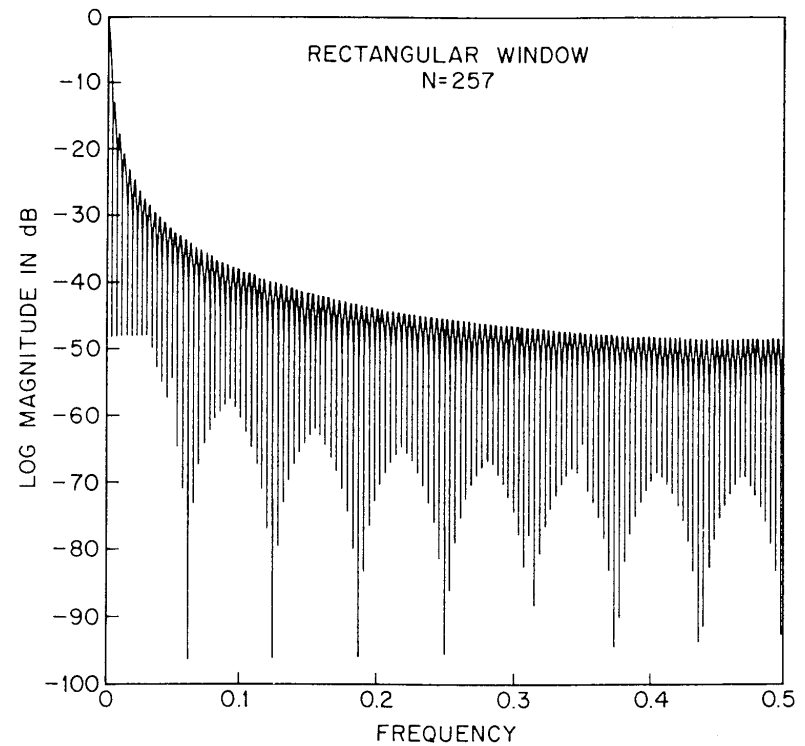
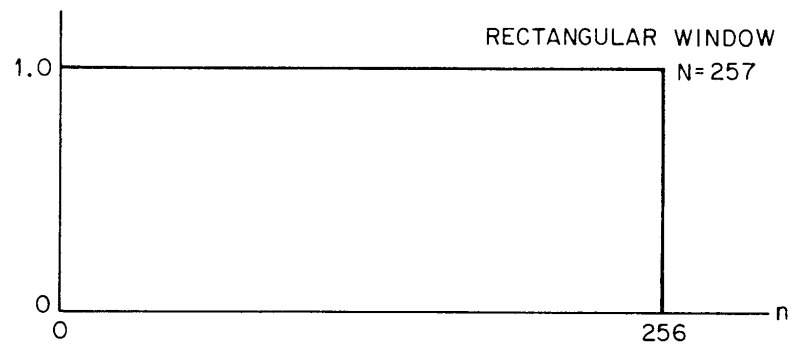
- If n is fixed, then it can be shown that:

$$X_n(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} W(e^{j\theta})e^{j\theta n}X(e^{j(\omega+\theta)})d\theta$$

- The above equation is meaningful only if we assume that $X(e^{j\omega})$ represents the Fourier transform of a signal whose properties continue outside the window, or simply that the signal is zero outside the window.
- In order for $X_n(e^{j\omega})$ to correspond to $X(e^{j\omega})$, $W(e^{j\omega})$ must resemble an impulse with respect to $X(e^{j\omega})$.

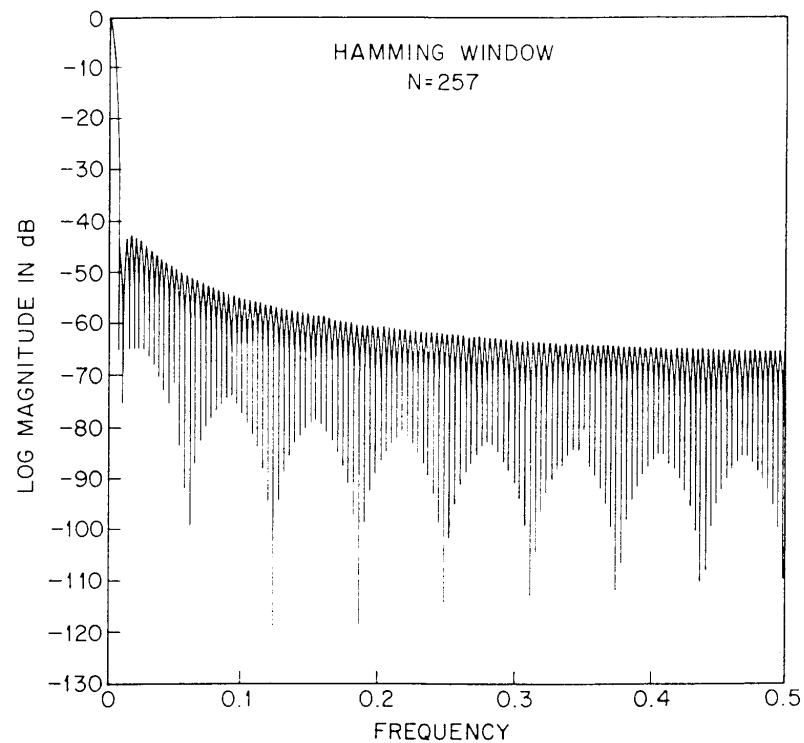
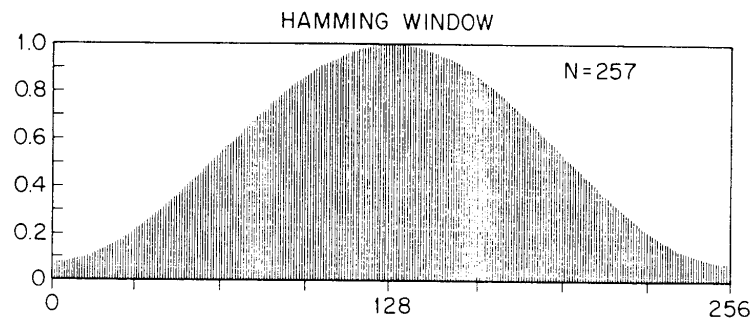
Rectangular Window

$$w[n] = 1, \quad 0 \leq n \leq N - 1$$



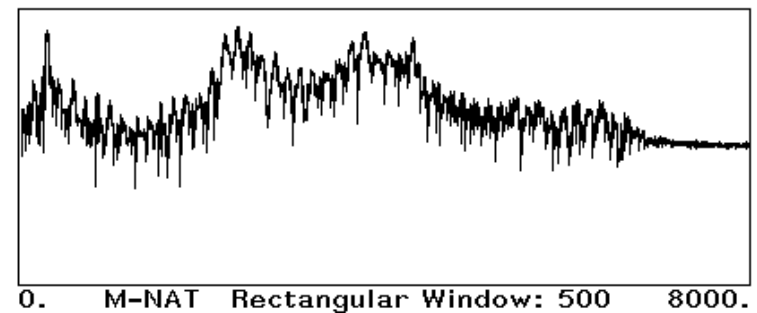
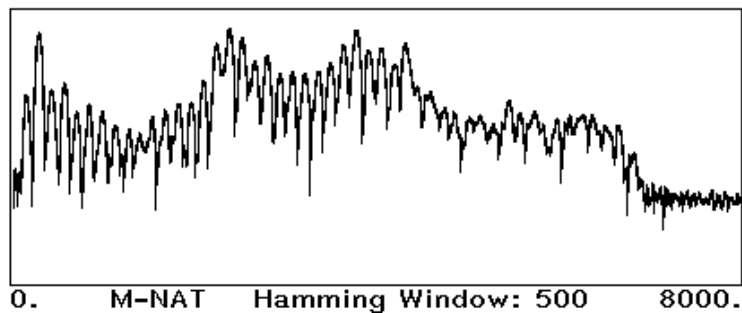
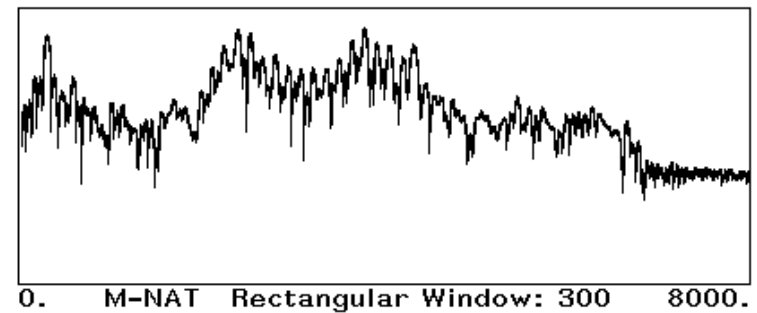
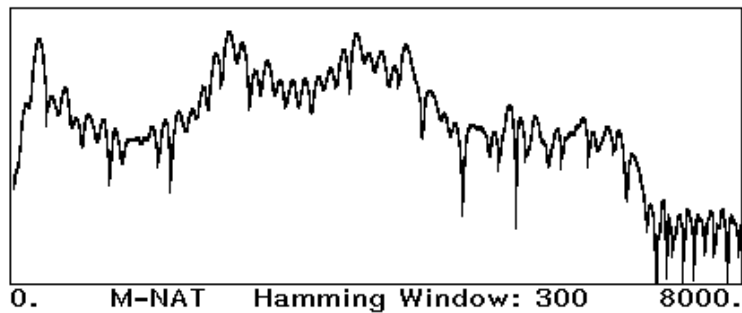
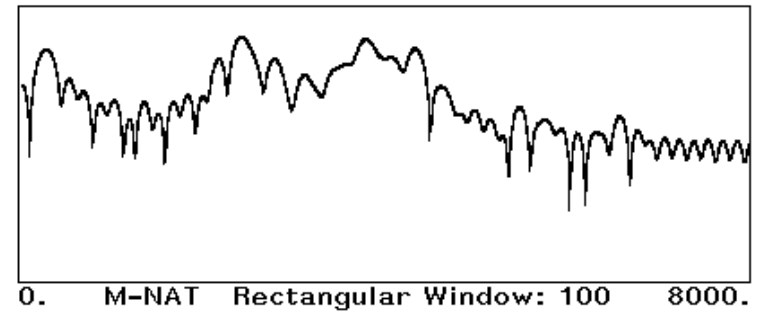
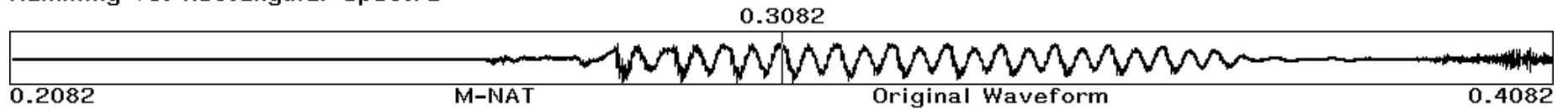
Hamming Window

$$w[n] = 0.54 - 0.46 \cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \leq n \leq N-1$$

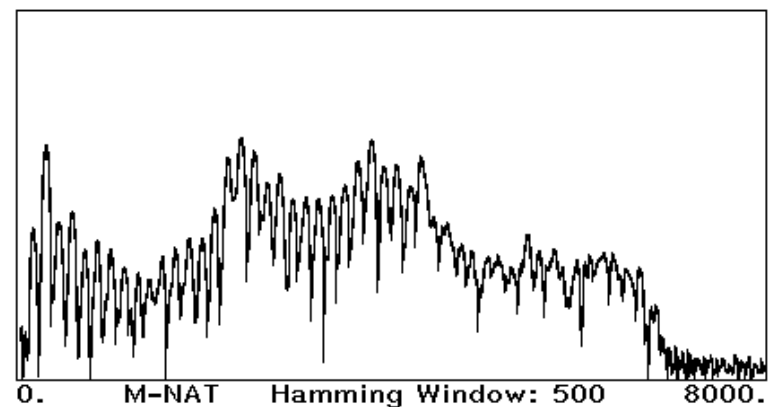
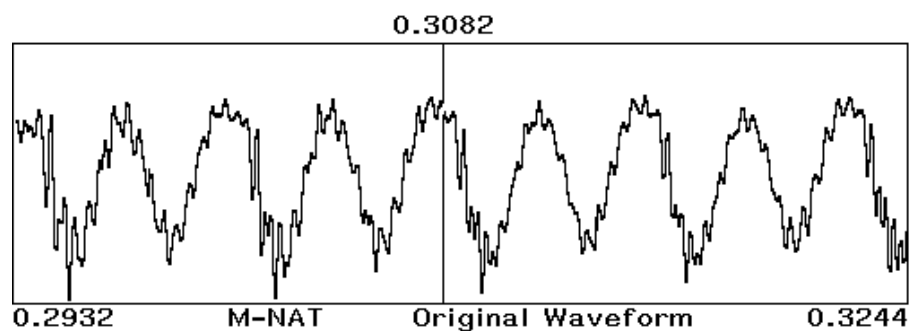
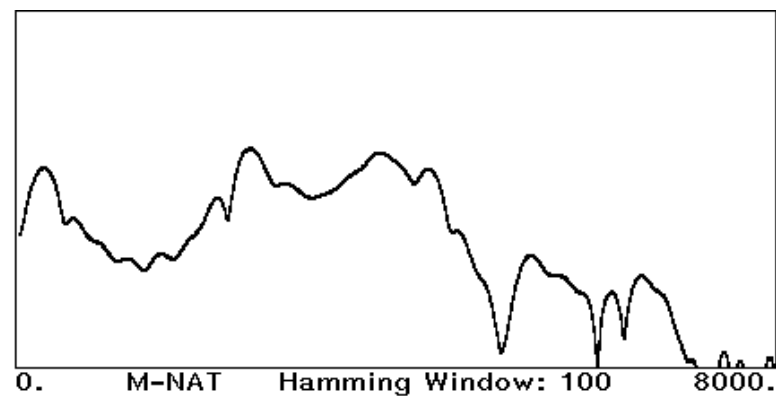
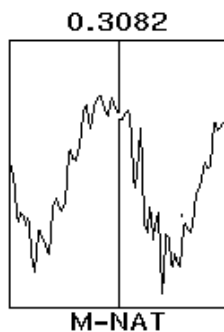


MIT Comparison of Windows

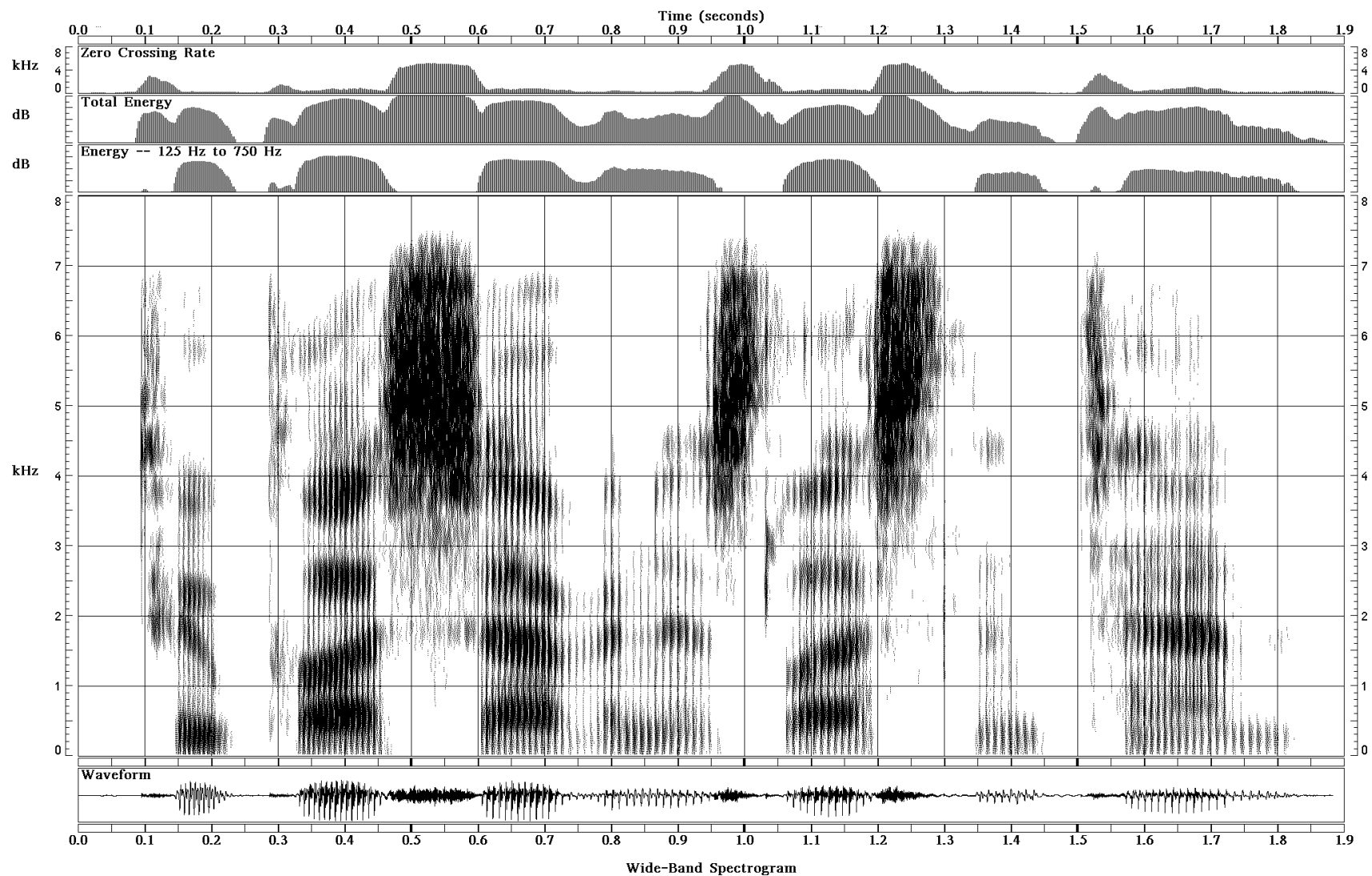
Hamming Vs. Rectangular Spectra



Comparison of Windows (cont'd)

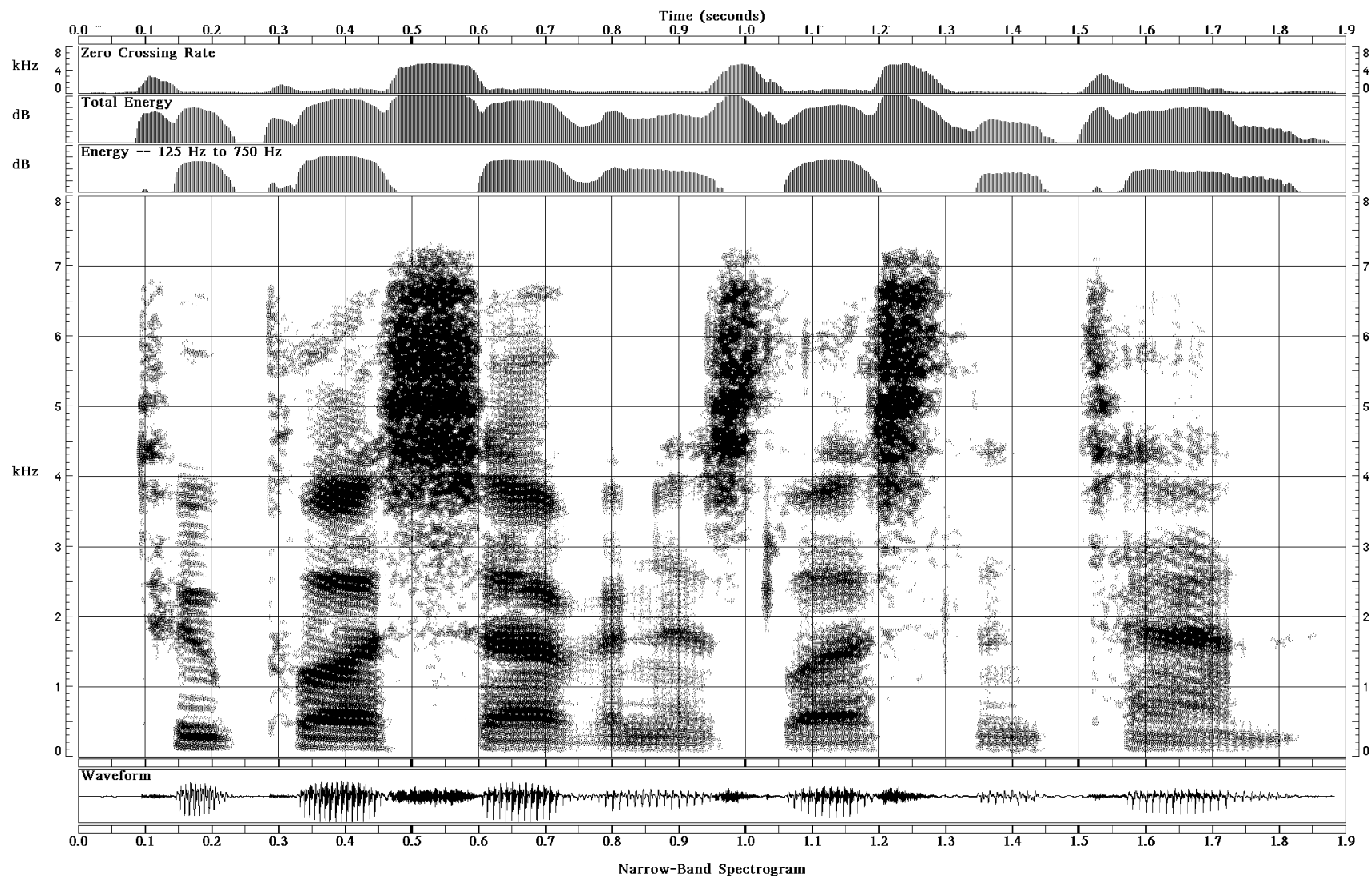


MIT A Wideband Spectrogram



Two plus seven is less than ten

A Narrowband Spectrogram



Two plus seven is less than ten

Discrete Fourier Transform

$$x[n] \iff X[k] = X(z) \Big|_{z=e^{j\frac{2\pi k}{M}n}}$$

N points

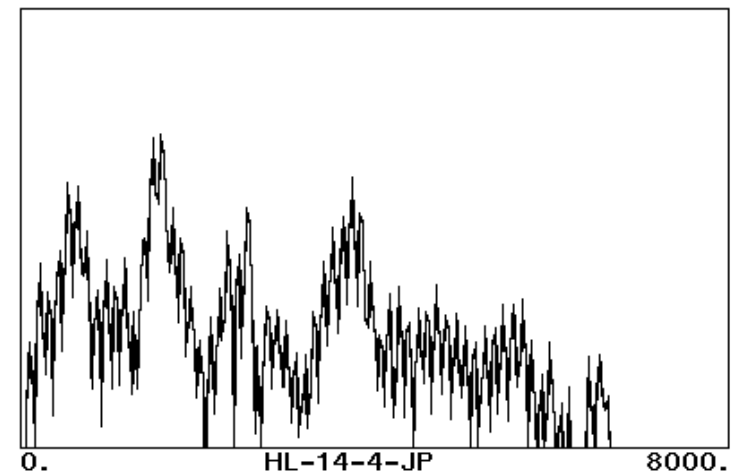
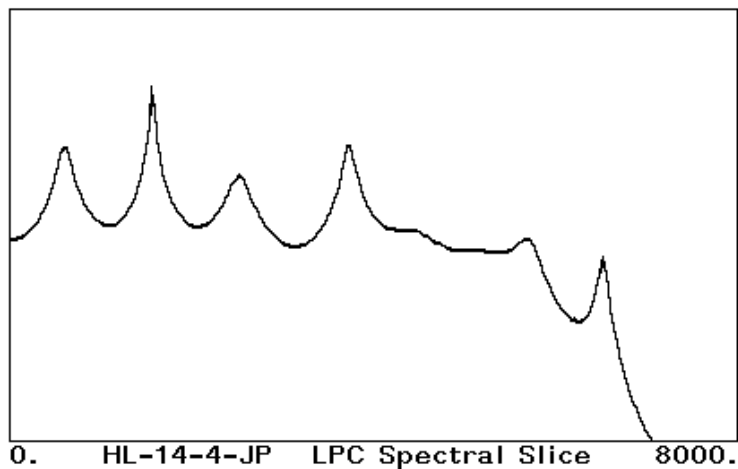
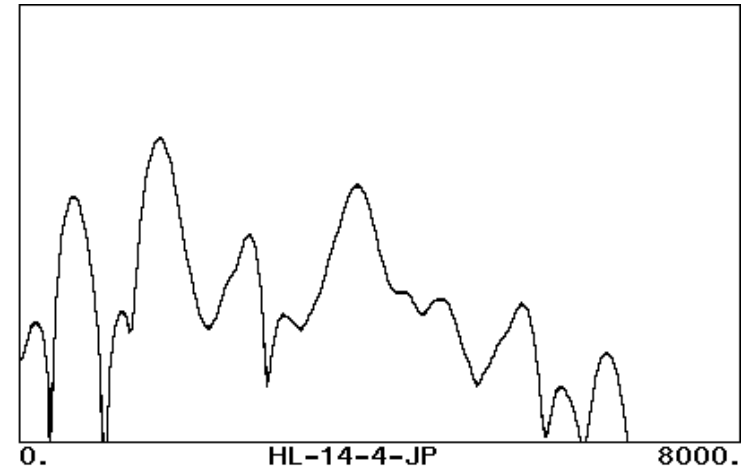
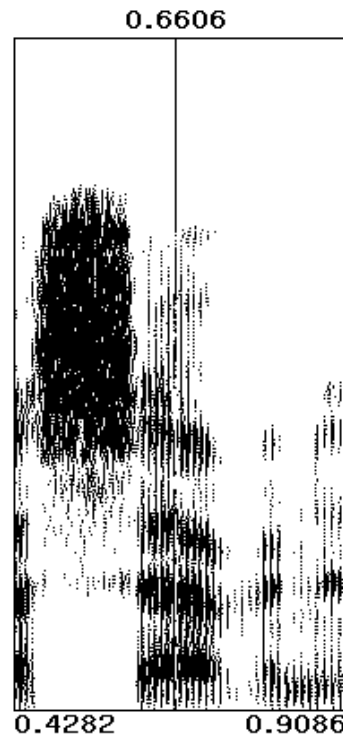
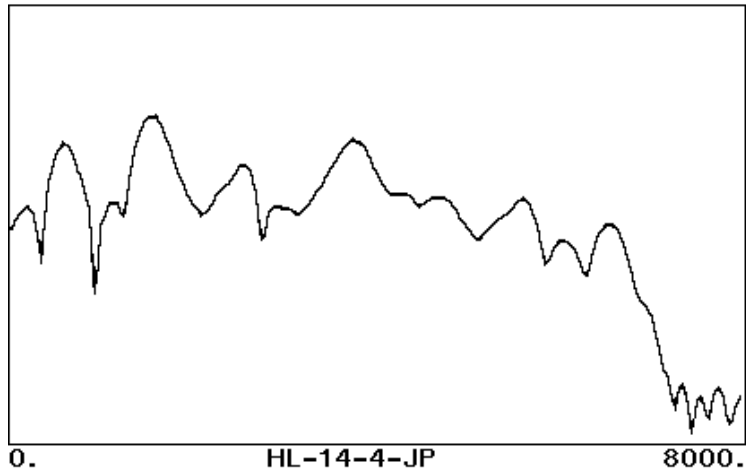
M points

$$\begin{cases} X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi k}{M}n} \\ x[n] = \frac{1}{M} \sum_{k=0}^{M-1} X[k] e^{j\frac{2\pi k}{M}n} \end{cases}$$

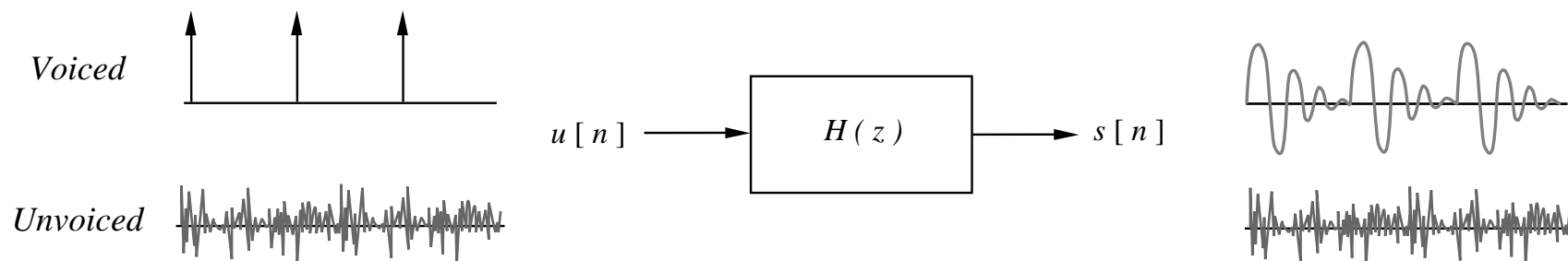
In general, the number of input points, N , and the number of frequency samples, M , need not be the same.

- If $M > N$, we must zero-pad the signal
- If $M < N$, we must time-alias the signal

Examples of Various Spectral Representations



Cepstral Analysis of Speech



- The speech signal is often assumed to be the output of an LTI system; i.e., it is the convolution of the input and the impulse response.
- If we are interested in characterizing the signal in terms of the parameters of such a model, we must go through the process of de-convolution.
- Cepstral analysis is a common procedure used for such de-convolution.

- Cepstral analysis for convolution is based on the observation that:

$$x[n] = x_1[n] * x_2[n] \iff X(z) = X_1(z)X_2(z)$$

By taking the *complex* logarithm of $X(z)$, then

$$\log\{X(z)\} = \log\{X_1(z)\} + \log\{X_2(z)\} = \hat{X}(z)$$

- If the complex logarithm is unique, and if $\hat{X}(z)$ is a valid z-transform, then

$$\hat{x}(n) = \hat{x}_1(n) + \hat{x}_2(n)$$

The two convolved signals will be additive in this new, cepstral domain.

- If we restrict ourselves to the unit circle, $z = e^{j\omega}$, then:

$$\hat{X}(e^{j\omega}) = \log |X(e^{j\omega})| + j \arg\{X(e^{j\omega})\}$$

It can be shown that one approach to dealing with the problem of uniqueness is to require that $\arg\{X(e^{j\omega})\}$ be a continuous, odd, periodic function of ω .

Cepstral Analysis (cont'd)

- To the extent that $\hat{X}(z) = \log\{X(z)\}$ is valid,

$$\left\{ \begin{array}{l} \hat{x}[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \hat{X}(e^{j\omega}) e^{j\omega n} d\omega \\ \quad = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \log\{X(e^{j\omega})\} e^{j\omega n} d\omega \quad \text{complex cepstrum} \\ c[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} \log |X(e^{j\omega})| e^{j\omega n} d\omega \quad \text{cepstrum} \end{array} \right.$$

- It can easily be shown that $c[n]$ is the even part of $\hat{x}[n]$.
- If $\hat{x}[n]$ is real and causal, then $\hat{x}[n]$ be recovered from $c[n]$. This is known as the Minimum Phase condition.

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An Example

$$p[n] = \delta[n] + \alpha\delta[n - N] \quad 0 < \alpha < 1$$

$$P(z) = 1 + \alpha z^{-N}$$

$$\hat{P}(z) = \log [P(z)] = \log [1 + \alpha z^{-N}]$$

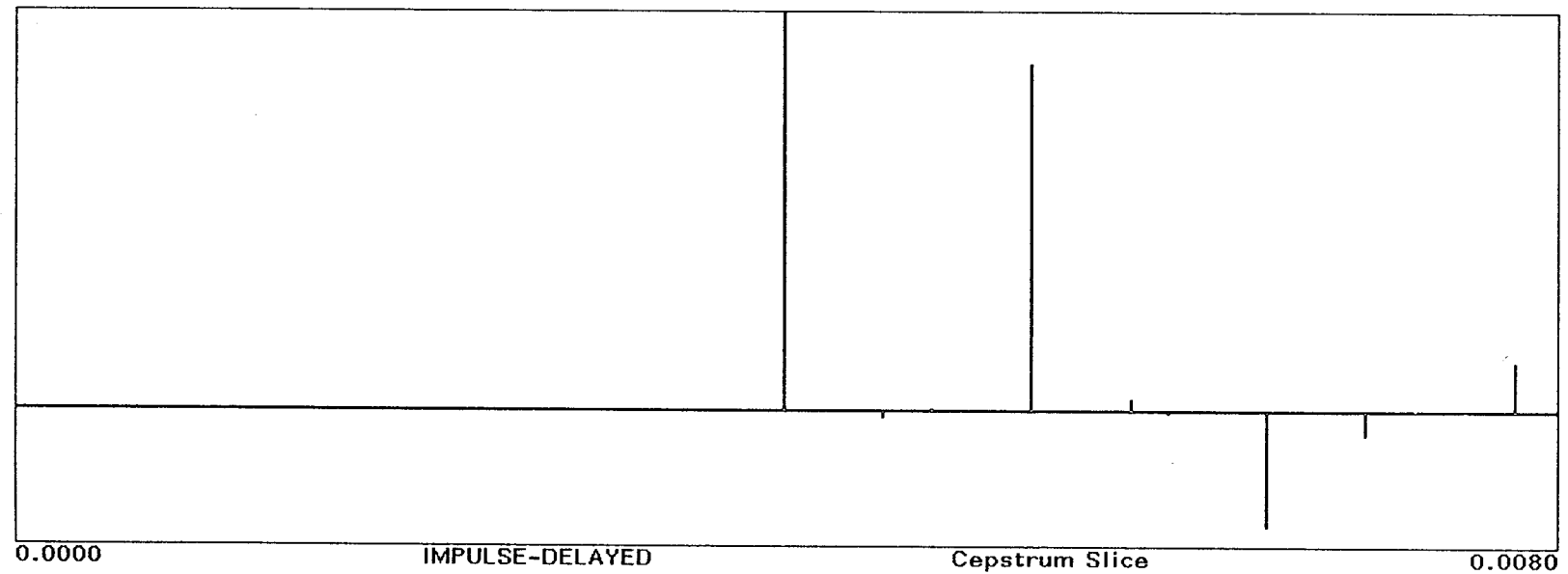
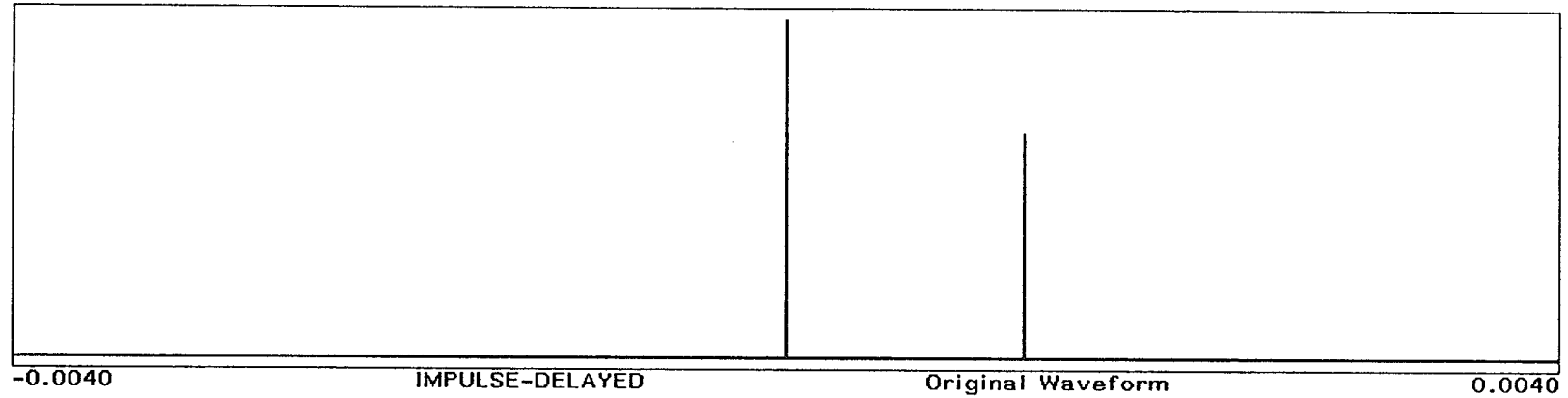
$$= \log [1 - (-\alpha)(z^N)^{-1}]$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\alpha^n}{n} z^{-nN}$$

$$\hat{P}(z) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{\alpha^n}{n} (z^N)^{-n}$$

$$\hat{p}[n] = \sum_{r=1}^{\infty} (-1)^{r+1} \frac{\alpha^r}{r} \delta[n - rN]$$

An Example (cont'd)



Computational Considerations

- We now replace the Fourier transform expressions by the discrete Fourier transform expressions :

$$\begin{cases} X_p[k] &= \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi}{N} kn} & 0 \leq k \leq N-1 \\ \hat{X}_p[k] &= \log\{X_p[k]\} & 0 \leq k \leq N-1 \\ \hat{x}_p[n] &= \frac{1}{N} \sum_{k=0}^{N-1} \hat{X}_p[k] e^{j \frac{2\pi}{N} kn} & 0 \leq n \leq N-1 \end{cases}$$

- $\hat{X}_p[k]$ is a sampled version of $\hat{X}(e^{j\omega})$. Therefore,

$$\hat{x}_p[n] = \sum_{r=-\infty}^{\infty} \hat{x}[n + rN]$$

- Likewise:

$$c_p[n] = \sum_{r=-\infty}^{\infty} c[n + rN]$$

where,

$$c_p[n] = \frac{1}{N} \sum_{k=0}^{N-1} \log |X_p[k]| e^{j \frac{2\pi}{N} kn} \quad 0 \leq n \leq N-1$$

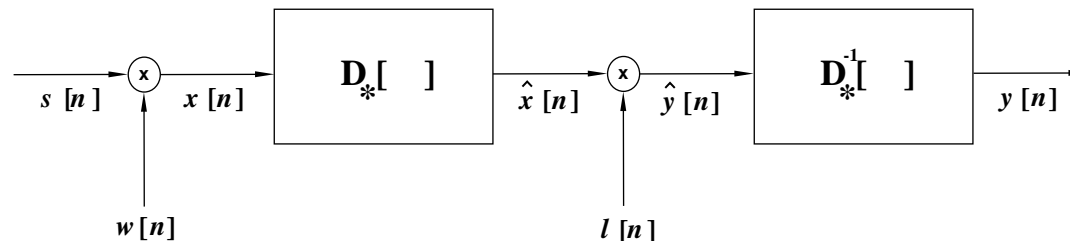
- To minimize aliasing, N must be large.

Cepstral Analysis of Speech

- For voiced speech:

$$s[n] = p[n] * g[n] * v[n] * r[n] = p[n] * h_v[n] = \sum_{r=-\infty}^{\infty} h_v[n - rN_p].$$

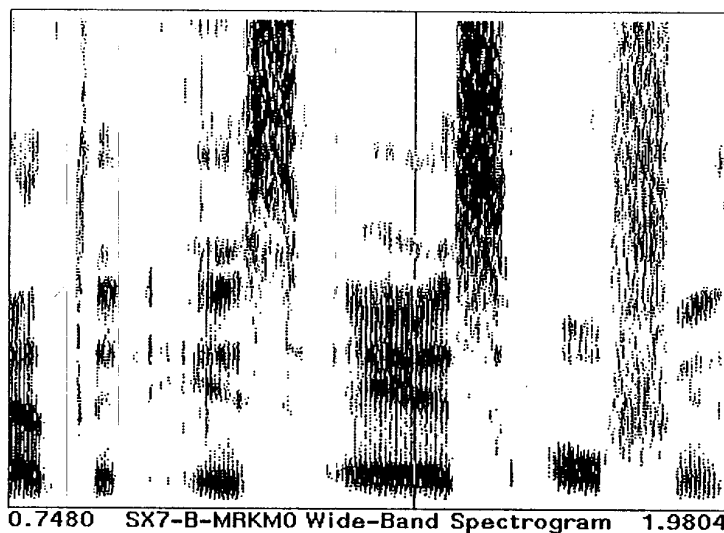
- For unvoiced speech: $s[n] = w[n] * v[n] * r[n] = w[n] * h_u[n]$.
- Contributions to the cepstrum due to periodic excitation will occur at integer multiples of the fundamental period.
- Contributions due to the glottal waveform (for voiced speech), vocal tract, and radiation will be concentrated in the low *quefrequency* region, and will decay rapidly with n .
- Deconvolution can be achieved by multiplying the cepstrum with an appropriate window, $l[n]$.



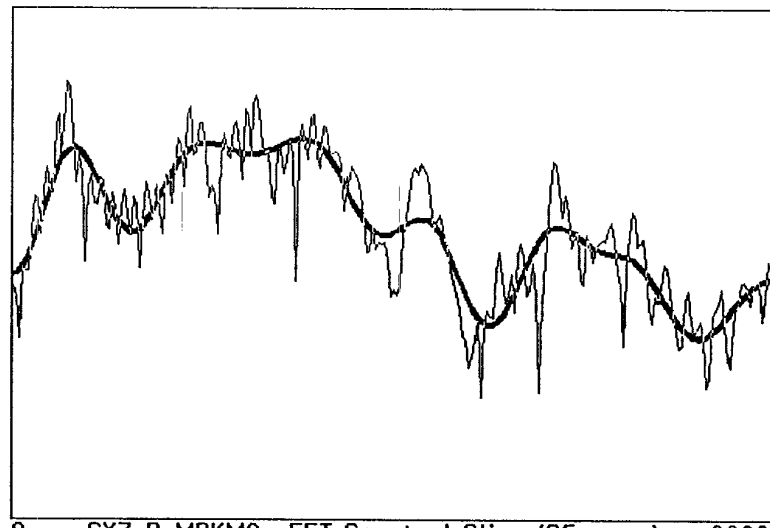
where D_* is the characteristic system that converts convolution into addition.

- Thus cepstral analysis can be used for pitch extraction and formant tracking.

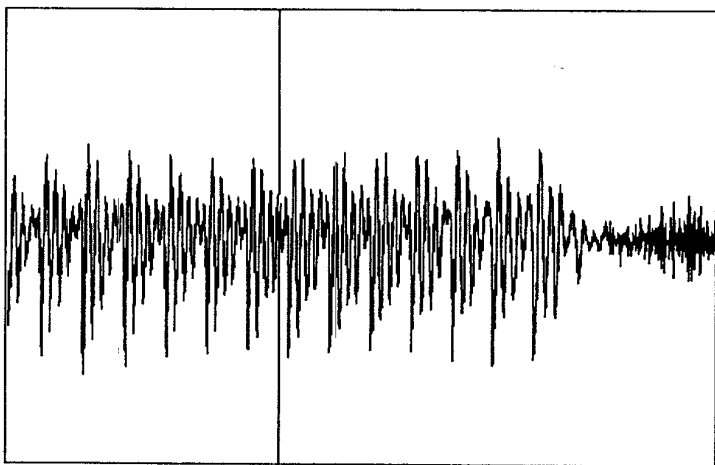
Example of Cepstral Analysis of Vowel (Rectangular Window)



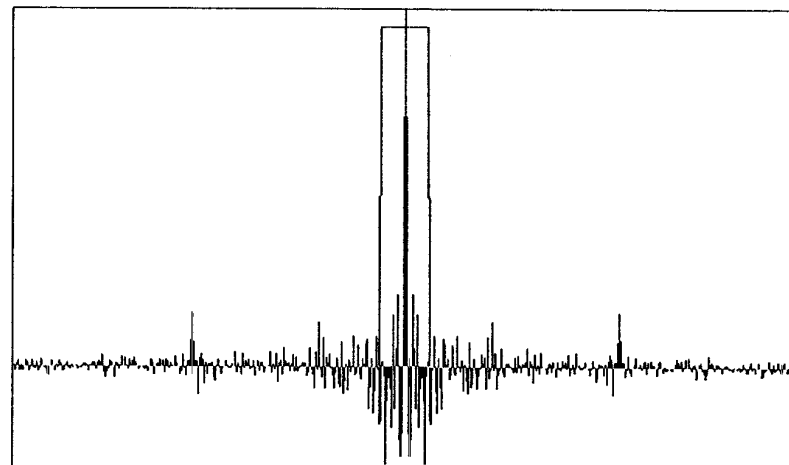
0.7480 SX7-B-MRKM0 Wide-Band Spectrogram 1.9804



0. SX7-B-MRKM0 FFT Spectral Slice (25 msec) 8000.

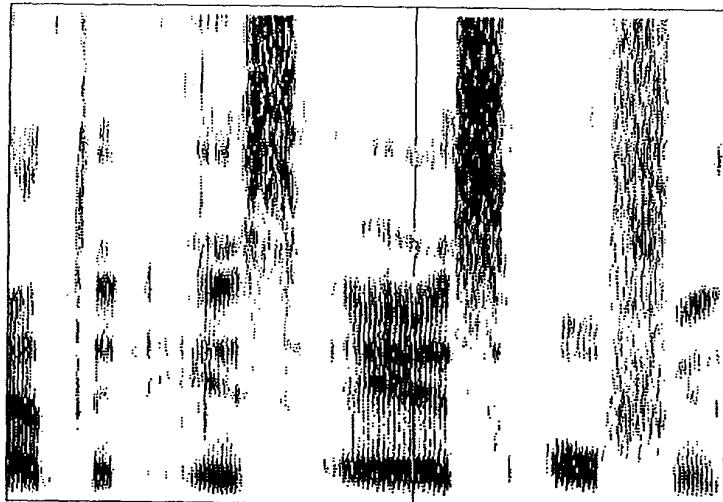


1.3850 SX7-B-MRKM0 Original Waveform 1.5350

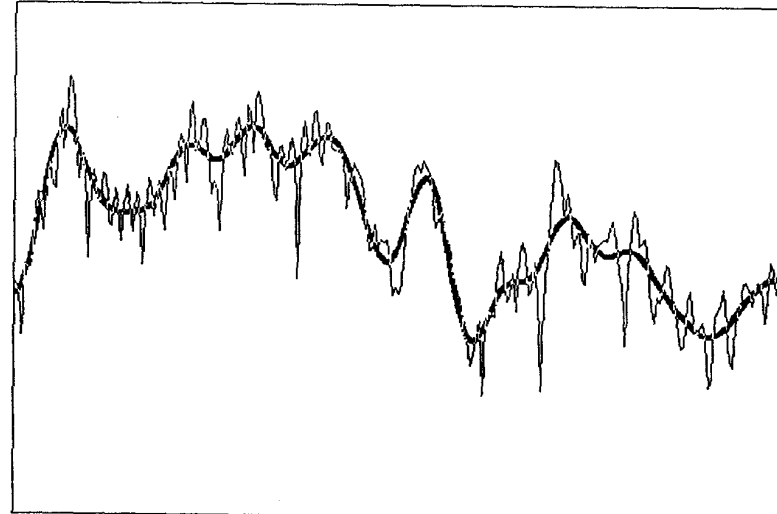


0.0000 SX7-B-MRKM0 Cepstrum Slice 0.0320

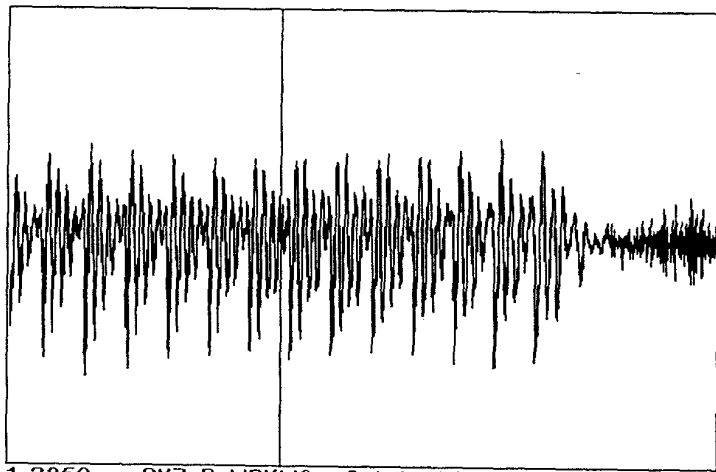
Example of Cepstral Analysis of Vowel (Tapering Window)



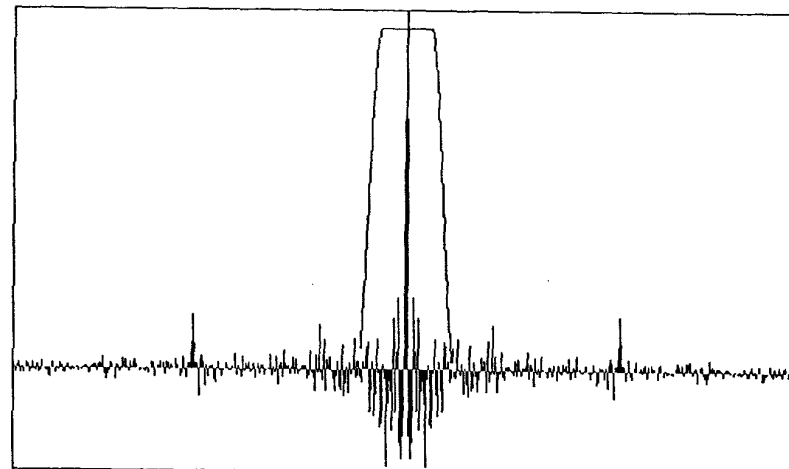
0.7480 SX7-B-MRKM0 Wide-Band Spectrogram 1.9804



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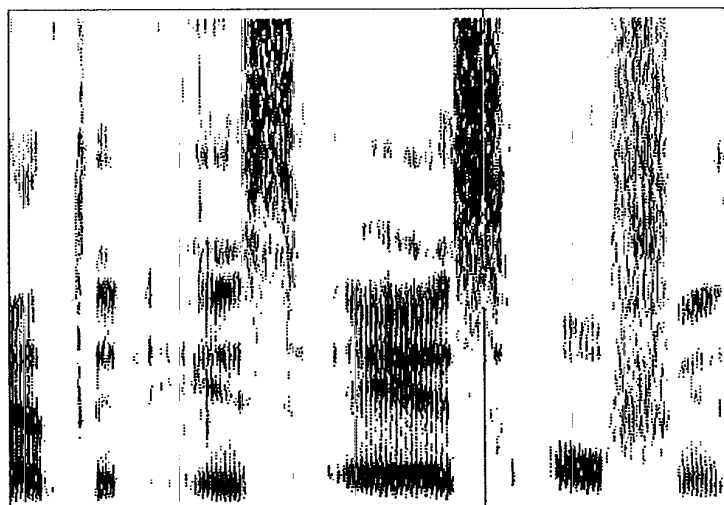


1.3850 SX7-B-MRKM0 Original Waveform 1.5350

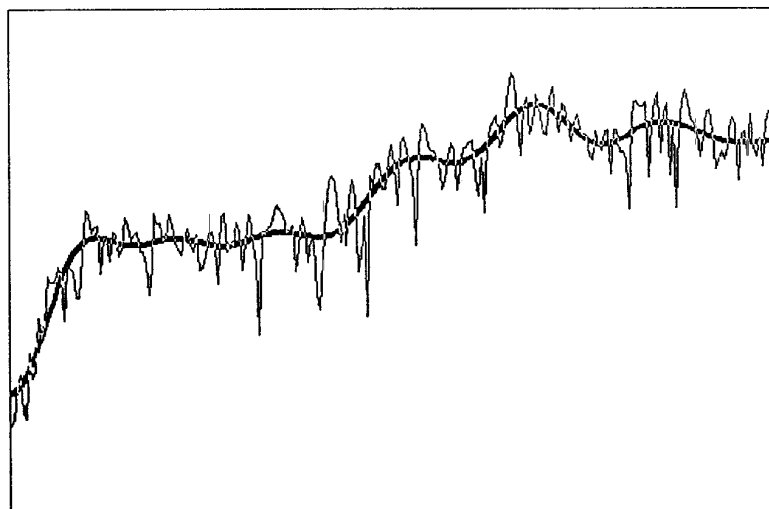


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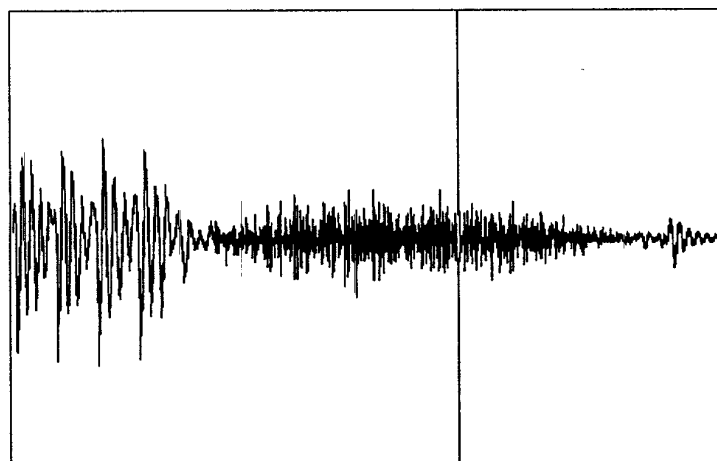
Example of Cepstral Analysis of Fricative (Rectangular Window)



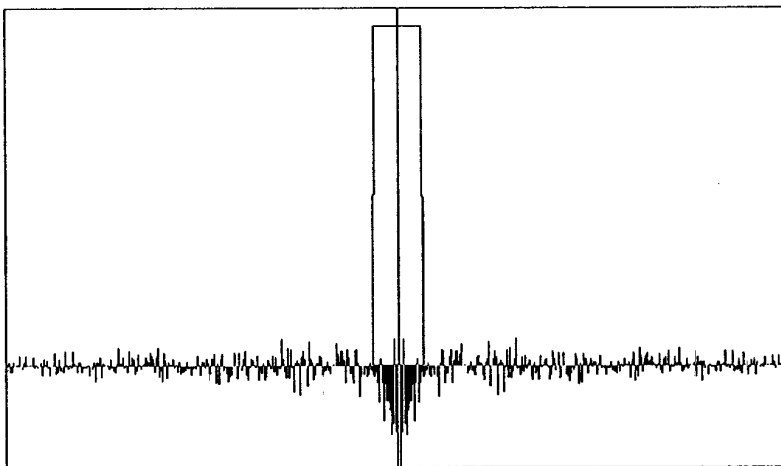
0.7480 SX7-B-MRKM0 Wide-Band Spectrogram 1.9804



0. SX7-B-MRKM0 FFT Spectral Slice (25 msec) 8000.

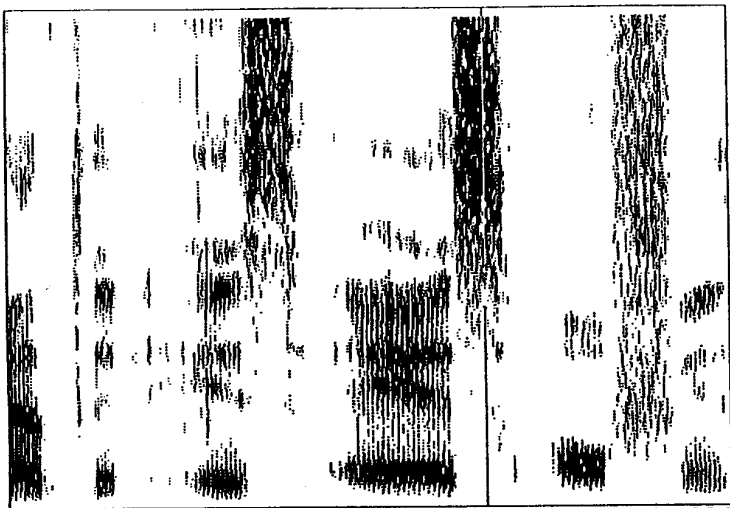


1.4694 SX7-B-MRKM0 Original Waveform 1.6194

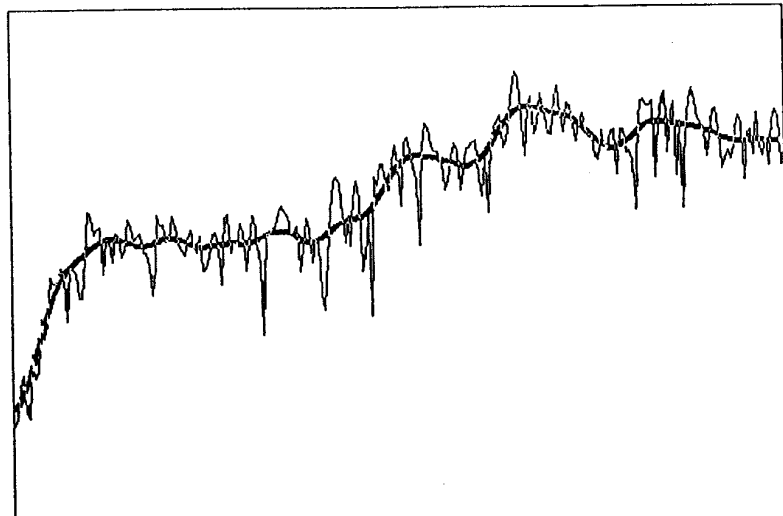


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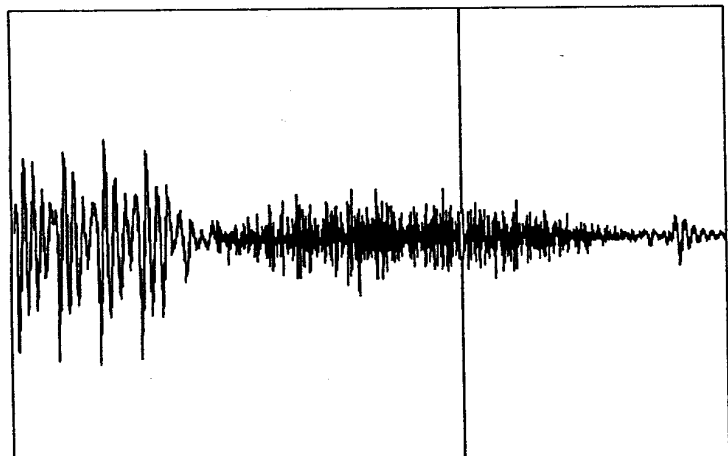
Example of Cepstral Analysis of Fricative (Tapering Window)



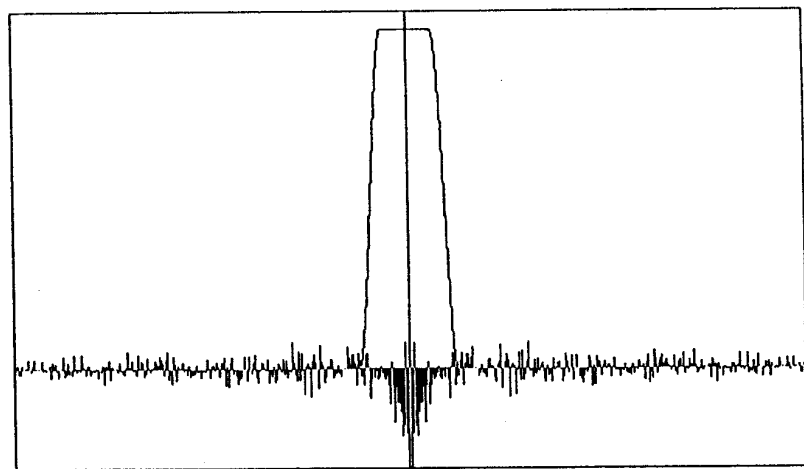
0.7480 SX7-B-MRKM0 Wide-Band Spectrogram 1.9804



0. SX7-B-MRKM0 FFT Spectral Slice (25 msec) 8000.



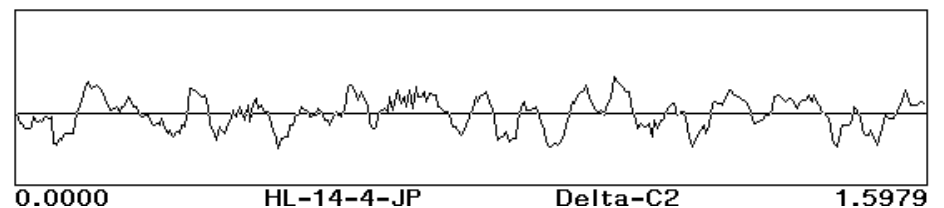
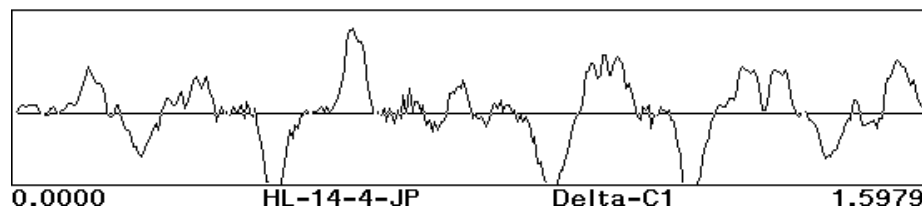
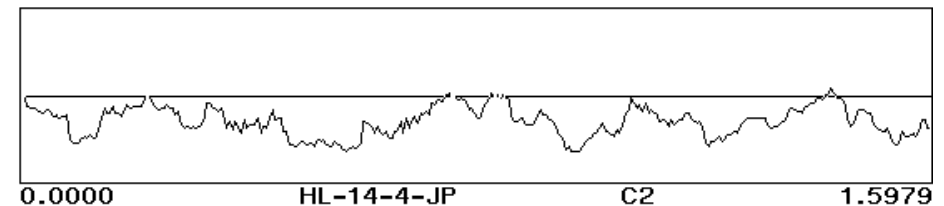
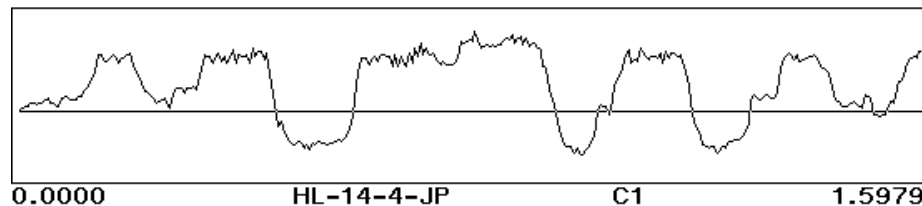
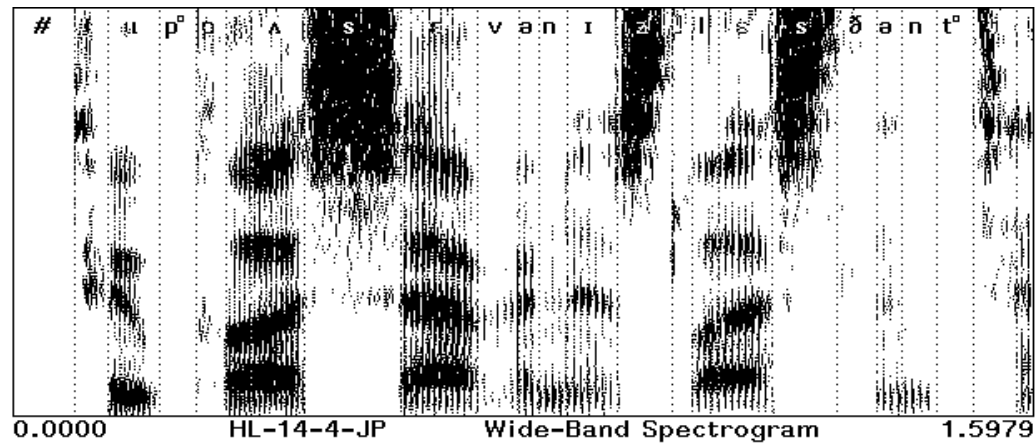
1.4694 SX7-B-MRKM0 Original Waveform 1.6194



0.0000 SX7-B-MRKM0 Cepstrum Slice 0.0320

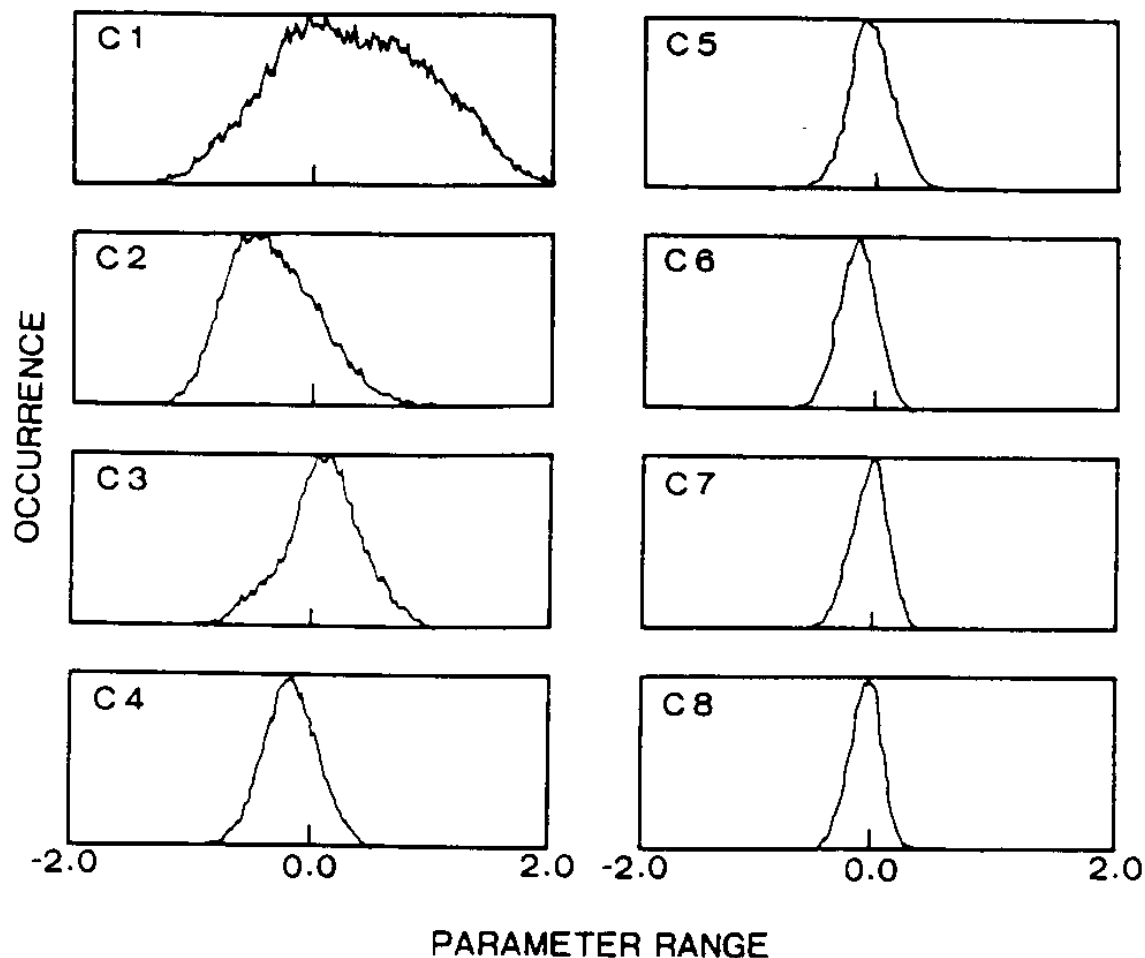
The Use of Cepstrum for Speech Recognition

Many current speech recognition systems represent the speech signal as a set of cepstral coefficients, computed at a fixed frame rate. In addition, the time derivatives of the cepstral coefficients have also been used.



Statistical Properties of Cepstral Coefficients (Tohkura, 1987)

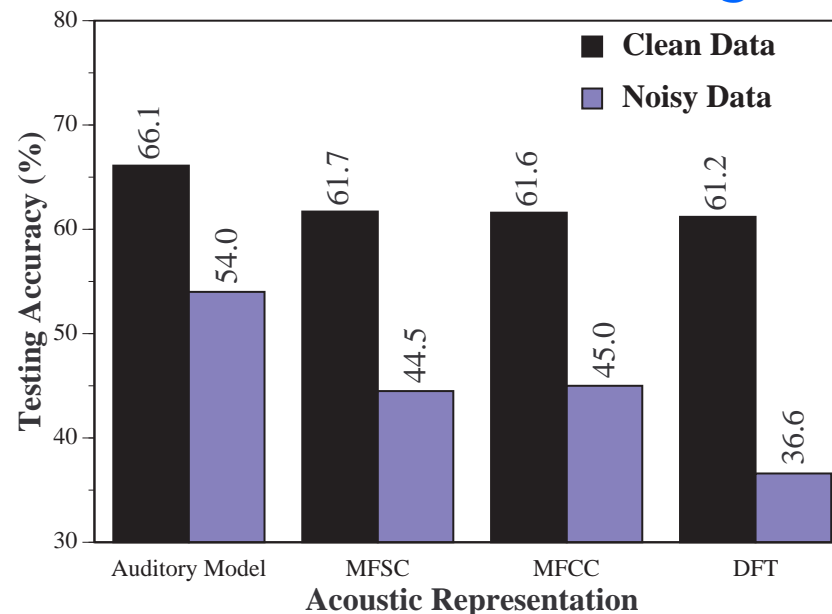
From a digit database (100 speakers) over dial-up telephone lines.



Signal Representation Comparisons

- Many researchers have compared cepstral representations with Fourier-, LPC-, and auditory-based representations.
- Cepstral representation typically out-performs Fourier- and LPC-based representations.

Example: Classification of 16 vowels using ANN (Meng, 1991)



- Performance of various signal representations cannot be compared without considering how the features will be used, i.e., the pattern classification techniques used. (Leung, et al., 1993).

Things to Ponder...

- Are there other spectral representations that we should consider (e.g., models of the human auditory system)?
- What about representing the speech signal in terms of phonetically motivated attributes (e.g., formants, durations, fundamental frequency contours)?
- How do we make use of these (sometimes heterogeneous) features for recognition (i.e., what are the appropriate methods for modelling them)?

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References

1. Tohkura, Y., "A Weighted Cepstral Distance Measure for Speech Recognition," *IEEE Trans. ASSP*, Vol. ASSP-35, No. 10, 1414-1422, 1987.
2. Mermelstein, P. and Davis, S., "Comparison of Parametric Representations for Monosyllabic Word Recognition in Continuously Spoken Sentences," *IEEE Trans. ASSP*, Vol. ASSP-28, No. 4, 357-366, 1980.
3. Meng, H., *The Use of Distinctive Features for Automatic Speech Recognition*, SM Thesis, MIT EECS, 1991.
4. Leung, H., Chigier, B., and Glass, J., "A Comparative Study of Signal Representation and Classification Techniques for Speech Recognition," *Proc. ICASSP*, Vol. II, 680-683, 1993.