Midterm

- You have 110 minutes (9:05-10:55 am) to complete the test.
- This is a closed-book test, except that three $8.5'' \times 11''$ sheets of notes are allowed.
- Calculators are allowed (provided that erasable memory is cleared).
- There are two problems on the quiz. The first is a six-part problem, each part worth 10 points. The second problem consists of four unrelated true-false questions, each worth 10 points.
- The problems are not necessarily in order of difficulty.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- If we can't read it, we can't grade it.
- If you don't understand a problem, please ask.

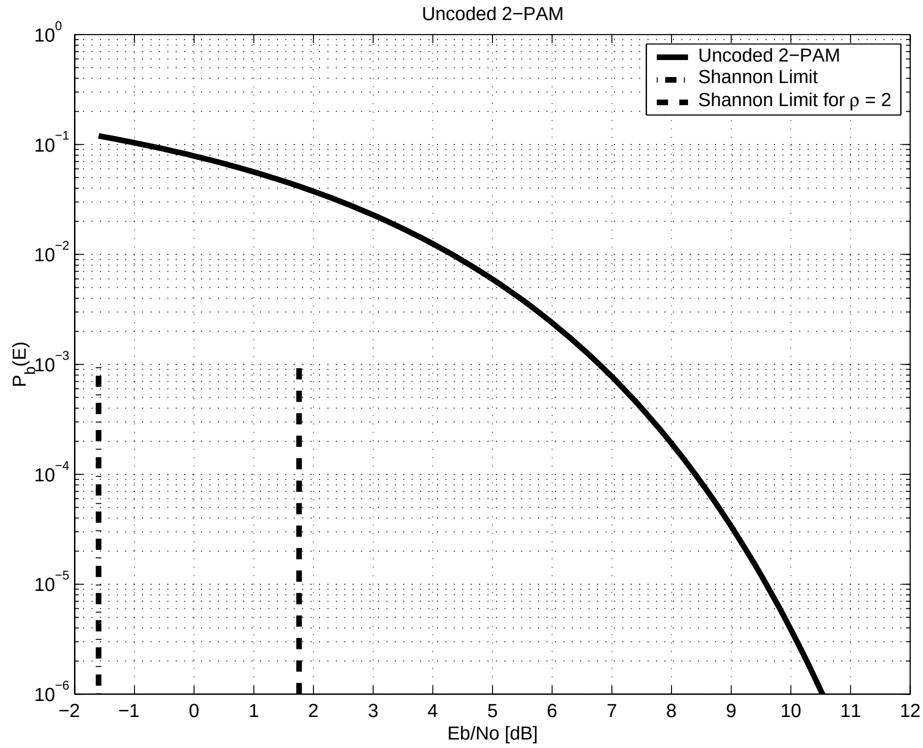

Figure 1. $P_b(E)$ *vs.* E_b/N_0 for uncoded binary PAM.

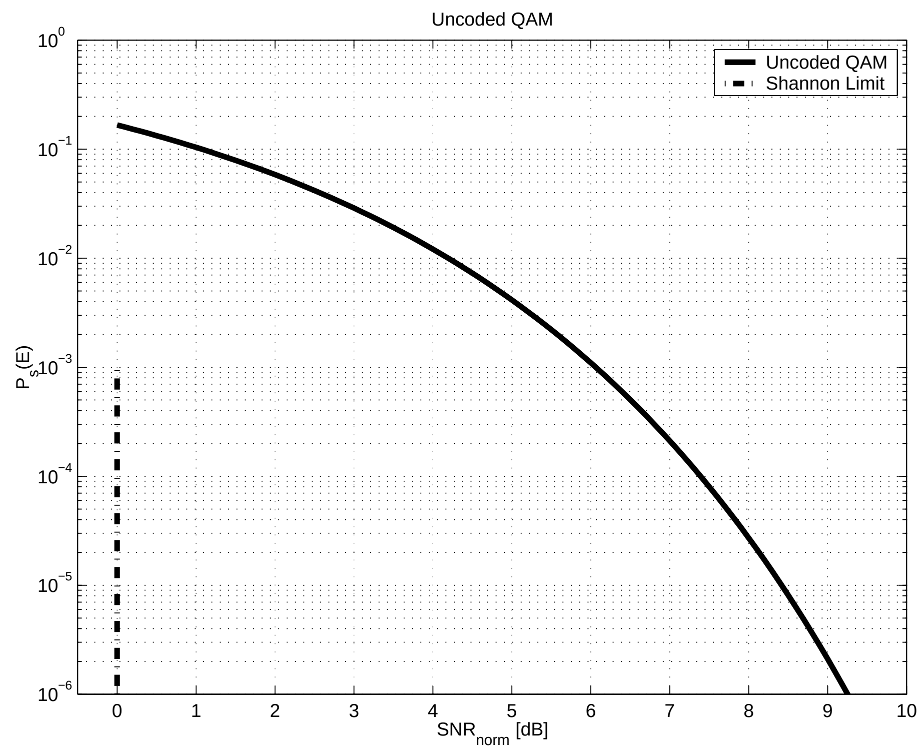

Figure 2. $P_s(E)$ *vs.* $\mathrm{SNR}_{\mathrm{norm}}$ for uncoded $(M \times M)$-QAM.

α	dB (round numbers)	dB (two decimal places)
1	0	0.00
1.25	1	0.97
2	3	3.01
2.5	4	3.98
e	4.3	4.34
3	4.8	4.77
π	5	4.97
4	6	6.02
5	7	6.99
8	9	9.03
10	10	10.00

Table 1. Values of certain small factors α in dB.

code	ρ	γ_c	(dB)	N_d	K_b	γ_{eff} (dB)	s	t
(8,7,2)	1.75	7/4	2.43	28	4	2.0	1	2
(8,4,4)	1.00	2	3.01	14	4	2.6	2	3
(16,15,2)	1.88	15/8	2.73	120	8	2.1	1	2
(16,11,4)	1.38	11/4	4.39	140	13	3.7	3	5
(16, 5,8)	0.63	5/2	3.98	30	6	3.5	3	4
(32,31, 2)	1.94	31/16	2.87	496	16	2.1	1	2
(32,26, 4)	1.63	13/4	5.12	1240	48	4.0	4	7
(32,16, 8)	1.00	4	6.02	620	39	4.9	6	9
(32, 6,16)	0.37	3	4.77	62	10	4.2	4	5
(64,63, 2)	1.97	63/32	2.94	2016	32	1.9	1	2
(64,57, 4)	1.78	57/16	5.52	10416	183	4.0	5	9
(64,42, 8)	1.31	21/4	7.20	11160	266	5.6	10	16
(64,22,16)	0.69	11/2	7.40	2604	118	6.0	10	14
(64, 7,32)	0.22	7/2	5.44	126	18	4.6	5	6

Table 2. Parameters of RM codes with lengths $n \leq 64$.

Problem M.1 (60 points)

Your boss wants you to do a feasibility study for a digital communication system with the following characteristics.

You are allowed to use the frequency band B between 953 and 954 MHz. The allowed signal power is $P = 10^6$ power units. The noise in the band is additive white Gaussian noise with single-sided power spectral density $N_0 = 1$ power units per Hz.

For the purposes of the feasibility study, you may assume optimally bandwidth-efficient modulation, ideal brick-wall (zero-rolloff) filters, perfect receiver synchronization, etc.

(a) What is the Shannon limit on the achievable data rate R in bits per second (b/s)?

(b) What is the maximum data rate R that can be achieved with uncoded modulation, if the target error rate is of the order of 10^{-5}?

(c) Suppose that for complexity reasons you are restricted to using Reed-Muller codes with block length $n \leq 64$. What is the maximum data rate R that can be achieved, if the target error rate is of the order of 10^{-5}?

Now let the allowed signal power be only $P = 10^5$ power units, with all else the same.

(d) What is the Shannon limit on the achievable data rate R in bits per second (b/s)?

(e) What is the maximum data rate R that can be achieved with uncoded modulation, if the target error rate is of the order of 10^{-5}?

(f) Suppose that you are allowed to use any code that has been introduced in this course so far. What is the maximum data rate R that can be achieved, if the target error rate is of the order of 10^{-5}?

Problem M.2 (40 points)

For each of the propositions below, state whether the proposition is true or false, and give a proof of not more than a few sentences, or a counterexample. No credit will be given for a correct answer without an adequate explanation.

(a) Let $\mathcal{A} = \{a_j(t), 1 \leq j \leq M\}$ be a set of M real $\mathcal{L}_2$ signals, and let the received signal be $r(t) = x(t) + n(t)$, where $x(t)$ is a signal in $\mathcal{A}$, and $n(t)$ is additive (independent) white Gaussian noise. Then, regardless of whether the signals in $\mathcal{A}$ are equiprobable or not, it is possible to do optimal detection on $r(t)$ by first computing from $r(t)$ a real M-tuple $\mathbf{r} = (r_1, r_2, \ldots, r_M)$, and then doing optimal detection on $\mathbf{r}$.

(b) Let $\mathcal{A}$ be an arbitrary M-point, N-dimensional signal constellation, and let $\mathcal{A}' = \alpha U \mathcal{A}^K$ be the constellation obtained by taking the K-fold Cartesian product $\mathcal{A}^K$, scaling by $\alpha > 0$, and applying an orthogonal transformation U. Then the effective coding gain of $\mathcal{A}'$ is the same as that of $\mathcal{A}$.

(c) Let $\{\mathcal{C}_j, j = 1, 2, \ldots\}$ be an infinite set of binary linear (n_j, k_j, d_j) block codes $\mathcal{C}_j$ with $n_j \to \infty$ as $j \to \infty$. Then in order for the performance of these codes in AWGN to approach the Shannon limit as $j \to \infty$, it is necessary that either $\lim_{j\to\infty} k_j/n_j > 0$ or $\lim_{j\to\infty} d_j/n_j > 0$.

(d) The Euclidean-space image $s(\mathcal{C})$ of a binary linear block code $\mathcal{C}$ under the 2-PAM map $\{s(0) = +\alpha, s(1) = -\alpha\}$ has zero mean, $\mathbf{m}(s(\mathcal{C})) = \mathbf{0}$, unless there is some coordinate position in which all codewords in $\mathcal{C}$ have value 0.