ACOUSTICS

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CHAPTER 1

INTRODUCTION AND TERMINOLOGY

PART I *Introduction*

1.1. A Little History. Acoustics is entering a new era—the precision-engineering era. One hundred years ago acoustics was an art. For measuring instruments, engineers in the field used their ears primarily. The only controlled noise sources available were whistles, gongs, and sirens. Microphones consisted of either a diaphragm connected to a mechanical scratcher that recorded the shape of the wave on the smoked surface of a rotating drum or a flame whose height varied with the sound pressure. About that time the great names of Rayleigh, Stokes, Thomson, Lamb, Helmholtz, König, Tyndall, Kundt, and others appeared on important published papers. Their contributions to the physics of sound were followed by the publication of Lord Rayleigh's two-volume treatise, "Theory of Sound" (1877 and 1878). Acoustics rested there until W. C. Sabine, in a series of papers (1900–1915), advanced architectural acoustics to the status of a science.

Even though the contributions of these earlier workers were great, the greatest acceleration of interest in the field of acoustics followed the invention of triode-vacuum-tube circuits (1907) and the advent of radio-broadcasting (1920). With vacuum-tube amplifiers available, loud sounds of any desired frequency could be produced, and the intensity of very faint sounds could be measured. Above all it became feasible to build measuring instruments that were compact, rugged, and insensitive to air drafts.

The progress of communication acoustics was hastened, through the efforts of the Bell Telephone Laboratories (1920*ff*), by the development of the modern telephone system in the United States.

Architectural acoustics received a boost principally from the theory and experiments coming out of Harvard, the Massachusetts Institute of Technology, and the University of California at Los Angeles (1930–1940), and several research centers in England and Europe, especially Germany.

In this period, sound decay in rectangular rooms was explained in detail, the impedance method of specifying acoustical materials was shown to be useful, and the computation of sound attenuation in ducts was put on a precise basis. The advantages of skewed walls and of using acoustical materials in patches rather than on entire walls were demonstrated. Functional absorbers were introduced to the field, and a wider variety of acoustical materials came on the market.

The science of psychoacoustics was also developing. At the Bell Telephone Laboratories, under the splendid leadership of Harvey Fletcher, the concepts of loudness and masking were quantified, and many of the factors governing successful speech communication were determined (1920–1940). Acoustics, through the medium of ultrasonics, entered the fields of medicine and chemistry. Ultrasonic diathermy was being tried, and acoustically accelerated chemical reactions were reported.

Finally, World War II came, with its demand for the successful detection of submerged submarines and for highly reliable speech communication in noisy environments such as aircraft and armored vehicles. Great laboratories were formed in England, Germany, France, and in the United States at Columbia University, Harvard, and the University of California to deal with these problems. Research in acoustics reached proportions undreamed of a few years before and has continued unabated.

Today, acoustics is passing from being a tool of the telephone industry, a few enlightened architects, and the military into being a concern in the daily life of nearly every person. International movements are afoot to legislate and to provide quiet housing. Labor and office workers are demanding safe and comfortable acoustic environments in which to work. Architects in rapidly increasing numbers are hiring the services of acoustical engineers as a routine part of the design of buildings. In addition there is the more general need to abate the great noise threat from aviation—particularly that from the jet engine, which promises to ruin the comfort of our homes. Manufacturers are using acoustic instrumentation on their production lines. Acoustics is coming into its own in the living room, where high-fidelity reproduction of music has found a wide audience.

This book covers first the basic aspects of acoustics: wave propagation in the air, the theory of mechanical and acoustical circuits, the radiation of sound into free space, and the properties of acoustic components. Then follow chapters dealing with microphones, loudspeakers, enclosures for loudspeakers, and horns. The basic concepts of sound in enclosures are treated next, and practical information on noise control is given. The text deals finally with measurements and psychoacoustics. Throughout the text we shall speak to *you*—the student of this modern and interesting field.

1.2. What Is Sound? In reading the material that follows, your goal should be to form and to keep in mind a picture of what transpires when

the diaphragm on a loudspeaker, or any surface for that matter, is vibrating in contact with the air.

A sound is said to exist if a disturbance propagated through an elastic material causes an alteration in pressure or a displacement of the particles of the material which can be detected by a person or by an instrument. Because this text deals primarily with devices for handling speech and music, gases (more particularly, air) are the only types of elastic material with which we shall concern ourselves. Fortunately, the physical properties of gases are relatively easy to express, and we can describe readily the nature of sound propagation in such media.

Imagine that we could cut a tiny cubic "box" out of air and hold it in our hands as we would a block of wood. What physical properties would it exhibit? First, it would have weight and, hence, mass. In fact, a cubic meter of air has a mass of a little over one kilogram. If a force is applied to it, the box will then accelerate according to Newton's second law, which says that force equals mass times acceleration.

If we exert forces compressing two opposing sides of the little cube, the four other sides will bulge outward. The incremental pressure produced in the gas by this force will be the same throughout this small volume. This obtains because pressure in a gas is a scalar, *i.e.*, a nondirectional quantity.

Imagine the little box of air to be held tightly between your hands. Still holding the box, move one hand forward so as to distort the cube into a parallelepiped. You find that no opposition to the distortion of the box is made by the air outside the two distorted sides. This indicates that air does not support a shearing force.†

Further, if we constrain five sides of the cube and attempt to displace the sixth one, we find that the gas is elastic; *i.e.*, a force is required to compress the gas. The magnitude of the force is in direct proportion to the displacement of the unconstrained side of the container. A simple experiment will convince you of this. Close off the hose of an automobile tire pump so that the air is confined in the cylinder. Push down on the plunger. You will find that the confined air behaves as a simple spring.

The spring constant of the gas varies, however, with the method of compression. A force acting to compress a gas necessarily causes a displacement of the gas particles. The incremental pressure produced in the gas will be directly proportional to the incremental change in volume. If the displacement takes place slowly one can write

$$\Delta P = -K\,\Delta V \qquad \text{slow process}$$

where K is a constant. If, on the other hand, the displacement, and

† This is only approximately true, as the air does have viscosity, but the shearing forces are very small compared with those in solids.

hence the change in volume, takes place rapidly, and further if the gas is air, oxygen, hydrogen, or nitrogen, the incremental pressure produced is equal to $1.4K$ times the incremental change in volume.

$$\Delta P = -1.4K\,\Delta V \qquad \text{fast process, diatomic gas}$$

Note that a positive increment (increase) in pressure produces a negative increment (decrease) in volume. Processes which take place at intermediate rates are more difficult to describe, even approximately, and fortunately need not be considered here.

What is the reason for the difference between the pressure arising from changes in volume that occur rapidly and the pressure arising from changes in volume that occur slowly? For slow variations in volume the compressions are *isothermal*. By an isothermal variation we mean one that takes place at constant temperature. There is time for the heat generated in the gas during the compression to flow to other parts of the gas or, if the gas is confined, to flow to the walls of the container. Hence, the temperature of the gas remains constant. For rapid variations in volume, however, the temperature rises when the gas is compressed and falls when the gas is expanded. There is not enough time during a cycle of compression and expansion for the heat to flow away. Such rapid alternations are said to be *adiabatic*.

In either isothermal or adiabatic processes, the pressure in a gas is due to collisions of the gas molecules with container walls. You will recall that pressure is force per unit area, or, from Newton, time rate of change of momentum per unit area. Let us investigate the mechanism of this momentum change in a confined gas. The container wall changes the direction of motion of the molecules which strike it and so changes their momentum; this change appears as a pressure on the gas. The *rate* at which the change of momentum occurs, and so the magnitude of the pressure, depends on two quantities. It increases obviously if the number of collisions per second between the gas particles and the walls increases, or if the amount of momentum transferred per collision becomes greater, or both. We now see that the isothermal compression of a gas results in an increase of pressure because a given number of molecules are forced into a smaller volume and will necessarily collide with the container more frequently.

On the other hand, although the adiabatic compression of a gas results in an increase in the number of collisions as described above, it causes also a further increase in the number of collisions and a greater momentum transfer per collision. Both these additional increases are due to the temperature change which accompanies the adiabatic compression. Kinetic theory tells us that the velocity of gas molecules varies as the square root of the absolute temperature of the gas. In the adiabatic process then, as contrasted with the isothermal, the molecules get hotter;

they move faster, collide with the container walls more frequently, and, having greater momentum themselves, transfer more momentum to the walls during each individual collision.

For a given volume change ΔV, the rate of momentum change, and therefore the pressure increase, is seen to be greater in the adiabatic process. It follows that a gas is stiffer—it takes more force to expand or compress it—if the alternation is adiabatic. We shall see later in the text that sound waves are essentially adiabatic alternations.

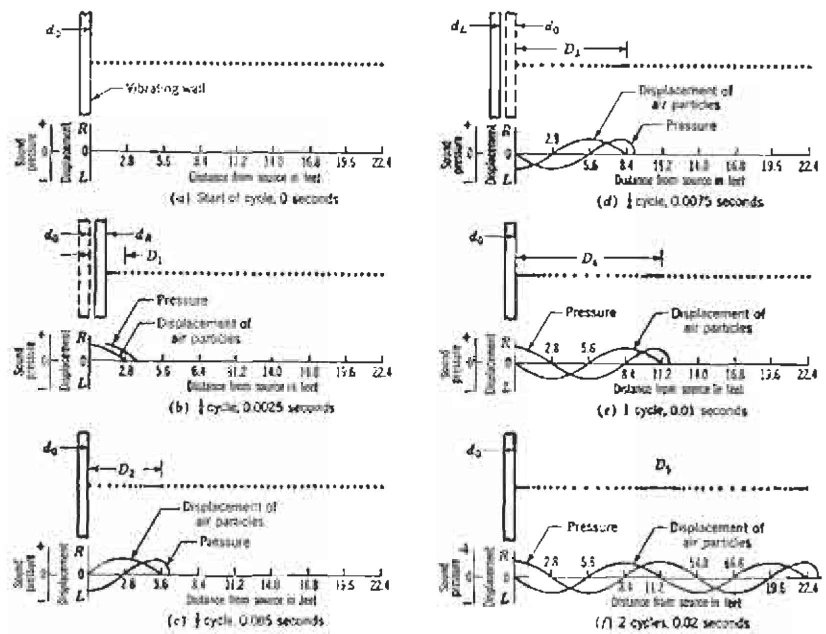

Fig. 1.1 Pressure and displacement in a plane sound wave produced by a sinusoidally vibrating wall. D_1 = one-fourth wavelength; D_2 = one-half wavelength; D_3 = three-fourths wavelength; D_4 = one wavelength; D_5 = two wavelengths. R means displacement of the air particles to the right, L means displacement to the left, and O means no displacement. Crowded dots mean positive excess pressure and spread dots mean negative excess pressure. The frequency of vibration of the piston is 100 cycles per second.

1.3. Propagation of Sound through Gas. The propagation of sound through a gas can be fully predicted and described if we take into account the factors just discussed, *viz.*, the mass and stiffness of the gas, and its conformance with basic physical laws. Such a mathematical description will be given in detail in later chapters. We are now concerned with a qualitative picture of sound propagation.

If we put a sinusoidally vibrating wall in a gas (see Fig. 1.1*a*), it will accelerate adjacent air particles and compress that part of the gas nearest to it as it moves forward from rest. This initial compression is shown in Fig. 1.1*b* as a crowding of dots in front of the wall. The dots represent air particles. These closely crowded air particles have, in addition to their random velocities, a forward momentum gained from the wall. They collide with their neighbors to the right and during the collision transfer forward momentum to these particles, which were at rest. These particles in turn move closer to their neighbors, with which they collide, and so on. Progressively more and more remote parts of the medium will be set into motion. In this way, through successive collisions, the force built up by the original compression may be transferred to distant parts of the gas.

When the wall reverses its motion, a rarefaction occurs immediately in front of it (see Figs. 1.1*c* and 1.1*d*). This rarefaction causes particles to be accelerated backward. and the above process is now repeated in the reverse direction, and so on, through successive cycles of the source.

It is important to an understanding of sound propagation that you keep in mind the relative variations in pressure, particle displacement, and particle velocity. Note that, at any one instant, the maximum particle displacement and the maximum pressure do not occur at the same point in the wave. To see this, consider Fig. 1.1*c*. The maximum pressure occurs where the particles are most tightly packed, *i.e.*, at $D_2 = 5.6$ ft. But at D_2 the particles have not yet moved from their original rest position, as we can see by comparison with Fig. 1.1*a*. At D_2, then, the pressure is a maximum, and the particle displacement is zero. At this instant, the particles next to the wall are also at their zero-displacement position, for the wall has just returned to its zero position. Although the particles at both D_2 and d_0 have zero displacement, their environments are quite different. We found the pressure at D_2 to be a maximum, but the air particles around d_0 are far apart, and so the pressure there is a minimum. Halfway between d_0 and D_2 the pressure is found to be at the ambient value (zero incremental pressure), and the displacement of the particles at a maximum. At a point in the wave where pressure is a maximum, the particle displacement is zero. Where particle displacement is a maximum, the incremental pressure is zero. Pressure and particle displacement are then 90° out of phase with each other.

At any given point on the wave the pressure and particle displacement are varying sinusoidally in time with the same frequency as the source. If the pressure is varying as $\cos 2\pi ft$, the particle displacement, 90° out of phase, must be varying as $\sin 2\pi ft$. Particle velocity, however, is the time derivative of displacement and must be varying as $\cos 2\pi ft$. At any one point on the wave, then, pressure and particle velocity are in phase.

We have determined the relative phases of the particle displacement, velocity, and pressure at a point in the wave. Now we ask, What phase

relationship exists between values of, say, particle displacement measured at two different points on the wave? If the action originating from the wall were transmitted instantaneously throughout the medium, all particles would be moving in phase with the source and with each other. This is not the case, for the speed of propagation of sound is finite, and at points increasingly distant from the source there is an increasing delay in the arrival of the signal. Each particle in the medium is moved backward and forward with the same frequency as the wall, but not at the same time. This means that two points separated a finite distance from each other along the wave in general will not be moving in phase with each other. Any two points that are vibrating in exact phase will, in this example of a plane wave, be separated by an integral number of wavelengths. For example, in Fig. 1.1*f* the 11.2- and 22.4-ft points are separated by exactly one wavelength. A disturbance at the 22.4-ft point occurs at about 0.01 sec after it occurs at the 11.2-ft point. This corresponds to a speed of propagation of about 1120 ft/sec. Mathematically stated, a *wavelength* is equal to the speed of propagation divided by the frequency of vibration.

$$\lambda = \frac{c}{f} \tag{1.1}$$

where λ is the wavelength in meters (or feet), c is the speed of propagation of the sound wave in meters (or feet) per second, and f is the frequency in cycles per second.

It is an interesting fact that sound waves in air are longitudinal; *i.e.*, the direction of the vibratory motion of air particles is the same as the direction in which the wave is traveling. This can be seen from Fig. 1.1. Light, heat, or radio waves in free space are transverse; *i.e.*, the vibrations of the electric and magnetic fields are perpendicular to the direction in which the wave advances. By contrast, waves on the surface of water are circular. The vibratory motion of the water molecules is in a small circle or ellipse, but the wave travels horizontally.

1.4. Measurable Aspects of Sound. Consider first what measurements might be made on the medium before a sound wave or a disturbance is initiated in it. The gas particles (molecules) are, on the average, at rest. They do have random motion, but there is no net movement of the gas in any direction. Hence, we say that the *particle displacement* is zero. It follows that the *particle velocity* is zero. Also, when there is no disturbance in the medium, the *pressure* throughout is constant and is equal to the *ambient pressure*, so that the *incremental pressure* is zero. A value for the ambient pressure may be determined from the readings of a barometer. The *density*, another measurable quantity in the medium, is defined as usual as the mass per unit volume. It equals the *ambient density* when there is no disturbance in the medium.

When a sound wave is propagated in the medium, several measurable changes occur. The particles are accelerated and as a result are displaced from their rest positions. The particle velocity at any point is not zero except at certain instants during an alternation. The temperature at a point fluctuates above and below its ambient value. Also, the pressure at any point will vary above and below the ambient pressure. This incremental variation of pressure is called the *sound pressure* or the *excess pressure*. A pressure variation, in turn, causes a change in the density called the *incremental density*. An increase in sound pressure at a point causes an increase in the density of the medium at that point.

The speed with which an acoustical disturbance propagates outward through the medium is different for different gases. For any given gas, the speed of propagation is proportional to the square root of the absolute temperature of the gas [see Eq. (1.8)]. As is the case for all types of wave motion, the speed of propagation is given by Eq. (1.1).

In later chapters of this book, we shall describe instruments and techniques for measuring most of the quantities named above. Sound will then seem real, and you will have a "feel" for intensity, sound-pressure level, power level, and the other terms of acoustics that we are about to define.

PART II *Terminology*

You now have a general picture of the nature of a sound wave. To proceed further in acoustics, you must learn the particular "lingo," or accepted terminology. Many common words such as pressure, intensity, and level are used in a special manner. Become well acquainted with the special meanings of these words at the beginning. They will be in constant use throughout the text. The list of definitions below is by no means exhaustive, and some additional terminology will be presented as needed in later chapters.[1] If possible, your instructor should have you make measurements of sounds with a sound-level meter and a sound analyzer so that the terminology becomes intimately associated with physical phenomena.

The mks system of units is used throughout this book. Although the practicing acoustical engineer may believe that this choice is unwarranted in view of the widespread use of cgs units, it will be apparent in Chap. 3, and again in Chap. 10, that great simplicity results from the use of the mks

[1] A good manual of terminology is "American Standard Acoustical Terminology," Z24.1—1951, published by the American Standards Association, Inc., New York, N.Y.

system. In the definitions that follow, the units in the cgs system are indicated in parentheses following the mks units. Conversion tables are given in Appendix II.

1.5. General. *Acoustic.* The word "acoustic," an adjective, means intimately associated with sound waves or with the individual media, phenomena, apparatus, quantities, or units discussed in the science of sound waves. Examples: "Through the acoustic medium came an acoustic radiation so intense as to produce acoustic trauma. The acoustic filter has an output acoustic impedance of 10-acoustic ohms." Other examples are acoustic horn, transducer, energy, wave, mobility, refraction, mass, component, propagation constant.

Acoustical. The word "acoustical," an adjective, means associated in a general way with the science of sound or with the broader classes of media, phenomena, apparatus, quantities, or units discussed in the science of sound. Example: "Acoustical media exhibit acoustical phenomena whose well-defined acoustical quantities can be measured, with the aid of acoustical apparatus, in terms of an acceptable system of acoustical units." Other examples are acoustical engineer, school, glossary, theorem, circuit diagram.

1.6. Pressure and Density. *Static Pressure* (P_0). The static pressure at a point in the medium is the pressure that would exist at that point with no sound waves present. At normal barometric pressure, P_0 equals approximately 10^5 newtons/m^2 (10^6 dynes/cm^2). This corresponds to a barometer reading of 0.751 m (29.6 in.) Hg mercury when the temperature of the mercury is 0°C. Standard atmospheric pressure is usually taken to be 0.760 m Hg at 0°C. This is a pressure of 1.013×10^5 newtons/m^2. In this text when solving problems we shall assume $P_0 = 10^5$ newtons/m^2.

Microbar (μb). A microbar is a unit of pressure commonly used in acoustics. One microbar is equal to 0.1 newton per square meter or 1 dyne per square centimeter. In this text its use is not restricted to the cgs system.

Instantaneous Sound Pressure [$p(t)$]. The instantaneous sound pressure at a point is the incremental change from the static pressure at a given instant caused by the presence of a sound wave. The unit is the microbar (0.1 newton per square meter or 1 dyne per square centimeter).

Effective Sound Pressure (p). The effective sound pressure at a point is the root-mean-square (rms) value of the instantaneous sound pressure, over a time interval at that point. The unit is the microbar (0.1 newton per square meter or 1 dyne per square centimeter). In the case of periodic sound pressures, the interval should be an integral number of periods. In the case of nonperiodic sound pressures, the interval should be long enough to make the value obtained essentially independent of small changes in the length of the interval.

Density of Air (ρ_0). The ambient density of air is given by the formulas

$$\rho_0 = 1.29 \frac{273}{T} \frac{P_0}{0.76} \qquad \text{kg/m}^3 \text{ (mks)} \qquad (1.2)$$

$$\rho_0 = 0.00129 \frac{273}{T} \frac{P_0}{0.76} \qquad \text{gm/cm}^3 \text{ (cgs)} \qquad (1.3)$$

where T is the absolute temperature in degrees Kelvin and P_0 is the barometric pressure in meters of mercury. At normal room temperature of $T = 295°\text{K}$ (22°C or 71.6°F), and for a static pressure $P_0 = 0.751$ m Hg, the ambient density is $\rho_0 = 1.18$ kg/m³. This value of ρ_0 will be used in solving problems unless otherwise stated.

1.7. Speed and Velocity. *Speed of Sound* (c). The speed of sound in air is given approximately by the formulas

$$c = 331.4 + 0.607\theta \qquad \text{m/sec (mks)} \qquad (1.4)$$

$$c = 33{,}140 + 60.7\theta \qquad \text{cm/sec (cgs)} \qquad (1.5)$$

$$c = 1087 + 1.99\theta \qquad \text{ft/sec (English-centigrade)} \qquad (1.6)$$

$$c = 1052 + 1.106F \qquad \text{ft/sec (English-Fahrenheit)} \qquad (1.7)$$

where θ is the ambient temperature in degrees centigrade and F is the same in degrees Fahrenheit. For temperatures above 30°C or below −30°C, the velocity of sound must be determined from the exact formula

$$c = 331.4 \sqrt{\frac{T}{273}} = 331.4 \sqrt{1 + \frac{\theta}{273}} \qquad \text{m/sec} \qquad (1.8)$$

where T is the ambient temperature in degrees Kelvin. At a normal room temperature of $\theta = 22°(F = 71.6°)$, $c \doteq 344.8$ m/sec, or 1131.2 ft/sec. These values of c will be used in solving problems unless otherwise stated.

Instantaneous Particle Velocity (*Particle Velocity*) [$u(t)$]. The instantaneous particle velocity at a point is the velocity, due to the sound wave only, of a given infinitesimal part of the medium at a given instant. It is measured over and above any motion of the medium as a whole. The unit is the meter per second (in the cgs system the unit is the centimeter per second).

Effective Particle Velocity (u). The effective particle velocity at a point is the root mean square of the instantaneous particle velocity (see *Effective Sound Pressure* for details). The unit is the meter per second (in the cgs system the unit is the centimeter per second).

Instantaneous Volume Velocity [$U(t)$]. The instantaneous volume velocity, due to the sound wave only, is the rate of flow of the medium perpendicularly through a specified area S. That is, $U(t) = Su(t)$, where $u(t)$ is the instantaneous particle velocity. The unit is the cubic meter per second (in the cgs system the unit is the cubic centimeter per second).

1.8. Impedance. *Acoustic Impedance* (*American Standard Acoustic Impedance*). The acoustic impedance at a given surface is defined as the complex ratio† of effective sound pressure averaged over the surface to effective volume velocity through it. The surface may be either a hypothetical surface in an acoustic medium or the moving surface of a mechanical device. The unit is newton-sec/m⁵, or the mks acoustic ohm.‡ (In the cgs system the unit is dyne-sec/cm⁵, or acoustic ohm.)

$$Z_A = \frac{p}{U} \qquad \text{newton-sec/m}^5 \text{ (mks acoustic ohms)} \qquad (1.9)$$

Specific Acoustic Impedance (Z_s). The specific acoustic impedance is the complex ratio of the effective sound pressure at a point of an acoustic medium or mechanical device to the effective particle velocity at that point. The unit is newton-sec/m³, or the mks rayl.§ (In the cgs system the unit is dyne-sec/cm³, or the rayl.) That is,

$$Z_s = \frac{p}{u} \qquad \text{newton-sec/m}^3 \text{ (mks rayls)} \qquad (1.10)$$

Mechanical Impedance (Z_M). The mechanical impedance is the complex ratio of the effective force acting on a specified area of an acoustic medium or mechanical device to the resulting effective linear velocity through or of that area, respectively. The unit is the newton-sec/m, or the mks mechanical ohm. (In the cgs system the unit is the dyne-sec/cm, or the mechanical ohm.) That is,

$$Z_M = \frac{f}{u} \qquad \text{newton-sec/m (mks mechanical ohms)} \qquad (1.11)$$

Characteristic Impedance ($\rho_0 c$). The characteristic impedance is the ratio of the effective sound pressure at a given point to the effective particle velocity at that point in a free, plane, progressive sound wave. It is equal to the product of the density of the medium times the speed of sound in the medium ($\rho_0 c$). It is analogous to the characteristic impedance of an infinitely long, dissipationless transmission line. The unit is the mks rayl, or newton-sec/m³. (In the cgs system, the unit is the rayl, or dyne-sec/cm³.)

In the solution of problems in this book we shall assume for air that $\rho_0 c = 407$ mks rayls (or $\rho_0 c = 40.7$ rayls) which is valid for a temperature of 22°C (71.6°F) and a barometric pressure of 0.751 m (29.6 in.) Hg.

1.9. Intensity, Energy Density, and Levels. *Sound Intensity* (I). The sound intensity measured in a specified direction at a point is the average

† "Complex ratio" has the same meaning as the complex ratio of voltage and current in electric-circuit theory.

‡ This notation is taken from Table 12.1 of American Standard Z24.1—1951.

§ Named in honor of Lord Rayleigh.

rate at which sound energy is transmitted through a unit area perpendicular to the specified direction at the point considered. The unit is the watt per square meter. (In the cgs system the unit is the erg per second per square centimeter.) In a plane or spherical free-progressive sound wave the intensity in the direction of propagation is

$$I = \frac{p^2}{\rho_0 c} \qquad \text{watts/m}^2 \tag{1.12}$$

NOTE: In the acoustical literature the intensity has often been expressed in the units of watts per square centimeter, which is equal to $10^{-7} \times$ the number of ergs per second per square centimeter.

Sound Energy Density (D). The sound energy density is the sound energy in a given infinitesimal part of the gas divided by the volume of that part of the gas. The unit is the watt-second per cubic meter. (In the cgs system the unit is the erg per cubic centimeter.) In many acoustic environments such as in a plane wave the sound energy density at a point is

$$D = \frac{p^2}{\rho_0 c^2} = \frac{p^2}{\gamma P_0} \tag{1.13}$$

where γ is the ratio of specific heats for a gas and is equal to 1.4 for air and other diatomic gases. The quantity γ is dimensionless.

Electric Power Level, or Acoustic Intensity Level. The electric power level, or the acoustic intensity level, is a quantity expressing the ratio of two electrical powers or of two sound intensities in logarithmic form. The unit is the decibel. Definitions are

$$\text{Electric power level} = 10 \log_{10} \frac{W_1}{W_2} \qquad \text{db} \tag{1.14}$$

$$\text{Acoustic intensity level} = 10 \log_{10} \frac{I_1}{I_2} \qquad \text{db} \tag{1.15}$$

where W_1 and W_2 are two electrical powers and I_1 and I_2 are two sound intensities.

Extending this thought further, we see from Eq. (1.14) that

$$\begin{aligned} \text{Electric power level} &= 10 \log_{10} \frac{E_1^2}{R_1} \frac{R_2}{E_2^2} \\ &= 20 \log_{10} \frac{E_1}{E_2} + 10 \log_{10} \frac{R_2}{R_1} \qquad \text{db} \end{aligned} \tag{1.16}$$

where E_1 is the voltage across the resistance R_1 in which a power W_1 is being dissipated and E_2 is the voltage across the resistance R_2 in which a power W_2 is being dissipated. Similarly,

$$\text{Acoustic intensity level} = 20 \log_{10} \frac{p_1}{p_2} + 10 \log_{10} \frac{R_{S2}}{R_{S1}} \qquad \text{db} \tag{1.17}$$

where p_1 is the pressure at a point where the specific acoustic resistance (*i.e.*, the real part of the specific acoustic impedance) is R_{S1} and p_2 is the pressure at a point where the specific acoustic resistance is R_{S2}. We note that $10 \log_{10} (W_1/W_2) = 20 \log_{10} (E_1/E_2)$ only if $R_1 = R_2$ and that $10 \log_{10} (I_1/I_2) = 20 \log_{10} (p_1/p_2)$ only if $R_{S2} = R_{S1}$.

Levels involving voltage and pressure alone are sometimes spoken of with no regard to the equalities of the electric resistances or specific acoustic resistances. This practice leads to serious confusion. It is emphasized that the manner in which the terms are used should be clearly stated always by the user in order to avoid confusion.

Sound Pressure Level (SPL). The sound pressure level of a sound, in decibels, is 20 times the logarithm to the base 10 of the ratio of the measured effective sound pressure of this sound to a reference effective sound pressure. That is,

$$\text{SPL} = 20 \log_{10} \frac{p}{p_{\text{ref}}} \qquad \text{db} \tag{1.18}$$

In the United States p_{ref} is either

(*a*) $\qquad p_{\text{ref}} = 0.0002$ microbar (2×10^{-5} newton/m²)

or

(*b*) $\qquad p_{\text{ref}} = 1$ microbar (0.1 newton/m²)

Reference pressure (*a*) has been in general use for measurements dealing with hearing and for sound-level and noise measurements in air and liquids. Reference pressure (*b*) has gained widespread use for calibrations of transducers and some types of sound-level measurements in liquids. The two reference levels are almost exactly 74 db apart. The reference pressure must always be stated explicitly.

Intensity Level (IL). The intensity level of a sound, in decibels, is 10 times the logarithm to the base 10 of the ratio of the intensity of this sound to a reference intensity. That is,

$$\text{IL} = 10 \log_{10} \frac{I}{I_{\text{ref}}} \tag{1.19}$$

In the United States the reference intensity is often taken to be 10^{-16} watt/cm² (10^{-12} watt/m²). This reference at standard atmospheric conditions in a plane or spherical progressive wave was originally selected as corresponding approximately to the reference pressure (0.0002 microbar).

The exact relation between intensity level and sound pressure level in a plane or spherical progressive wave may be found by substituting Eq. (1.12) for intensity in Eq. (1.19).

$$\mathrm{IL} = \mathrm{SPL} + 10 \log_{10} \frac{p_{ref}^2}{\rho_0 c I_{ref}} \quad \mathrm{db} \tag{1.20}$$

Substituting $p_{ref} = 2 \times 10^{-5}$ newton/m² and $I_{ref} = 10^{-12}$ watt/m² yields

$$\mathrm{IL} = \mathrm{SPL} + 10 \log_{10} \frac{400}{\rho_0 c} \quad \mathrm{db} \tag{1.21}$$

It is apparent that the intensity level IL will equal the sound pressure level SPL only if $\rho_0 c = 400$ mks rayls. For certain combinations of temperature and static pressure this will be true, although for $T = 22°\mathrm{C}$ and $P_0 = 0.751$ m Hg, $\rho_0 c = 407$ mks rayls. For this common case then, the intensity level is smaller than the sound pressure level by about 0.1 db. Regardless of which reference quantity is used, it must always be stated explicitly.

Acoustic Power Level (PWL). The acoustic power level of a sound source, in decibels, is 10 times the logarithm to the base 10 of the ratio of the acoustic power radiated by the source to a reference acoustic power. That is,

$$\mathrm{PWL} = 10 \log_{10} \frac{W}{W_{ref}} \quad \mathrm{db} \tag{1.22}$$

In this text, W_{ref} is 10^{-13} watt. This means that a source radiating 1 acoustic watt has a power level of 130 db.

If the temperature is 20°C (67°F) and the pressure is 1.013×10^5 newtons/m² (0.76 m Hg), the sound pressure level in a duct with an area of 1 ft² cross section, or at a distance of 0.282 ft from the center of a "point" source (at this distance, the spherical surface has an area of 1 ft²), is, from Eqs. (1.12) and (1.18)

$$\begin{aligned}\mathrm{SPL}_{1\,\mathrm{ft}^2} &= 10 \log_{10} \frac{I \rho_0 c}{p^2_{ref}} = 10 \log_{10} \frac{W \rho_0 c}{S p^2_{ref}} \\ &= 10 \log_{10} \left[\frac{W}{0.093} \times 412.5 \times \frac{1}{(2 \times 10^{-5})^2} \right] \\ &= 10 \log_{10} \frac{W}{10^{-13}} + 0.5\end{aligned}$$

where W = acoustic power in watts
$\rho_0 c$ = characteristic impedance = 412.5 mks rayls
S = 1 ft² of area = 0.093 m²
p_{ref} = reference sound pressure = 2×10^{-5} newton/m²

In words, the sound pressure level equals the acoustic power level plus 0.5 db under the special conditions that the power passes uniformly through an area of 1 ft², the temperature is 20°C (67°F), and the barometric pressure is 0.76 m (30 in.) Hg.

Sound Level. The sound level at a point in a sound field is the reading in decibels of a sound-level meter constructed and operated in accordance with the latest edition of "American Standard Sound Level Meters for the Measurement of Noise and Other Sounds."[2]

The meter reading (in decibels) corresponds to a value of the sound pressure integrated over the audible frequency range with a specified frequency weighting and integration time.

Band Power Level (PWL_n). The band power level for a specified frequency band is the acoustic power level for the acoustic power contained within the band. The width of the band and the reference power must be specified. The unit is the decibel. The letter n is the designation number for the band being considered.

Band Pressure Level (BPL_n). The band pressure level of a sound for a specified frequency band is the effective sound pressure level for the sound energy contained within the band. The width of the band and the reference pressure must be specified. The unit is the decibel. The letter n is the designation number for the band being considered.

Power Spectrum Level. The power spectrum level of a sound at a specified frequency is the power level for the acoustic power contained in a band one cycle per second wide, centered at this specified frequency. The reference power must be specified. The unit is the decibel (see also the discussion under Pressure Spectrum Level).

Pressure Spectrum Level. The pressure spectrum level of a sound at a specified frequency is the effective sound pressure level for the sound energy contained within a band one cycle per second wide, centered at this specified frequency. The reference pressure must be explicitly stated. The unit is the decibel.

DISCUSSION. The concept of pressure spectrum level ordinarily has significance only for sound having a continuous distribution of energy within the frequency range under consideration.

The level of a uniform band of noise with a continuous spectrum exceeds the spectrum level by

$$C_n = 10 \log_{10} (f_b - f_a) \quad \mathrm{db} \tag{1.23}$$

where f_b and f_a are the upper and lower frequencies of the band, respectively. The level of a uniform noise with a continuous spectrum in a band of width $f_b - f_a$ cps is therefore related to the spectrum level by the formula

$$L_n = C_n + S_n \tag{1.24}$$

where L_n = sound pressure level in decibels of the noise in the band of width $f_b - f_a$, for C_n see Eq. (1.23), S_n = spectrum level of the noise, and n = designation number for the band being considered.

[2] "American Standard Sound Level Meters for the Measurement of Noise and Other Sounds," Z24.3—1944, American Standards Association, Inc., New York, N.Y. This standard is in process of revision.

The *beam width* of a directivity pattern is used in this text as the angular distance between the two points on either side of the principal axis where the sound pressure level is down 6 db from its value at $\theta = 0$.

4.1. Spherical Sources.[1,2] A spherical source is the simplest to consider because it radiates sound uniformly in all directions. As we saw from Eq. (2.62), the sound pressure at a point a distance r in any direction from the center of a spherical source of any radius in free space is equal to

$$p(r,t) = \frac{\sqrt{2}\,A_+}{r} e^{j(\omega t - kr)} \tag{4.1}$$

where A_+ is the magnitude of rms sound pressure at unit distance from the center of the sphere.

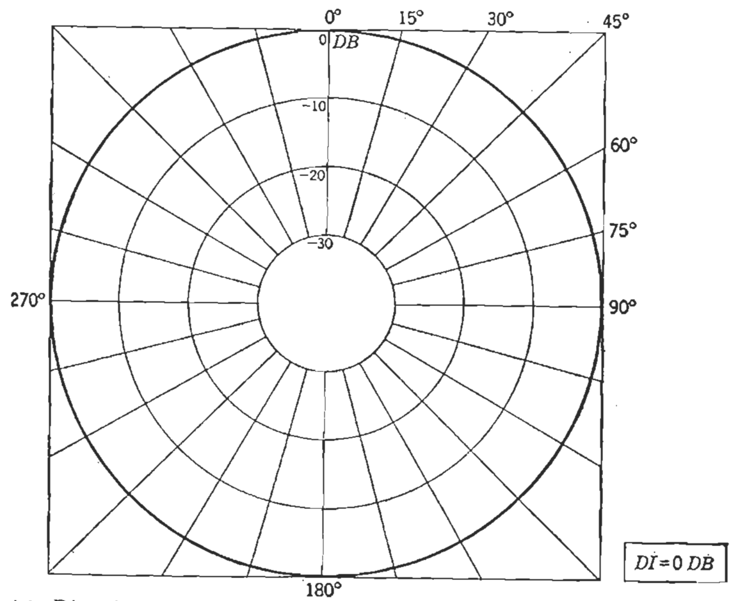

Fig. 4.1. Directivity pattern for a nondirectional source. Such a pattern is drawn on a particular plane intersecting the center of the source. The directivity index DI (defined in Part XI) equals 0 db at all angles.

Directivity Pattern. On a polar diagram, the directivity pattern on any plane surface intersecting the center of such a spherical source is given in Fig. 4.1. It is obviously a nondirectional source.

Simple Source (Point Source).[3] For the special case of a very small source, whose radius a is small compared with one-sixth wavelength (that

[1] P. M. Morse, "Vibration and Sound," 2d ed., pp. 311–326, McGraw-Hill Book Company, Inc., New York, 1948.

[2] L. E. Kinsler and A. R. Frey, "Fundamentals of Acoustics," pp. 163–173, John Wiley & Sons, Inc., New York, 1950.

[3] Morse, *op. cit.*, pp. 312–313.

is, $ka \ll 1$), the velocity at the surface of the sphere is [see Eq. (2.63)]

$$u(a,t) \doteq \frac{\sqrt{2}\,A_+}{\rho_0 c a}\,\frac{c}{j2\pi f a}\, e^{j(\omega t - ka)} \tag{4.2}$$

A source for which this formula is valid is called a *simple source.*

Substitution of Eq. (4.2) into Eq. (4.1) yields

$$p \doteq j\,\frac{U_0 f \rho_0}{2r}\, e^{-jk(r-a)} \tag{4.3}$$

where U_0 = rms volume velocity in cubic meters per second of the very small source and is equal to $(4\pi a^2)u_{rms}$

p = rms sound pressure in newtons per square meter at a distance r from the simple source

Strength of a Simple Source.[3] The rms magnitude of the total air flow at the surface of a simple source in cubic meters per second (or cubic centimeters per second in the cgs system) is given by U_0 and is called the *strength of a simple source.*†

Intensity at Distance r. At a distance r from the center of a simple source the intensity is given by

$$I = \frac{|p|^2}{\rho_0 c} = \frac{U_0^2 f^2 \rho_0}{4r^2 c} \quad \text{watts/m}^2 \tag{4.4}$$

When the dimensions of a source are *much smaller* than a wavelength, the radiation from it will be much the same no matter what shape the radiator has, as long as all parts of the radiator vibrate substantially in phase. The intensity at any distance is directly proportional to the square of the volume velocity and the frequency.

4.2. Combination of Simple Sources.[4] The basic principles governing the directivity patterns from loudspeakers can be learned by studying combinations of simple sources. This approach is very similar to the consideration of Huygens wavelets in optics. Basically, our problem is to add, vectorially, at the desired point in space, the sound pressures arriving at that point from all the simple sources. Let us see how this method of analysis is applied.

Two Simple Sources in Phase. The geometric situation is shown in Fig. 4.2. It is assumed that the distance r from the two point sources to the point A at which the pressure p is being measured is large compared with the separation b between the two sources.

The spherical sound wave arriving at the point p from source 1 will have

† In some texts the peak magnitude of the total air flow instead of the rms magnitude is used. In these texts, the "strength of a simple source" is $\sqrt{2}\,(4\pi a^2)u_{rms}$.

[4] H. F. Olson, "Elements of Acoustical Engineering," 2d ed., pp. 31–34, D. Van Nostrand Company, Inc., New York, 1947.