

[SQUEAKING]

[RUSTLING]

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**DAVID
PERREAULT:**

OK, glad everybody made it through the snow. Why don't we get started? So what we're going to do is today is we're going to start talking about magnetics, because in the middle of most power converters, there's some magnetic component or more than one magnetic component, so we really ought to understand them and know how to design them.

What I'm going to be talking about today in particular is in *Principles of Power Electronics* chapter 18, so I'll try to make sure that's in the correct reading for the homework set. And what I'd like to do is start by just reminding everybody of how this was likely treated in your introduction to physics class, straight from Maxwell's equations. And then I want to sort of step back and come up with some models that simplify things, magnetic circuit models that let you understand the behavior of magnetic components without having to write Maxwell's equations left, right, and center.

OK. So let's just remind ourselves of the integral form of Maxwell's equations, right. So we have Ampere's law, right, which basically says the integral of $\mathbf{H} \cdot d\mathbf{l}$ is equal to the integral of $\mathbf{J} \cdot d\mathbf{A}$ plus d/dt the integral of $\epsilon_0 \mathbf{E} \cdot d\mathbf{A}$. All right. This is displacement current.

We're going to do the MQS approximation, magnetoquasistatic approximation, where we ignore the effect of changing electric fields. We're ignoring displacement current, right, so basically, this just says that if I integrate the magnetic field around a loop, it basically equals the net current that's passing through that loop.

OK. The next thing we are interested in is Faraday's law, right, which says that the integral of $\mathbf{E} \cdot d\mathbf{l}$ around a loop, the integral of the electric field around a loop, is minus the derivative of the integral of $\mathbf{B} \cdot d\mathbf{A}$ -- that is, minus the derivative of the total magnetic flux linked by that loop. OK, that's sort of the source of induced voltage.

OK. The third one we're going to be interested in is flux continuity, which is basically saying there is no magnetic charge-- that is, the integral of $\mathbf{B} \cdot d\mathbf{A}$, the net flux coming out of some volume, is zero. OK. So these are what we're going to be concerned with in terms of designing magnetic components. There's also Gauss's law, but we're just not going to use Gauss's law.

OK. Just as a reminder here, \mathbf{H} is magnetic field intensity, which we usually use as amperes per meter as units. \mathbf{B} is magnetic flux density, and that's usually in units of tesla. OK. And there's some relationship between \mathbf{B} and \mathbf{H} , right. More generally, right, \mathbf{B} is equal to $\mu_0 \mathbf{H}$ plus the magnetization that relates to what's going on in the material.

For our purposes, we're going to consider just soft, permeable materials, and we're going to say, OK, we can write \mathbf{B} as being equal to $\mu \mathbf{H}$, where μ is the permeability. OK. And so we're assuming \mathbf{B} and \mathbf{H} are proportional. OK. And remember that μ_0 , the permeability of free space, is $4\pi \times 10^{-7}$ henries per meter.

OK. In general, I should say that this is a simplification, right, that happens when you have magnetic domains that can align due to an imposed field. More generally, one might have B is some function of H , and in fact, it might even be a function of the instantaneous H field and history. OK. So in general, the relationship between B and H can be somewhat complex, and we'll see that moving forward. But for simplicity, let's just think about having some permeability μ which is some relative permeability times μ_0 .

OK. So those are our basic rules that we're going to work with, and hopefully, you recall these from introductory physics or any more advanced magnetism class. Let's consider calculation of an inductance, right. So this is something that usually most introductory physics classes do. And I want to go through it in part to remind you of how we use these rules to get to it and because it'll also set up sort of the better way to do it-- or not the better way, the simpler way to handle all these things.

OK. So let's suppose we just have a toroid. OK. So here we have some toroidal structure. OK. And I'm going to come up and I'm going to put a set of windings on it, right.

So here's some set of turns that I'm going to put on it, of wire, right, like this. And let's suppose I put N turns of wire on it, and I have some current flowing into it, i . Right, so there's N turns of wire, each with-- N turns of wire that has a current i through it.

OK. And what I want to do is essentially find out what the inductance of this structure is, OK, and I'm going to make some approximations. I'm going to assume that this core, magnetic core, OK, has some permeability μ which is much greater than μ_0 . It has some cross-sectional area that I'm going to call area of the core, some length that I'm going to call the length of the core. OK. And I'm going to assume that sort of this radius is fairly large compared to the dimension of the core, such that the fields inside the core are roughly uniform, right. So I'm kind of taking the simplest case just so we can see what's going on.

OK. What do I want to do to find the inductance of this inductor? OK. I'll kind of outline it as three steps. One, I'm going to find the H field in the core, then I'm going to find the flux in the core. OK. Then I'm going to find the flux linkage in the core, λ , and we have λ is equal to Li .

In other words, the flux linkage in the core is proportional to the current, OK, and I want to find that proportionality constant. And if you don't recall what flux linkage is, we'll come back to it, all right. So any questions about what we're trying to do?

OK, so let's start about finding the H field in the core, right. So what I'm assuming is that there is going to be some H field owing to this-- a set of turns on the core with the winding, there's going to be some H field inside this core, this way. OK. So what do I know?

I know that the integral of $H \cdot dl$ is equal to the integral of $J \cdot dA$, right. Well, I'm going to integrate around exactly this loop like this, and how much current is piercing that loop? Right, if the loop's going through the middle of the core, well, I have i and i and i , so there's N times i going through that loop, right. So the integral of $J \cdot dA$ ought to just be equal to Ni .

OK. So if I then take that, I can say that, OK, H integrated around the loop, if I assume it's uniform, right, what I'm going to get is H times the length of the core is equal to Ni , or H is equal to Ni over the length of the core, OK, where this is, like, $2\pi r$ of the middle of the core. Any questions about that?

AUDIENCE: So is the core kind of like that edge in this circular pattern?

DAVID
PERREAULT: Yeah, I'm assuming that the permeability of the core is really high and-- yes, so actually-- and in fact, it doesn't, in some sense, matter, right. I mean, the only assumption isn't that the core bends it. I'm just assuming it's everywhere around it.

If you were worried about that assumption, you could say, well, suppose I distributed the turns exactly evenly around it the whole way, right, then everything would be sort of rotationally symmetric, so H ought to be the same everywhere, right. And then even if I didn't accept that the field I cared about was going around the core, then it would be legit. Great question.

OK. So what do I got to find next? I got to find the magnetic flux. OK. Well, to find the magnetic flux, I'm going to assume that B is equal to $\mu_{\text{core}} H$, right. So I'm interested in finding the magnetic flux in the core.

OK. And that magnetic flux is traveling around the core, OK, and it's proportional to the H field. OK. This is flux density, right, and the flux itself is equal to the B in the core times the cross-sectional area of the core, right. The net flux is the flux density times the area, OK, so the total flux traveling through this, if you want to think of the core as sort of a pipe for flux, is the B in the core times the area of the core. So I can work this out to be, OK, μ_c times N over l_c times the A_c .

OK. So here's the net flux in the core. OK. The flux in the core is proportional to the current with this proportionality constant. OK. Any questions about that? And so in total, I have some flux density, and so hence there's some net flux in the core that's kind of traveling around this loop.

OK. All right. So what do I need next? What I want to find next is, What's the flux linkage? OK, well, what is the flux linkage?

The flux linkage is the total flux linking this net winding. All right. What does that mean? Well, I can think about it a certain way. There's some flux coming around here, and it's piercing through the shape caused by this N -turn winding. And this helical winding, you could think of, if I dipped this whole helical winding in a bath of soapy water and I'd get some kind of surface that's spiraling up, how much net flux pierces that surface is what the flux linkage is.

Another way to think about it is, OK, I have this one turn coming around, right. So it kind of came around like this into here. And so I can think of sort of this point here is kind of a loop through which the flux pierces once, and then there's another turn, the next turn, the flux pierces that once, another turn, the flux pierces that once.

And so λ , the flux linkage, the flux linking the whole N -turn winding, is just simply N , the number of turns, times the flux in the core. OK, does that make sense to everybody? So what does that mean?

λ is, then, if I just substitute in here, I get λ is equal to $\mu_c A_c$ over $l_c N$ squared times i . OK. And I just said the relationship between the flux linkage and the current is L , so this thing here is inductance. Any questions about that?

Now, what do we mean by that, or what is the use of that? Well, voltage, the voltage on the winding is equal to $L di/dt$, but it's also equal to $d\lambda/dt$. Right, the voltage that I see at that winding, so if I have i here, this voltage here is going to be equal to $d\lambda/dt$, where $d\lambda$ is the-- λ is the flux linking the winding.

OK. Well, why is that the case? That comes back to Faraday's law, right. We said Faraday's law said, OK, if I had, for example, one turn here coming around, the total integral of $E \cdot dl$ around that gives me an MMF which is equal to the flux linking through that winding.

So the voltage on one turn is $d\phi/dt$, right, because the integral of $B \cdot dA$ is ϕ , right. The voltage on two turns total is $2 d\phi/dt$, et cetera. So for N turns, it's $N d\phi/dt$, so it ends up being $d\lambda/dt$. That makes sense to everybody?

Each turn's going to see a voltage-- this turn is going to see a v_1 turn, which is $d\phi/dt$. The voltage across the whole thing is $N d\phi/dt$, or $dN\phi/dt$, or $d\lambda/dt$. OK. So when I put that together, that just gives me v is equal to $L di/dt$, which is sort of because this is equal to d/dt of Li . And we get the formula for inductance that we know and love, or formula for how an inductor behaves that we know and love. Yeah?

AUDIENCE: Could you comment on the negative sign?

DAVID I'm sorry?

PERREAULT:

AUDIENCE: Could you comment on the negative sign in Faraday's law?

DAVID Yeah, I mean, that's generated by Lenz's law. And what that essentially says is the sign of the voltage-- if I have a $d\phi/dt$ going through, say, one loop, the sign of the voltage is such that the current induced by that voltage in an external circuit would oppose the rate of change of flux. OK. And so what that says is that for a positive current into this, I'm going to get an induced voltage this way for the $d\lambda/dt$.

PERREAULT:

OK. So it basically comes back to Lenz's law. Excellent question. Any other questions? And we'll see how that plays out in the systems we're going to create.

OK. So that gives me inductance. Let me point out just a couple little things about inductance. The first thing I'll note is, notice that inductance is not proportional to the number of turns, but it's proportional to the number of turns squared.

Why is that? Because the H field I induce, I get N times i , so there's a factor of N there. And then the flux that gets linked, the flux linkage has another factor of N , so there's an N squared in the inductance. OK. So if I double the number of turns, I don't double the inductance, I quadruple it.

All right. And in fact, a lot of manufacturers, when they give you a core, will tell you the inductance of one turn. They'll give you something that's often called the A -sub- L value, or the specific inductance, right. So it's sort of the nanohenries for one turn, right, and if you want the L for your inductor, it's equal to N squared A sub L . Right, so if A sub L is a hundred nanohenries per turn, you've got to multiply-- a hundred nanohenries at one turn, you've got to multiply it by the square of the number of turns, not the number of turns.

OK. The other thing I'll note here is that notice that the inductance here is proportional to the permeability of the core, right. And we kind of made the approximation at the beginning that the permeability of the core is large compared to μ_0 , in order to make this simplified calculation work. The problem with having a high permeability-- and that could be a thousand or more-- is that it's not a very stable number, right.

It could be plus or minus 25%, even in the ideal case, for a lot of core materials, and that's before you talk about temperature or flux or anything else going and changing, right. So this is not a great way to make a very stable valued inductance, so we might want to think about, How do we deal with the variability that often turns up in the permeability of core materials when we design inductors? OK, and we'll see that. Any questions?

OK. Now, before I jump into, How do we do this quicker than writing Maxwell's equations? let's consider one more example, and what I'd like to consider now is an example with a gap in the core. OK. So I'm going to have-- I'll draw it as a C core this time. It doesn't have much to do with the actual shape of the core.

OK. So here's my core. OK. So I'm going to put N turns on this core. I'm going to put a current i in here again. I'm going to assume that this core, the actual core section, has a length of core and a cross-sectional area core and a permeability of the core which again I'm going to assume is very high.

Then I'm going to have some gap here, and maybe that gap has nothing in it. Maybe it's just air. It's the permeability of free space, right.

So maybe I'll assume that this is the area of the gap, this spacing, which might be the treated the same as the area of the core. It has a permeability which might be the permeability of free space, and something that's the length of the gap. All right, just this is the length of the gap here, l_g . OK. Everybody see that setup?

So how would I analyze this structure? OK. Well, I would follow the same exact pattern, OK, except that because part of the structure has high permeability and part of it has low permeability, finding the H field is a little bit trickier, or finding the flux is a little bit trickier. So if I were to follow my first step, with Ampere's law, right, what I'm going to do is I'm going to get the integral of $H \cdot dl$ is going to be the H in the core times the length of the core plus the H in the gap times the length of the gap. OK, and that's going to be equal to Ni .

OK. All right. The question is, How do I deal with the H in the core and the H in the gap, or how are they related? OK, where we get that is we come back to flux continuity, right.

Flux is neither created nor destroyed, right. There's no magnetic charge. So that means whatever flux here comes out of the core, whatever flux enters a little pillbox at the edge of this core, must also pop out the other side into the gap, right. So what that roughly means is that the B in the core times the area of the core must equal the B in the gap times the area of the gap.

OK. And if I then assume-- let me just assume, for sake of argument, that the way this flux goes, I'm just going to treat the area of the core and the area of the gap the same. OK. So then I can just say, OK, if A_c equals A_g , then that means the B in the core is equal to the-- B in the core is equal to B in the gap. OK. Actually, I don't even have to make that approximation. Let's just leave it like this.

Actually, it's even better if I leave it like this. OK. The B in the core times the area of the core is equal to B in the gap times the area of the gap, which is also equal to the total flux. If there's some flux running around this loop, it looks like this. Here's the total flux that's traveling around this loop. And so all I'm saying is whatever flux is coming down this pipe in the core is jumping out into the gap, right, so I can say that each of these is equal to the flux, OK, in the core.

All right. So then maybe I can write this as $\mu_{\text{core}} H_{\text{core}} \text{ times } A_{\text{core}}$ is equal to μ_{gap} or $\mu_0 \text{ times } H$ in the gap times the area of the gap is equal to ϕ in the core. OK. So I could then say, OK, H_{c} is equal to ϕ in the core divided by $\mu_{\text{c}} A_{\text{c}}$, and H in the gap is equal to the flux in the core divided by $\mu_0 \text{ area of the gap}$.

OK. So now let me just go substitute that in here, and what am I going to get? I'm going to get H_{c} , which is this, so I'm going to get ϕ_{c} -- I'll just call it ϕ -- eh, I'll call it ϕ_{c} -- $\phi_{\text{c}} \text{ times } L_{\text{c}} \text{ over } \mu_{\text{c}} A_{\text{c}}$ plus $\phi_{\text{c}} \text{ times } L_{\text{g}}$ over $\mu_0 A_{\text{g}}$ is equal to Ni .

OK. So let's use that. So now what I'm going to get is the following. I can write ϕ in the core is simply equal to Ni divided by this bracketed expression, $L_{\text{c}} \text{ over } \mu_{\text{c}} A_{\text{c}}$ plus $L_{\text{g}} \text{ over } \mu_0 A_{\text{g}}$.

OK. And then I can just say, OK, that's fine. That means the flux linkage, λ , is simply N times the flux in the core, right, and that's going to be N^2 divided by $L_{\text{c}} \text{ over } \mu_{\text{c}} A_{\text{c}}$ plus $L_{\text{g}} \text{ over } \mu_0 A_{\text{g}}$ times i . And this thing, again, OK, is what we call inductance, L . Any questions about that?

So we've again used Maxwell's equations to come up with the inductance of our structure. OK. I'll point out a couple of things about this. Suppose if-- as long as I have-- if I have A_{c} is on the same scale as A_{g} -- in other words, the gap area and the core area are about the same thing-- as if $L_{\text{g}} \text{ over } \mu_0$ is much greater than $L_{\text{c}} \text{ over } \mu_{\text{c}}$.

And remember that μ_{core} , I'm assuming a very high permeability relative to μ_0 . And if it's much higher than the length of the gap, then what I get is L is approximately equal to N^2 over $L_{\text{g}} \text{ over } \mu_0 A_{\text{g}}$. OK. In other words, this term no longer matters.

And then I get an inductance that doesn't really depend upon the exact value of the core permeability because it's dominated by this term. OK, so I'm setting my inductance based on geometry, say, based on the area and length of the gap rather than any material parameter, and that could be a very desirable thing because now some variation doesn't matter anymore. Any questions about that?

What does that mean as a practical matter if I do this, if I get into this situation? What I'm really saying is that the magnetic energy stored-- right, I get an energy storage of $1/2 Li^2$ -- right, we all know that the energy stored in an inductor, a linear inductor, is $1/2 Li^2$, right? Where is that energy stored? It's stored in the magnetic fields, right, but where in this structure is the energy stored?

Well, the magnetic energy storage density is equal to $1/2$ -- or, actually, the total magnetic energy stored is going to be $1/2$ the integral of the volume $B \cdot H \text{ dv}$. OK. In other words, it's the product of the B and H fields integrated gives me the total energy stored. OK. Now, if the area of the core and the area of the gap are the same, right, we know that the B in the core and the B in the gap are the same, right. But the H is $B \text{ over } \mu$, and that means that the H field in the core is going to be really small compared to the H field in the gap.

OK. So the flux is sort of continuous. It doesn't change as you're going around, OK, the flux density, but the magnetic field strength does change. And what that means is you'll have very high magnetic field strength in the gap.

So if I'm drawing my magnetic field H , I'll have a lot of field in the gap and very little in the core. OK. And as a consequence, all the energy is actually-- or the dominant portion of the energy is stored in the gap. And that's why I sort of don't care about what the core is doing, because all the energy's stored here, if I've met that approximation.

So we often do that, put a gap in it, firstly because it makes us insensitive to the actual material parameters, which can vary a lot. And, B , as it turns out, because of the limitations on how much flux you can have, you can get a lot more energy storage if you put a gap in the core. OK. And what we're really doing is we're kind of focusing the magnetic field down into this space and storing the energy here. Any questions about that?

Now, as you might imagine, we can do these calculations and use Maxwell's equations and get to all these results. And that's great, but it's kind of tedious, right. It would be nice if there was a much faster way to do these calculations, both to figure out things like inductances, but also to figure out, What are the magnetic fluxes here and there?

What are what are the flux linkages I get on different windings? I might have a transformer with more than one winding, right. How can I do that easily? And the way we do that is using something called a magnetic circuit model, OK.

And here's the idea. Let me come back to this equation here, right. This sort of looks like the following, roughly speaking.

This equation kind of looks like v is equal to-- I'm sorry, I is equal to v divided by R_1 plus R_2 , right. If I have a resistance, remember, if I have a resistance of a resistor, how do I calculate that? It's the length of the resistive structure divided by the cross-sectional area divided by the conductivity. It's length over conductivity times cross-sectional area.

That's just the sort of thing here, except that instead of having conductivity, I have permeability. All right. So this sort of looks like I is equal to v over R_1 plus the quantity R_1 plus R_2 . And notice that that flux is passing through the core and the gap in series.

So if I thought of flux as kind of like being like a current, maybe I could make an equivalent circuit model that talks about fluxes and currents-- or MMFs, as we'll see in a second-- and use that as a means of quickly calculating things. OK. So let's take a look at that. In our model, we're going to define Ni , quantities of Ni -- or maybe I should write this in yellow-- as being what's known as a-- and sometimes we use the symbol F , script F -- as being what we call an magnetomotive force, or MMF.

OK. In some sense, the N turns with i in there are sort of the thing that's driving flux in the system. Right, no i , no flux. No turns, no flux.

So the thing that's trying to push flux is this Ni . It's the magnetomotive force. This is sort of like our voltage.

Then I have flux. This is just simply flux, right, which I'm going to think of as something as I've sort of expressed it, is flowing. That's what's going through the magnetic circuit.

And then the last thing I'm going to have is something I'm going to call-- I'm going to use script x-- a reluctance, which is l_x , the length over μ times A_x . This, I'm going to call it a reluctance. This is sort of like a resistance, right, where in some sense, μ is sort of conductivity for magnetic flux, all right, because reluctance gets big-- or resistance gets big when I have higher length of the structure.

Resistance gets small when I get bigger area of the structure, and then there's the conductivity term in an electrical resistor. Here, its resistance to flux, and we would call this a reluctance. OK. Does that make sense to everybody?

So here's the idea. If I have this structure, where I'm going to have N and i , and that's going to drive a flux around this loop, the way I would model this is with an MMF, Ni . That's what's trying to push flux through the circuit.

It has to travel through the core, and I have a reluctance of the core which is l , the length of the core, over the permeability of the core and the area of the core. OK. Then it has to go through the gap, right.

The flux is going around the gap, and I get the reluctance of the gap is equal to the length of the gap over μ_0 divided by the area of the gap. OK. And what I'm going to get is, flowing around this loop is ϕ , is flux. Does that make sense to everybody?

So this is what's known as a magnetic circuit model. Right, so if you tell me how much Ni and total reluctance is flowing through, I can calculate flux, and it turns out exactly into that equation, right. Flux is Ni divided by the sum of the reluctance of the core and the reluctance of the gap.

It's exactly that equation. Everything else fell out from it. Does that make sense to everybody?

So to the extent that we can break our structures into pieces of sort of flux pipe, if you will, or high permeability-- they're sort of like our wires, if you will, they may have some resistance, but they're like our wires-- and then some high-reluctance, low-permeability regions which flux is being forced to cross, this lumped model is pretty good, and it's a lot easier than starting to write Maxwell's equations.

OK. Now, what does it take for a circuit model to work? Right, I mean, I've sort of told you what the across variable is. That's MMF.

The through variable is flux, and I have my reluctance elements. But what do I need to make a circuit model work? I mean, why should it work for the general case?

Well, what do I have in an electrical circuit? I have KVL and KCL, right. Well, let's come back over to Maxwell's equations. Basically, flux continuity says the integral of $B \cdot dA$ going into some region as zero, right. That's essentially saying the net flux going into any piece of space must add up to zero.

That is exactly the magnetic circuit equivalent of KCL, right. And in fact, if I think about KVL and KCL for electric circuits, they also derive from Maxwell's equations. They're actually approximations, right. KCL in the electric circuit, I'm thinking that there's just no charge buildup. It's a conservation-of-charge thing here.

Here, I'm using flux continuity to justify KCL. What am I doing here? I'm saying that the sum of the Ni 's, if you will, around the loop, or the sum of Ni times ϕ times R_c plus ϕ times R_g , is giving me Ni , that is essentially Ampere's law.

Right, Ampere's law is saying, How does the MMF that I generate relate to the flux I get? OK. So this sum around this loop is exactly what Ampere's law is doing. OK. So this is a pretty good tool for handling the general case.

OK. Let me stop there and just see what questions we have. Well, before I do that, so let me just clarify, right. In an electric circuit, I have an EMF, and I'm saying, in my magnetic circuit, what I'm going to have is-- or maybe I'll draw it this way, in yellow-- I have my MMF Ni .

In an electric circuit, I have current. Whereas, in my magnetic circuit, I have flux, and in my electric circuit, I have resistance. R is equal to l over σA for my resistor, and in my magnetic circuit, I have a reluctance which is l over $\mu \times A$.

OK. So these are just the mappings between my electric circuit and my magnetic circuit, and I have KVL and KCL in each case. OK. So let me pause there. Are there any questions about this model?

AUDIENCE: What exactly is the area of air gap that's in this?

DAVID PERREAULT: It doesn't have to be-- if I imagined that this gap was really small, OK, so I have a tiny gap and a big, say, cross-sectional area, most of the flux just kind of goes straight across, right, like this, and maybe I get a tiny bit of fringing out at the corners.

OK. So for the most part, if I ask, Where is flux going? there's not a lot-- I mean, flux always leaves the surface perpendicularly, right. So there's not a lot of flux that's bending out so that the effective area through which this is going in the gap is about the same as the core. Now, if I had a really big gap, what I get is a bunch of flux doing this kind of thing, right, so then the effective area through which it's traveling out in the free space is effectively bigger than it is in the core.

But if I consider this case where I have a very short gap-- you know, I can ignore fringing, and so I just treat area core as the area gap. If you said I suddenly have a tiny bit of core material and a lot of stuff going on, that's very hard to calculate.

Magnetic circuits tend to work well when I have sort of a lot of high-permeability material to carry my flux and little spaces like high-reluctance regions to which it's going to jump across short distances. That's the best case for these magnetic circuits. Great question. Other questions?

So what does this kind of thing let us do? Well, it means that you can say, for example, suppose I suddenly said, you know what, I'm going to build this structure. I'll say the whole thing has a cross-sectional area A . But here I go, and I'm going to put my winding on the center leg here.

And this is actually a kind of structure you might build, in fact, for something, right. So I have N , and I have i . And I might say, OK, I have some dimension A here, and I know the cross-sectional area of the whole thing, and so forth. How could I find the flux and inductance of this structure?

I might recognize that flux is going to come up here. Some of it's going to go this way, and I'll have ϕ_1 . And I'll have some flux that goes this way, and I'll have ϕ_2 . And this flux in the middle will be ϕ_1 plus ϕ_2 , right.

Can I calculate that structure? Yeah. Why? Because I could say, OK, I have N_i , OK, and then I say, there's this block of material and I'm only drawing this 2D. There's this block of material going up the middle, and maybe I'll call that sort of-- I don't know what I called it in my notes. Maybe I call that R_3 .

And then there's some path through the core that goes this way and this way. Maybe I'll call that R_{c1} . And there's some gap here, and I'll call that R_{g1} . Right, and the same on the other side, there's some R_{c2} and some R_{g2} , right.

And then I could say, OK, well, you know, KVL, KCL, and this magnetic circuit all apply, and I can find the total flux ϕ . So I could just say, look, the net reluctance looking up into here is simply equal to r_{net} , is simply equal to R_3 plus R_{c1} plus R_{g1} in parallel with R_{c2} plus R_{g2} .

Right, I now have the net reluctance. I then get ϕ as being-- the ϕ in the center leg as being equal to N_i divided by R_{net} . And then I can find L is equal to N^2 over R_{net} times i , and this is the inductance, right.

So, boom, just by knowing the geometry of each of these things, I calculate inductance, and I'm done. All right. And I didn't have to start writing Maxwell's equations or anything else. Does that make sense to everybody?

Now, this is fully as legitimate as electric circuits, deriving it down from Maxwell's equations. I will say that there are some kind of limitations that make it harder to use than an electric circuit. Why is that?

If I go build some electric circuit like this, I go get a battery and some resistors and stuff, it's a pretty good bet that my current's flowing around this loop, right. The difference in electrical conductivity between my conductors and my insulators might be, like, 12 orders of magnitude, right. So if I have "10 to the 12th" difference in conductivity between my wire and the stuff that's around it, all the current goes through the wire, right. So KCL is pretty darn good, right.

On the other hand, in a magnetic circuit, what's guiding the flux? What makes this reluctance element work? Well, instead of conductivity, which is guiding where electrical currents go, I have permeability determining where magnetic flux goes.

And typical magnetic materials that you use in power applications might be 10 to the 3 or 10 to the 4th times μ_0 , OK, and not 10 to the 12th. And so what that means is that the things that are my conductors, like my cores-- right, my conductors of the magnetic flux are my cores-- only have 10 to the 3 or 10 to the 4th times the magnetic conductivity as the space around it. So magnetic circuits are a kind of leaky, if you will, right.

It'd be like building an electric circuit where your insulators are crummy and charge is kind of like-- currents are kind of flowing around in other places, too. So magnetic circuits tend to be more approximate than electrical circuits in doing calculations.

OK. And it becomes particularly hard when you start to say, OK, I've got really big gaps and flux is going out here a lot, right. Then it's not that the model is wrong, it's just hard to figure out what the correct area is and what the correct reluctances and so forth are, right. So magnetic circuits, you've just got to think of them as being a little bit leaky.

OK. And so typically, what we're relying on is sections of core that are very low reluctance and then small pieces that are fairly high reluctance in order to make the calculations very accurate. But you can do them one way or another, and they're also a great thinking tool even when you can't.

The other thing I will say is that in an electrical circuit, our unit of conductivity is sigma, right. We have J equals sigma E , like Ohm's law, right. And materials that are conductive, they tend to follow that law over a pretty wide range of electric fields and current densities-- not always, right. Like, you could run into limits of that.

But in magnetic structures, what we'll see is, if I have the relationship between B and H , right, it tends to do something like this, that we'll see. If this is the μ of the core material, it's above some saturation flux density, right, this drops off and starts to become μ_0 . Basically, all the magnetic domains align, and it doesn't behave very well anymore.

And so my approximation of thinking of just something as some permeability μ core only applies over some ranges of flux densities. So I might have to go back and put that into my thinking, too. I can't use these out to infinite flux densities the way I might think-- I might be less hesitant to do in a conventional electrical circuit.

So there are some limitations on this model, but, boy, does it make your life easy, OK. And we're going to take great advantage of it moving forward. Any questions? OK, great. We will take up more on magnetics and magnetic circuits tomorrow-- sorry, Monday.