

[SQUEAKING]

[RUSTLING]

[CLICKING]

DAVID

PERREAULT:

OK why don't we get started. Welcome back from spring break. I hope you guys got a little rest in.

So what we're going to talk about the next few lectures is what is done when one gets to higher powers in power electronics. And in particular, we're going to talk about three-phase power, or more generally, polyphase AC systems.

And let's perhaps first start by motivating it. Why would one think of wanting a polyphase system? And here's one of the notions.

Suppose I had-- I was going to build a machine. So I'm going to have some rotor or some place where I'm going to make something that's going to try to spin. And so maybe I put a coil on it. And here, I'll draw some stator side coil, and I'll drive it with some voltage source. And if I do that, I can push some magnetic field here.

So maybe I can create an x-directed magnetic field-- B_x . So the notion is if I create-- in this case, I'm showing the magnetic flux density. If I create some x-directed flux density, I can cause some magnet or some piece of ferromagnetic material to align with that direction.

So if this was going to give me-- I could create here, maybe B_x is equal to $B_0 \cos(\omega t)$, for example. And then if something was spinning, I could transfer energy to that.

On the other hand that's only one direction. But imagine I came and said, all right. That's one phase. This could be a sinusoidal voltage I'm driving with. But perhaps I created a second one like this that's perpendicular to it. And maybe with this one, I could create some y-directed magnetic flux density.

Well, maybe the orange set of coils here could create B_y is equal to $B_0 \sin(\omega t)$. If I did that with both of those, what would I get for a net magnetic field? I could then get a total magnetic field, or a total magnetic flux density as I'm drawing it, that B_x is $B_0 \cos(\omega t)$ in the \hat{x} direction, plus $B_0 \sin(\omega t)$ in the \hat{y} direction.

So I'm using one coil to give me the x-directed and one coil to give me the y-directed. And notice that-- so these are-- the two coils are perpendicular in space or orthogonal in space. But the other thing is I'm making the two drives of them such that they're orthogonal in time as well. So they're perpendicular in time. And why would I do that?

Well, let's just think about the magnitude and phase of the net flux density in this space. What I would then get is I would get the magnitude of B is the sum of the square roots of these is just going to be B_0 because cosine squared plus sine squared is 1. So the net magnitude of this between x and y is just going to be 1

And the angle of the net flux density is going to be the arctangent-- the angle in space is going to be the arctangent of sine of ωt over cosine of ωt , which is going to be equal to ωt .

So what that says is if I play this game, what I'm going to get is a net magnetic field that has constant amplitude and rotates in space with an angular frequency ωt . So then if I put some of salient ferromagnetic material in here, or I put a magnet, a pole pair in here, I could make that rotate in space.

So the concept-- and this concept of using spatially and time orthogonal signals to generate a rotating magnetic field is-- the first person who really made it practical and used it for practical purposes was Nikola Tesla. And that's the first thing that made him famous was this is how you can build AC machines.

And so this is shown with two phase. And, in fact, Tesla's original systems were two phase. It turns out you can do the same thing with three phases.

So you could imagine putting in-- instead of having two that are 90 degrees out of phase, you could imagine having a first one that generated something like this direction, a second one that generated a second direction-- second set of coils to generate a second direction that was this. And a third set of coils-- let's see. I've got orange, pink-- a third set of coils overlapped in space that would generate a third direction.

And it turns out that if I, again, make the coils phase shifted in space and phase shifted in time, we can get a rotating magnetic field. Just to demonstrate this to you, Mansi has been very kind to set up a demonstration of this. And this is a modern version of a demonstration that Nikola Tesla himself did at the Columbian Exposition in 1893.

**MANSI
JOISHER:**

OK. So today we have a demo to visualize a rotating magnetic field. The demo here is an adaption of Tesla's Egg of Columbus demo from 1893. Tesla used a two-phase inductor, but we're going to use a three-phase toroidal inductor here, and that's going to be our stator. For our rotor, we're going to use this metal egg. And then on this side, we have our three-phase inverter with six switches. And this is our gating circuit.

So what's going to happen is that we're going to create some rotating magnetic field using our stator, and then that's going to induce some eddy currents in our rotor. That is our egg. And then it's going to force the egg to spin.

So let's start by trying to slowly increase the bus voltage and see what's happen-- what's going to happen. And then I'm plotting the three-phase currents there. So they should be phase-shifted by 120 degrees.

So I'm slowly increasing the bus voltage for the inverter, and we see that the phase currents slowly increase. And as I increase the voltage, we see the egg is slowly spinning. As I go higher, the egg starts spinning faster, faster. And then eventually, due to a gyroscopic action, the eggs going to start spinning at it's end, and it's going to start rotating here.

So what we have here is that we have a rotating magnetic field from our stator, and it's forcing our rotor to rotate. And that's the key concept behind an induction motor.

[APPLAUSE]

DAVID PERREAULT: Thank you, Mansi-- a while industry founded on that demo. So let's talk about three-phase more broadly. So if I have a three-phase voltage set, maybe I could have V_a of t is equal to $V_s \sin \omega t$. V_b of t is equal to $V_s \sin(\omega t - 2\pi/3)$. And V_c of t is equal to $V_s \sin(\omega t + 2\pi/3)$. So we're going to have three sinusoids in time shifted by 120 degrees, or $2\pi/3$, from each other.

Now, recall that we can write any of these sinusoidal time domain waveforms as phasors. So I can write V_x of t is equal to the real part of $\hat{V}_x e^{j\omega t}$, where \hat{V}_x is the phasor, which indicates the magnitude and phase of the sinusoid.

So in this case, I could write $V_s \sin \omega t$ as being equal to the real part of $V_s e^{-j\pi/2} e^{j\omega t}$. And in this case, if I plotted-- this is my-- this could be my phasor \hat{V}_A , which says it has a magnitude of V_s and a phase of minus $\pi/2$. So on the real and imaginary plane, on the complex plane, it might look like this, where it has an angle of minus 90 degrees with respect to 0 and an amplitude of V_s .

And if I then said if this was \hat{V}_A , then I could have, say, \hat{V}_B would be here, and \hat{V}_C would be here. And so that's a snapshot of the magnitude and phase of each of these. When I'm multiplying by $e^{j\omega t}$, that means I'm taking this set of elements and rotating it by an angle ωt . So I could think of these phasors as spinning in time.

And the real part, which is the projection to give me the time domain waveform, is really the instantaneous projection onto the real axis. So at time equals 0, V_A is $V_s \sin \omega t$ has 0 value because sine wave is 0 at time equals 0. If I waited until ωt equals 90, it would rotate up, and it would have a value V_s . So we often draw-- we often express our time domain waveforms as a set of phasors. Any questions about that?

AUDIENCE: So the phase on $\sin \omega t$ of $\pi/2$ -- that's just-- that make it so it's 0 equals 0?

DAVID PERREAULT: Yeah. This minus $\pi/2$ is just saying that if I'm a sine wave, a sine has 0 projection at t equals 0. And as I wait and ωt gets bigger, this kind of rotates around and becomes larger, which is what a sine wave does.

So I can create a set of three phases that way and represent them with a phasor. So I can talk only about this component if I desire, understanding that the instantaneous waveform is rotated and projected to get the instantaneous time domain value.

There's different ways we might connect these three voltage sources. So suppose I create three voltage sources this way. The most typical way you see a source connected is in a so-called y connection, also historically sometimes called a star connection. And one might do that this way.

Maybe I would have V_A , V_B of t , and V_C of t . And you notice that what I do is I-- for illustrative purposes, sometimes I draw the voltages' directions in my schematic in the way they're reflected in terms of their phasor diagram, just to show their relations to one another. That's kind of a convention that people follow.

But what I'm doing is I'm connecting one terminal of each source to one node that we're going to call the neutral. And that's the neutral point. And then I have three voltages that I can play with, each with respect to this neutral voltage.

So something where I brought out of this wire, this wire, and this wire are my three phase voltages. And the neutral would be called three-phase four wire. And that's a very common way to connect a source. You can also connect a load that way. And we'll talk about why.

Another way one could connect it is to have a delta connection. So I could have-- for example, I could connect something this way. I could have a voltage VBC connected this way, a voltage VAB connected here. And a voltage VCB-- I'm sorry-- VCA connected here.

And if I did that, then I could bring out these three leads. Now I have no neutral, but I still have A, B, and C, and I can just do the differential connections. So this is-- this is node A. This is still node B, and this is still node C.

Now, this is a less common way to connect a set of sources because if there's any imbalance between these, I would get circulating currents around this loop, So people tend not to connect generators this way. But sometimes you see it done that way. I've seen it done that way before. It's not uncommon to connect loads that way.

So these are two basic-- and this is called a delta connection, for obvious reasons. So you can either have your sources or your loads connected in delta or y. The only difference with delta connection is you have no neutral to trans-- to connect up to other things. Any questions about that?

So why would we do this? Why bother with three-phase systems? And I said people almost exclusively do this at higher power levels. What is the advantage? Well, it turns out there's a lot of advantages of doing a three-phase system instead of a single-phase system. And let's go through those advantages.

I've already shown you one, actually, and that is this one, in the sense that because I can create a rotating magnetic field, if I only had a single phase, if I started misaligned with my rotor, then I could get it going, and it would generate-- it would it would spin. I can transfer energy that way.

But if I started out aligned, it just would never start. It would just sit there. Because I can only generate-- with single-phase, I can only generate one physical direction. Whereas, with polyphase I can generate multiple directions. So that's one reason. That's a zeroth reason I might do it. But what's another reason?

Let's consider power transfer. If I just came up, and I gave you a single phase-- here we go, $V_s \sin \omega t$, and I hook it to a load, R, what is the power that I transfer? Well, this is V_A of t . So power would be equal to V squared over R. So it'd be just V_s^2 over R, \sin^2 of ωt , which through trigonometry I could rewrite as V_s^2 over $2R$ times $1 - \cos 2\omega t$.

So what would that look like in time? If I plotted the power transfer to the resistor in time, that would look like this. I'd have an average value of V_s^2 over $2R$, but it would pulsate because it pulsates because of this $\cos 2\omega t$. And it would do this. It would pulsate between 0 and twice the average value like this.

Every 0 crossing-- this is at twice the line frequency at $2\omega t$. Every 0 crossing is because the voltage went through 0. When I got 0 voltage, I'm not getting any power transfer. So no matter what I do, I'm going to have this pulsating power.

The way that reflects itself is even if I do have a single-phase motor and I'm doing that, I'm going to have torque ripple-- if it's running at constant speed, I'm going to have torque ripple at twice the line frequency.

Or I'm going to have-- if I'm going to have a rectifier, and I want to deliver constant power to somewhere else, I'm going to have to buffer these energy pulsations. And so I'm going to have to put energy storage out there to do that. Yeah?

AUDIENCE: So V_S squared over $2R$ is the average?

DAVID
PERREAULT: V_S squared over $2R$ is the average because the average of the cosine $2\omega t$ is 0. So if I took a time average of this, this term goes away and I get V_S squared over $2R$. And then I get this equal size pulsation. So the peak pulsation is equal to the average value.

So what happens-- what would happen if I walked out and I said, OK, well, let me go-- instead of having one load, suppose I had three. And I come up here and I'd say, OK, let me connect one resistor to here, one resistor to here, and one resistor to here. What would that do? What would that net transfer to those three loads do? And we can write that the same way.

So we could say, all right. Well, the power in this case is going to be equal to V_S squared over R times sine squared of ωt , plus sine squared of ωt minus 2π over 3, plus sine squared of ωt plus 2π over 3.

I can then rewrite these again as being equal to V_S squared over $2R$ times 3 minus cosine of $2\omega t$, minus cosine of $2\omega t$, minus 4π over 3, minus cosine of ωt , plus 4π over 3.

Now, why am I interested in that? If I look at these three terms, these things form a three-phase set at twice the line frequency. This should have been $2\omega t$.

So in other words, if I looked at the phasors for these three terms, they're at $2\omega t$, but they are adding up the same way that those three signals add up. In other words, they add up to 0. So these three things net to 0 all the time. And then I just get a pure DC power transfer of three V_S squared over $2R$ constant power.

So in principle, then, if I had a three-phase rectifier that treated each line the same, it could draw constant power from those three phases and need no energy storage to deliver constant power to the output. Or I would need to have no torque ripple in principle, at least, for power transfer reasons.

So this kind of constant power transfer capability of three-phase is a huge advantage. And the way to think about that intuitively is just that, OK, if I have three-phase shift in sinusoids, when one of them is crossing 0 and isn't delivering power, the other two can deliver power to make up for it and vice versa. So there's always somebody-- there's always a phase that can deliver power and make the power transfer constant.

Now, I should say, by the way, you can do this with two-phase, too. Because if I thought about it, if I had a sine and a cosine, I'd have power as being sine squared plus cosine squared, which also adds to 1. So you can do the constant power transfer trick and the rotating magnetic field trick with two-phase as well as three-phase.

So why might we do three-phase? And here's a very, very valuable reason why you might do it. Let's just think about suppose I go and I want to transfer power over some distance, So I have a generator. I want to send some power some distance, and then I want to throw it into a load.

So here's my sources. Here's my three-phase. I'll make it a y-connected system. And now suppose I wanted to go send this single-phase. Here's my wires. Here's my first wire. And I'll hook my load here.

And keep in mind, these wires have some resistance to them. So if I want to keep my loss down, I need to use a certain amount of size of wire to carry the current that I want to carry. So that's the single-phase system right there. I deliver the wire. I got to buy so much copper to deliver the wire within an allowed loss limit.

But suppose I go out and say, OK, let me go use two other phases. So I'll make-- here's my other conductor here. And maybe I'll connect them on this side as well. Here's another one, and I'll do my third one.

So I'm sending current out this wire and back this wire. And then I'll go do my last one. I'll send current out this wire. I'll connect, and I'll send current back through this wire as well.

So now I'm transferring three times as much power, if all the resistors were the same. But notice that I said all these three voltages and the resulting all these three currents add to 0. And that means that the net current in these center conductors is what? 0.

So suddenly, if what I cared about was the copper loss to send power down the line, I can basically throw away half of my conductors for a three-phase system instead of single-phase system. I need half the copper to carry that much current. And that is a huge advantage if your goal is to transfer power over long distances.

So three-- in fact, AC distribution, if you see what-- you look up, what you're going to see is basically three wires going along in all these towers. That's why is because basically they don't need to carry six conductors do it. They only need to carry three conductors, or three hefty sized conductors, in order to do that.

So that's a tremendous advantage. That advantage also applies to things like motors. So if you wind your motor, you can get away with less-- effectively, less copper to do the same thing. It also applies to fluxes and transformers.

We showed if you're going to have three-phase carrying of flux, essentially what we're doing here is that we're using the other phases to be our return conductors in the same way you can use your other legs in a three-phase transformer to be your flux returns. So there's tremendous advantage in reducing the size of equipment by using polyphase.

Now, I should say-- I'll give you one little tiny caveat to that, which is that if I thought about the insulation rating of these-- so the voltage between this conductor and the neutral is V_S . While I haven't calculated it for you yet, the voltage rating between this phase and, say, this phase is actually the square root of 3 V_S .

So it turns out you need a little bit more insulation if you're going to put this conductor right next to this conductor than if you put this conductor next to the ground. But if I cared about insulation, I could just turn down the voltage, and I'd still get an advantage of the 2 over the square root of 3 in terms of the amount of power I could handle for a given loss.

So essentially, depending upon whether you only care about your conductor loss or you care about insulation is what costs you everything, it's somewhere between 2 over the square root of 3 and a factor of 2 advantage to having polyphase. So it's a really big advantage in practice. So if you're at really low power, maybe you don't need the complexity. But if you want to transmit lots of power, polyphase is really your friend.

What else can we do? Here, I showed my source is connected in y, and my load is connected in y. And that's what lets me throw out a lot of the required heavy copper conductors. But there's nothing that says I couldn't come back here and create a polyphase source that was like this again. Here's VA, here's VB, and here's VC. And here's my neutral.

There's nothing that says I actually have to connect my loads the same way. I could do something different. I could, for example, go connect my loads like this. I could connect my source and delta-- my source in y and my load in delta, for example.

So that means that for each of these loads, if this is $V_{\text{sub } a}$ and this is $V_{\text{sub } b}$, I'm now going to have a different voltage across the actual load resistors than if I did this trick. Well, what would that look like? What does V_{ab} look like? Well, this is where phasors come in really handy here.

So if I said, OK, here's my complex plane, and I'm just going to draw the real and the imaginary axis. I could say that-- let me just keep consistent. VA is-- here's VA, and it has magnitude-- it has phase of minus 90 degrees and a magnitude of VS.

And here's VB. It also has a magnitude of VS. So here's-- this is-- I should say, be careful. This is \hat{V}_A . That's my phasor for VA. This is \hat{V}_B . What would my phasor for VAB look like?

Well, I can do subtraction in the phasor plane. I can just do it geometrically. So what that would look like is VA looks like that. VAB looks like that. VA minus VB, which is \hat{V}_{AB} , would be just VA plus the negative of VB. And it would look something like this. So this would be \hat{V}_{AB} .

And it turns out that the length of this vector is the square root of three VS. This is just basic trigonometry. And the phase of this vector is $\pi/6$. So VAB will lead VA by $\pi/6$. So what I would get is VAB of t would be equal to the square root of 3 VS times sine of $\omega t + \pi/6$ because it leads VA by $\pi/6$. Any questions about that? Yeah?

AUDIENCE: This is just semantics, but is there a reason why we use sine instead of cos?

DAVID PERREAULT: No. I just decided to start with sines today. Yeah. I mean, you could do it with cosines, and then everything would be rotated by 90 degrees.

You know what? The real reason I'm using sines is because they're 0 at $t = 0$, so they're easy to draw. But no. You could do-- in fact, if you don't like it, just weight $\pi/6$ and then you've got cosines. Yeah.

The thing that's important about three-phase, however, is making sure which order is which. And, in fact, we'll talk about this. But they often color-code the leads-- you need these different leads that you're wiring. And you've got to make sure you get your phase order right because otherwise, things rotate in the opposite direction from what you think.

And there's a very famous example of this, where there was there was a famous English researcher that used to make three-phase, essentially, railguns using polyphase power. And you use it to throw a projectile off the end. And he was doing a demo for the, I think, the Prince of Wales, like, the royalty in England.

And in between, he actually-- somebody had shut it down. And he reconnected, and he reconnected it backwards. So instead of throwing the projectile that way, it threw the projectile that way and almost took the head off of the Prince of Wales. So you do have to be careful about what your phase order is. But whether it's sine or cosine, take your pick.

So what this says is, if I did this, I get this scaling. If I looked at the other line-to-line phases-- this is VAB I just plotted-- I will get another three-phase set that essentially looks like this. So I get a scaling by the square root of 3, and I get a phase shift.

And it may be that maybe I have a load that wants to have higher resistance. So that scale of square root of 3 and voltage does me some good. And what we're going to see is there are certain converters where if I can generate a natural phase shift by a square root of 3, I get an advantage. So that can be handy in itself.

But here's the other piece of it. By adding up, I can take transformers-- if I had this three-phase set, I could take transformers, take a piece of this voltage, a piece of this voltage, and a piece of this voltage, and if I do that in a balanced way, I can actually create three-phase sets with any arbitrary phase shift that I want.

And that turns out to be handy for some kinds of converters you might want to build because we can do things like we talked about-- harmonic cancellation or ripple cancellation with interleaving. I have a natural way to generate phase-shifted sets of signals if I have a balanced three-phase set, and I can generate other balanced three-phase sets.

And when I say balanced, I mean the three of them add to 0 instantaneously at any given time. I guess I left off my third phase here, which should have been like this-- VC. Any questions about that?

So in terms of the benefit of doing polyphase, we've said we can transfer constant power, and hence, get constant torque, generate rotating magnetic fields. We can send energy at much lower transmission loss, which is why all long distance things are either done as polyphase AC or sometimes high voltage DC, depending upon what you're doing.

There is another huge advantage to polyphase, and we're going to see this as we start to look at converters, and that's the following. Suppose I went and built a system like this. But let me imagine-- in fact, what I'll do is let me connect up a y-connected load here. So I'll go-- and it doesn't actually matter if the load is y or delta, but it's clearer to see here.

So suppose I have some set of loads that looks like this. Actually, no. Let me back up on that. Let me do the delta-connected load. It's actually easier to see in delta. Sorry. It applies either way, but suppose I do this. Suppose I connect up. I have some load here, and then I connect another load here and then the last load here. So there's no neutral point connection in this thing.

And if I looked at these things-- now, these could be resistors, in which case everything is simple. But if I'm going to start to create rectifiers or other elements, just because my voltages are sine waves doesn't mean my currents are sine waves.

If you think about-- right at the beginning of class we said we could have a rectifier waveform whose current looked like a square wave or something like that. The currents don't have to follow and be linear compared to the voltages. They don't have to be proportional.

So let me just imagine that these are some kind of nonlinear elements. And the only thing I'm-- the only constraint I'm going to place on them is they act in a way that whatever this guy is doing, this guy is doing a third of a cycle later.

So what I'm saying is if this were some current-- I'll call it i_x of t , then this current-- and I'm not I'm not keeping to my earlier color conventions, but this current then would be i_x -- this would be i of x -- i sub x of t minus capital T over 3. I'm getting a phase shift in my currents.

So then what is this current going to be? If I looked at the net line current-- so I'll look at this current-- I'm going to call this i_b of t is simply going to be i_x of t minus i_x of t minus capital T over 3. I have a third of a cycle phase shift, a third of a fundamental cycle phase shift. And he's the difference between something and something shifted by a third of a cycle.

Well, why is that important? Suppose I could write i_x of t as a summation. Let's assume he has no DC component. So this could be n equal to 1 to infinity of i_n sine of $n\omega t$ plus ϕ_n .

That means that i_b of t would be equal to this minus a shift of a third of a cycle, which I could rewrite as n equals 1 to infinity of i_n times sine of $n\omega t$ plus ϕ_n , minus sine of $n\omega t$ plus ϕ_n , plus nT over 3.

So it's the difference between a signal and a signal where I'm shifting by T over 3 in time. But for the n -th harmonic, that is a phase of nT over 3. And what that means is-- actually, that should be-- I'm sorry. This should be $n\omega T$ over 3-- $n\omega T$ over 3. And ωT is just 2π . So it's n times 2π over 3.

So that means every time n is a multiple of 3, I'm taking a difference between something and something with the same phase. So this is exactly the harmonic cancelation game we were talking about in previous classes.

We said with inverters, if I could get two waveforms and shift them by the right angle, I would start to cancel harmonics. Well, this cancels all triple n harmonics. So that means if I have half wave symmetric waveforms, and that depends upon this non-linearity or the rectifier I use, I get no evens. And then I cancel 3 and 6 and 9, and then the lowest harmonic content I would have in the actual line waveform would be the 5th and the 7th.

So suddenly, even though I may have some rectifier that draws very funky current waveforms, I start to eliminate, or actually to be accurate, cancel out harmonic content I don't like. And the three-phase waveform set will give you that very beautifully.

So of all the games I'm talking about, the first couple ones you can do with two-phase. You can get a rotating magnetic field with two-phase. You can get constant power transfer with two-phase.

If I add a phase, I suddenly start to get these balanced waveforms that will cancel triple n harmonics. It will also give me much lower conduction loss in my conductors. So I get much better performance out of three-phase than I get out of two-phase.

Now, you said, well, what if I had four-phase? Four-phase also adds up to 0. And I can get all the benefits I talked about in four-phase, except that I'd cancel different sets of harmonics. So it wouldn't be quite as good in that regard, especially if I'm going to have things that don't have even harmonics anyways.

But the beauty of it is, is that 3 is the simplest number that gives you the most benefit. So people talk about higher numbers of polyphases and people have worked on it. But in practice, you want the simplest thing that's really good, and three-phase is it.

So two-phase gives you some of the benefits. You go up to three-phase, and you get all the benefits you want. And you go to more phases, well, yeah, you can get it, but it's more complicated. So why do it?

So that's a lightning introduction to all the-- or many of the benefits of having a polyphase signal. So if you're in your home, and you're up to a couple of kilowatts, you're going to see single-phase systems.

When you get to higher power levels for motor drives or electric vehicle drives or whatever you have, a lot of that is all done in three-phase. And all of AC power transmission is all three-phase. So if you look up on the-- look up at the towers, you're going to see three lines-- three big lines going across, and those are your three phases.

So I'm going to pause there. Are there any questions about just this introduction to three-phase before we start to talk about the actual power electronics content? All right. We'll wrap it there, and we will pick this up next class. Have a great day.