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DAVID PERREAULT: OK, why don't we get started. So I'm going to start today by recapping our discussion of magnetics and transformers. And we're going to expand on it. I'll just remind you that the sections of the textbook, *Principles of Power Electronics,* second edition that this is in, is chapters 18. And then some of the material I'm going to talk about today is in chapters 19 through 19.4.1. So that's one thing.

The other thing I'd like to remind you is now we're going to have assessments, which are effectively short little mini-quizzes. Just to remind you of the guidelines, you're not allowed to discuss them with anybody else. And the only assistance that the course staff can give you is really to answer the same kinds of questions that we would answer in an exam.

And for the most part, feel free to direct any questions, clarifications just to me directly, preferably by email instead of Piazza. But I'll try to respond to both. But that's just to clarify that. So unlike homeworks, where you guys are welcome to collaborate, ask the TAs or teaching staff anything you want, please just mainly direct those questions to me. Although, the other staff can answer as well. So first of all, any questions before we get going?

AUDIENCE: Is that going to be posted on Canvas after class?

DAVID PERREAULT: Yes, it'll come up on Canvas or Gradescope, I guess. An by the way, these are not designed-- if you're spending 12 hours solving this problem, it's not intended to be that way. They're supposed to be short, relatively short kinds of things to check your understanding on the topic. And usually, the assessment number is of aligned with the material of the homework number. So assessment 1 covers the material in homework 1.

So let's continue talking about transformers or multi-winding magnetic components. We said last time that if I took some core with a permeability and a core cross-section, a length of core, and I put two windings on it, one with N1 turns and another one with N2 turns, I would mainly get-- especially if this is a very high permeability core, I would get flux and flux density traveling around inside the core.

But I might also have some flux that links winding 1, but doesn't make it over to winding 2, and links winding 2, but doesn't make it over to winding 1. Now, when the permeability of the core is really high, the amount these other components matter is less. But nonetheless, we incorporated them.

And then what we did was we created a magnetic circuit model, and then worked back to what the flux linkages were for these two MMF sources. And from that, you can work back to what-- the derivative of the flux linkage is the voltage. And we can work back to the relationships between the voltages and the currents at the two terminals.

And that worked out to look like an inductance matrix. So if you have an inductance, that's one parameter that models everything. If you have a two-port structure-- two inputs, two output-- or two different terminal pairs, then you get a 2 by 2 matrix. And the matrix turns out to be symmetric, as you can see. We derived this in class.

And the interpretation-- and then we took this one step further. And we went quickly. We said, OK, this inductance matrix describes the terminal relations I get ideally between this voltage and current and this voltage and current. And I made the argument in class that we can make a circuit model for this.

So here is the circuit model. This is what we would consider a physically based circuit model for this system. And what did we say? We said, OK, well, firstly, let's ignore the external leakage fluxes that are going outside of the core. If I have only the yellow flux component, if the core is infinitely permeable, the flux density in the core is simply related to the flux and hence the volt seconds at the windings.

The voltage here and voltage here would be scaled versions of one another because they're all just linking the same flux scaled by the number of turns. But what we said was, well, H in the core is equal to B over mu in the core. So if mu in the core is infinite, then H is 0. And hence, we don't store any energy in the core.

But if mu is finite, or not too big, then what happens is we get some H field in the core and B dot H is the energy density in the core. And we store energy in the physical core. And the way that pops into the circuit model is through this inductance that appears across the ideal transformer.

And so basically, the difference in having infinite permeability, where you have no energy storage or equivalently an infinite magnetizing inductance that carries no current, is I get a finite magnetizing inductance that has a current that ends up being a mismatch between the ideal turns ratio of the transformer or the ideal transformation ratio of an ideal transformer.

And this magnetizing inductance is really the reason why we can't apply DC, because I can only have some finite flux density before the transformer saturates. Then we said OK, because I have flux that links winding 1 but may not make it over to winding 2, I get some leakage inductance on this side. And because I have flux linking winding 2 that doesn't link winding 1, I have some leakage inductance on this side.

And basically, 1/2 L leakage 1 I1 squared is precisely the energy that's stored in space by the magnetic field associated with leakage flux 1. And likewise, 1/2 L leakage 2 I2 squared is precisely the energy that's stored out in space of fields that link winding 2 but do not link winding 1.

Basically, if I have some ideal transformation ratio-- I'll call this V1-prime. And this is V2-prime-- which would be exactly proportional to one another by the turns ratio of the transformer, the leakage inductance has caused some mismatch. So first of all, any questions about that?

So this model is a standard physical model for a two-winding transformer. And these parasitics each represent something very specific physically in the world. The other thing I will note about this is that we can get these parameters of the circuit model.

And how do I know the circuit model is correct? Well, if you go back and find the relationships between V1 and V2 using the circuit model, it'll turn into exactly this, so long as you use these parameters. So what this says is that this circuit model can represent this inductance matrix. And the inductance matrix represents the terminal relationships of the device, neglecting loss and everything else.

I didn't say this last time, but you can also go naturally relate the circuit model parameters back to L11, L22, and the mutual inductance, LM. And again, you'll just see that this representation and this representation are precisely the same, as long as you have the right parameters.

And then the last thing I'll note is you ought to be careful whenever you're doing your own analysis like this to recognize that you have to be careful about relating-- remember, the flux that's of interest for the flux linkage to relate to voltage on the electrical side is related to the flux coming out of the positive terminal of the MMF source. So you've got to pay attention to your signs, especially when you have multiple windings. And that comes back to representing Lenz's law. So let me stop there and just ask if there's any questions.

So that was a lightning review of last class. What I'd like to do today is expand out our discussion of transformers in a couple of different directions. One relates to how we might model a transformer. This circuit model, as I said, is a physically based model, in that every parasitic component represents some physical energy stored somewhere in space in the device. And that's mostly what we want to do. I mean, that would be a typical thing we'd want to understand.

But recognize that in terms of-- if all I cared about was matching the terminal relations, I just wanted a black box inside my white box there that mapped voltage and current at terminal port 1 and voltage and current at port 2, what that's captured by is that's fully captured by this inductance matrix description. Now, how many independent parameters does my inductance matrix description have?

It really has three parameters. It has L11, L22, and then LM because an inductance matrix, because a magnetic component like this is a reciprocal element, the matrix describing it ends up having to be symmetric. And you can see that these two elements-- it's not just random that they ended up being the same thing. It has to turn out that way because of reciprocity.

Well, how many independent parameters does my circuit model have? Well, if I picked physical turns ratios-- I know I had 10 turns in the primary and 20 turns on the secondary. I pick 1 to 2 as the turns ratio of the ideal transformer-- well then this has just the right amount. It has L11-- I'm sorry. It has L leakage 1, L leakage 2, and the magnetizing inductance. And I could have put the magnetizing either on the primary and the secondary and scaled it in the right way.

But it, as long as I'm using the physical turns ratio, has the right amount of parameters. There's a 1 to 1 mapping between the two models. However, you might not always want to use the physical turns ratio. I might walk up to you and hand you the transformer. Here's this transformer. I bought it on DigiKey. Come back with the model for it. But you're not allowed to cut it apart because it's expensive. And I don't want to break it.

Well, how do you know what the actual turns ratio, like physically the number of wires on each side are? Well, you don't. So in terms of this model, the turns ratio is also a parameter. So in this circuit model, I really have four parameters I can fiddle with. And if I knew this one to be the physical one, then all the other things turn into physical quantities.

But in terms of modeling things, I could always go back and choose some non-physical value and still create a circuit model that correctly matched the inductance matrix model because here I have four parameters. There, I only have three. And in fact, sometimes, we do that.

We may do that because we don't know some parameter like this one. Or we may do it for convenience purposes. And what I'm doing is I'm throwing out the knowledge of where energy is being stored exactly. And I'm just coming up with something that mathematically gives me the right terminal relations.

So why might I do that? So I might argue that I could create a circuit model that was really just like this-- LA, LB, an ideal transformer-- I don't know what I used. I think I just did to Nx to Ny-- and then LC. And I would call this V1 and I1 and this V2 and I2.

And now, if I put this in a black box, I can just say, OK, I'm not going to worry about whether this physically represents things. But now, I have four parameters to play with. And I can use those four parameters any way I want. Well, what might I do? Maybe I might just want to say, it's easy for me to think about the voltage scaling for a 1 to 1 transformer.

The ideal transformer in the middle here has some turns ratio on it. Maybe I just make that 1 to 1. If I do that, I can then just create a model that has a 1 to 1 transform, even if my real device does not. I could create a circuit model where I force this parameter. I arbitrarily pick this to be 1 to 1.

And then I say, I'm going to have three other parameters that now I'm going to pick those three parameters to match the terminal relations. What would I get in this case? What I would get is L11 minus LM would be LA. LM would be LB. And L22 minus LM would be LC.

So the nice thing about this circuit model is if I know the inductance matrix and all I know is the inductance matrix, this is a perfectly good model for a transformer. The parameters may not represent anything physical. But it still gives me the right relationships between V1, I1, and V2, I2. Any questions about that?

So this is a trick that's often played to make circuits easier to think about. And then I can go remap it. If I'm actually designing a transformer, I can go remap it and get the actual physical model. But I don't need to use a physical model all the time. This is sometimes called the T model. Or they're all called the T model. But this is the T model. This is 1 to 1. There are other choices we might also make. And we'll see an example of that shortly. So any questions about that?

So let's start talking about constraints on the parameters. I said I have three independent parameters. But can the parameters be anything I want? Well, no, they can't, not for a real device anyways. Let's just start thinking about if I just handed you an inductor, L, off-shelf, real inductor, what can I say about the inductance of this thing?

Well, I would argue that, in a real inductor, at least large signal wise, the inductance has to be positive. Why is that? Because if I apply a positive voltage So suppose I applied a step in voltage. Here's V. Or here's V1. What should I1 do? Well, when I step the voltage from 0 to 1, I1 should go from 0 and ramp up. And since V is positive and I is positive, I've got energy going into this device. And this slope, dI/dT, is 1 over L.

What would happen if the inductance was negative? If the inductance was negative, instead of doing this, it would do this. This would be 1 over L. It would be sloping down because L is negative. So this is L less than 0. And energy, voltage times current, would be negative. And energy would be coming out. So I'd apply a positive voltage and my inductor would start squirting energy back at me. I'd love to have one of those things. I could make quite a bit of money off that.

But unfortunately, conservation of energy tells me I can't get that, at least large signal. So what do we know about an inductor? We know the inductance is a practical matter. The inductance has to be positive for anything you're going to wind up with wires and cores, that doesn't have a source of energy inside it.

Well, what could I say about my transformer parameters? Well, you can make a similar argument because what I really did here was I said dI/dT is equal to 1 over L V. That's the argument I just made about my signal terminal inductor.

What could I say about my inductance matrix? If I have a two-terminal device, what I get is V is equal to L dI/dT. This equation here is simply V, a vector equation, is equal to L, where this is an inductance matrix, times I, where that's a vector of currents. So it's I dot, I should say, dI/dT.

I can do the same thing. And I could write, OK, that means that d/dT of I1 and I2 is equal to L inverse times the voltage of vectors V1 and V2. And what does that look like? This is going to look like the inverse of that matrix. So that would be 1 over the determinant, L11 L22 minus LM quantity squared times L22 L11 minus LM minus LM times the vector of voltages V1 and V2.

So this is the [INAUDIBLE]. Suppose I made V2 negative, which would be basically V20, which would be me dropping a short on the secondary side, and then I apply a voltage over here. What should happen? Well, the same thing. If I have a short over here and I apply a voltage over here, I just ought to see some inductance there. And I should see energy ramping up into it.

Well, that only happens-- L22 is a positive number because that's the inductance measured on the secondary side with the primary open. This thing has to be a positive number. Or another way to put it is this inductance matrix has to be positive semi-definite.

So what I get is a requirement that LM, the magnitude of LM, has to be less than the square root of L11 L22 for any physical transformer. In the more winding case that we'll see, it just means the matrix has to be positive semidefinite.

Sometimes, we define a coupling coefficient, k. And k is defined as equal to LM over the square root of L11 L22. LM being equal to the square root of L11 L22 just is basically perfect coupling. What does that mean? That means there is essentially no leakage flux. All the flux in the primary is linking the secondary.

So k is sometimes used to say, in some sense, how much leakage is in the transformer. But more broadly, what we can say is, yeah, I've got three parameters-- L11, L22, and LM. But there are some limits on what you can get in terms of the relationships in the real world. Any questions about that? OK.

Let's start to expand upon this. We started with an inductor, which is basically a one-terminal device, one-port device. Then we did a transformer, a basic transformer, which is a two-port device. But we're not really limited to only two ports. We can have as many ports as we want. And as we'll see, we often do.

So let's think about what might happen if I have more ports. And it turns out now, naturally, because you have more terminal pairs, there's many more ways you can configure things. But I'm just going to show you two that are very basic versions. And what I'm going to show you is the equivalent of the ideal transformer model for these two basic versions that are very common.

Suppose I did this. All I did to create this transformer is I took something that had one winding on and added a second winding on the core path. Well, what happens if I add a third winding on the core path? So I could say, OK, here's my transformer, here's my core. And I'll have N1 turns here. I'll have N2 turns here.

And notice that positive voltage here throws flux around the core this way. So I have a dot here. Likewise, by the right-hand rule, I have a dot over here. Let me add one more. I'll have N3 turns on this one. I'll have I3 and V3.

Well, what would be the magnetic circuit model for this thing if this had some reluctance of the core? It would look like this. I would have N1 I1 N2 I2 and N3 I3. And then I would have some reluctance of the core, which is equal to the length of the core over mu of the core A of the core. And then I would have some flux in the core, which is doing this.

So all I've done is added one more MMF source. What would be the terminal relations of this thing? Well, if I do this approximation, I'm pretending there's no leakage flux at all. That means that lambda 1 is equal to N1 phi core. Lambda 2 is equal to N2 phi core. Lambda 3 is equal to 3 phi core.

So what I get is then phi core is equal to lambda 1 over N1, which is equal to lambda 2 over N2, which is equal to lambda 3 over N3. Or if I differentiated this, what I would get is V1 over N1 has to equal V2 over N2, which is equal to V3 over 3.

So what I've got here is, by adding my third winning, I get another-- just another scale voltage. And we often do that because we often want some ratiometric sets of voltages that our transformer might create. What about the currents? Well, let me do the simplest case here.

If mu core goes to infinity-- or actually, let me just do magnetic circuit KVL around this loop. What I get is N1 I1 plus N2 I2 plus N3 I3-- that's sum of these voltages-- must equal phi core R core. Everybody buy that? OK.

Well, if reluctance of the core equals 0-- or equivalently, that's because the permeability went to infinity. So the reluctance of the core is 0-- then the right-hand side of this becomes 0. And at reluctance of the core goes to 0, what I get is N1 I1 plus N2 I2 plus N3 I3 is equal to 0.

So the voltage relationship just scales from 2 to 3, but the current relationship doesn't. The current relationship just says the sum of the NI's into the windings have to be 0. And what that turns out to mean is all the energy flowing in two of the ports must come out the third of the port instantaneously for this idealized version. Questions about that?

What would I get if I started including parasitics? So what would I get if I started having a non-zero reluctance or I started having leakage fluxes from the windings, that kind of thing? Well, what I would get is instead of a 2 by 2 inductance matrix, I would get a 3 by 3 inductance matrix. And that would capture the parasitic behavior.

This version, this idealized version, we might often draw this way. I'll have multiple windings, all illustrated on the same core. So if you see this symbol-- and this is N1, N2, N3 with V1, V2, V3-- that's what-- this is the ideal relations for this structure, where all the windings are illustrated on this single core path.

And because we can talk about which way each of the currents in each of these windings throw flux around the core, we can draw the dots of the individual windings. And you see this like this. Any questions about that? What would happen if I included non-idealities? I'd surround this thing-- perhaps, one way to do it is I could surround this thing with additional inductances to match with the parasitics. And we'll talk more about that. Questions?

So let's think about other possibilities, however. This is perhaps the most common way to build a three-winding, or five-winding, or whatever transformer. Well, what we're going to get is a bunch of voltages at the terminals that are ideally ratiometrically related because that's the most common thing of what we want. We'll see next week cases where we might want to do that for converter design purposes.

But it's not the only thing you could do. Imagine this alternative structure. I might call this series-wound. They're all wound on the same series core path. What if I did something-- instead of that, suppose I did something like this. Suppose I built a core structure that looked like this.

And I put here's winding 1. And here's V1, I1. Here's winding 2, V2, I2. And here's winding 3, V3, I3. So my drawing is not very good here. But what you can see is now I have three legs. And each leg has his own winding.

That means that the flux going through this here, or the flux linkage here, isn't directly related to these two flux linkages. Or it's not 1 to 1. It's not just scaled because here I only had one flux path going around the core. Ideally, here was my flux path. There was only one flux path.

Here, there's a flux path here, and a different one here, and a different one here. And I get some constraints, but they're not the same fluxes. What does that mean in terms of the terminal relationships? Well, here's a magnetic circuit that I could draw for this thing.

I could say, OK, maybe I will have N1 I1 and some reluctance 1. Here's some reluctance that's the middle path, reluctance 2 and N2 I2. And here's the right half, N3 I3. And I could have phi 1, phi 2, and phi 3. What would be the terminal relations of this guy?

Well, in this case, if I ignore leakage flux, what do I know about these three fluxes? Well, all these three fluxes come up into this top node. And the fluxes must add to 0. So what I get is phi 1 plus phi 2 plus phi 3 has to equal 0 by magnetic KCL. There's no magnetic charge.

Well, that means that lambda 1 over N1 plus lambda 2 over N2 plus lambda 3 over N3 has to equal 0. Or if I differentiate this equation, what I'm going to get is V1 over N1 plus V2 over N2 plus V3 over N3 equals 0. So I no longer get the voltages on the winding scaling. What I get is the sum of the scaled voltages have to be 0.

What can I say about the currents? Well, if I let reluctance 1, reluctance 2, and reluctance 3 go to 0, if I assume I have an infinite permeability core, then all I get is these three MMF sources. And if I'm going to avoid infinite fluxes in the core, what would that require? That would require that N3 I3, N2 I1, and N1 I1 equal each other.

So I would get N1 I1 equals N2 I2 is equal to N3 I3. So in this parallel-wound transformer structure-- or parallel magnetic paths instead of series magnetic paths, the currents in the winding scale equally, ideally, and the sum of the scaled voltages adds to 0. It's exactly the dual of this structure. In fact, these are structural duals, so between series and parallel, so you get dual relationships between voltages and currents. Any questions about that?

AUDIENCE: So it's not intuitive why you had a reluctance for each source and have them between the two sources. [INAUDIBLE]. good for?

DAVID PERREAULT: Well, I just-- yeah, that's a good question. I just said, OK, let me take this first section, like maybe this section of the core, and ascribe one reluctance that this winding's on. And let me ascribe a second section of the core, like here, that this has wound on.

And then the third section of the core is over here. And that's the third reluctance. You could break it up a different way, but that's a one way you could think about it. So I've got three windings and three flux pipes that are all going between the top and the bottom. Does that make sense?

This structure is less used than the other structure, but it's actually quite widely used. Why? Because there are certain cases where maybe I have different parts of my circuit and I want to force them to all have the same current. So if I have N1 equal N2 equal N3, I can force currents to split or be matched.

So if I want to match voltages, that's a great transformer structure. If I want to force currents to be matched, this is a great transformer structure. It just depends what you want to do. And of course, with many windings, there's all kinds of stuff you can do. There's all kinds of magnetic structures you can and might build for different applications.

I should say, by the way, however, notice that, because there's no single flux path, there's not really a good way to indicate the dot convention here. There's something where the currents into the winding scale, but we don't usually represent those with dots. People usually draw the structure or they do something else. There's not a really perfect schematic, widely accepted schematic relationship between those.

Let's talk about the non-ideal. These are the equivalent of ideal transformer relationships for a three-winding transformer. You could extend that for N windings. If I had N windings on one core or N parallel core sections, you just expand those relationships naturally.

What would I get if I included parasitics? I would get a 3 by 3 inductance matrix. So what do I get, in this case? Well, what I would get in the general case is V1 V2 V3 is equal to some inductance matrix, L11 L22 L33.

Because it's reciprocal, I would have a symmetric matrix. So what I would have is L21 would equal L12. L31 would equal L13. And L32 would equal L23 times d/dT I1 I2 I3. Maybe I would write it as I1 I2 I3 dot. So this is V is equal to L matrix I dot for dI/dT. How many independent parameters do I have in this structure?

AUDIENCE: [INAUDIBLE].

DAVID PERREAULT: Six, yes. I have my three self-inductances. Those self-inductances are-- this is the inductance at port 1 with nothing else connected to it. So if I just measured it like it was an inductor from one port with everything else open circuited. I get L11. Same thing for measuring L2, same thing for measuring port 3. Then I have L21, which is equal to L12, L31, which is equal to L13, and L32, which is equal to L23.

So if I have a single inductor, I have one free parameter. If I have a 2 by 2, I have three parameters. If I have a 3 by 3, I have six parameters. What would happen if I had a four-winding transformer in the general case? Well, I'd add a V4 and an I4. And then I'd basically add one more bottom row of independent parameters, four parameters. And I'd have 10.

So it turns out that an N-winding transformer ends up with N times N plus 1 over 2 independent parameters. So it's 1, 3, 6, 10, and so forth. That gets kind of ugly, kind of quick. But that's the way it is. That's the general case. And this inductance matrix, still, for the same reason I said, it has to be positive semi-definite. Any questions about that?

So now, what might I want? I might want the equivalent. I've already said, OK, I have an inductance matrix. And if I come up to my device and I measure V1, and I1, and have all the other I0 open circuits, I can get L11. I can do a bunch of measurements on my device and figure out what these independent parameters are. That can be tricky. If you don't do it carefully, you'll get stuff that's not quite right. But you can get the terminal relationships. And so I can always get the inductance matrix by doing measurements on a given device.

How would I get a circuit model? What would my circuit model look like? Well, that's a trickier question. It's possible to do a physical model like this model with actual physical parameters. But that turns out to very often be quite tricky. And as you can see, this already has a bunch of parameters to it.

But what might we do? Suppose I wanted to just say, hey, I'm not going to worry about getting a physical model. Just give me a model that has just enough independent parameters to represent the three-port, or four-port, or five-port network I have.

Now, there's, in general, a lot of ways you could do that. I'm going to show you one that I particularly like. It's called the extended cantilever model. It's by no means the only way to do it, but it's a very clean means of doing it. So when we do circuit modeling for all kinds of devices, we often use the extended cantilever model. And it goes a little bit like this.

Suppose I had a one-terminal device. Maybe I would come up here and I'd say, OK, here's my one-terminal device. The inductance-- and I'm going to-- maybe I'll just draw this with a ground reference here for simplicity. This is my inductance L. But maybe I'll call it L11. And I'm going to use little l11 to distinguish it from my inductance matrix value.

But if I just had a single device with V1 and I1, I'd need one parameter that I'm going to call little l11, which is just the inductance of that one-turn winding. So I have one parameter here that captures what I want. How many more parameters would I need in general to model a two-port device, two-winding transformer?

AUDIENCE: Three.

DAVID PERREAULT: Three, right? So I somehow need to extend this model so that I would have two additional free parameters. And if I want to keep it as simple as possible, I want only two additional free parameters. And again, I'm stepping back and saying, I don't care if their physical parameters. I just want them to capture the terminal relations of the device.

So maybe I would come up and say, OK, I'm going to go add one more inductance. I'll call this L12. And then I'm going to add an ideal transformer and give it a transformation ratio of 1 to N2. And then I'll call this V2 and I2.

So now, this structure has a second parameter and a third parameter. Does that make sense to everybody? So clearly, I have three parameters here. If I pick the parameters right, I can match a 2 by 2 inductance matrix. Does that make sense to everybody?

What is this model? Well, this model, how does this model just-- recognize this is connected here. I'm just drawing it for simplicity that way. This model is just like this model, except that all I've done is I've made L leakage 1 0. So then I have this. This is one inductance. And then I took L leakage 2 and I put it on the other side of the transformer.

So my three parameters are this, the turns ratio, and this guy. And I've set this one to 0. So it's a non-physical thing. But it gives me the right set of parameters to model any 2 by 2. So that's sometimes called the cantilever model, the so-named because you have this L of inductances.

That's fine. I've got three total parameters. I can model any two-winding transformer that way. They're not going to be physical parameters. And by the way, I should have warned you before, if I have this model for a 2 by 2, what I know is that all of L leakage 1 L mu 1 and L leakage 2 all have to be positive inductances.

Why? Because they each represent some energy stored in space. So those are physical values of inductance representing physical energy storage. And this turns ratio is the physical turns ratio. If I do anything else, if I arbitrarily pick one of these models, there's no guarantee that the inductances in my parameter end up being positive or anything else. They're going to be whatever they need to match the terminal behavior.

So you can easily get some negative inductance value in your model if you're not using a physical model. And so don't get concerned if you see that. And that's just because you're trying to pick parameters to really match what the inductance matrix description does. And whatever those circuit parameters turn out to be, you'll get the right inductance matrix description.

So this is great for a two-winding transformer. I have three parameters for my two-winding transformer. How many do I need for my three-winding transformer? I need six total. That means I need three new ones. So if I'm going to add on a V3 and I3, I need three more parameters.

So what do I add on? I add on one transformer ratio as a parameter, 1 to N3. So here's one new parameter. And now, I need two more. Well, I can model the other two just with an inductance from here to here, which I'll call L23, and inductance from here to here, which I'll call L13. And that gives me my six parameters. And I can tie this one to the same reference point.

So you can see where this is going. Every time I want to add a new term, a new port, I add in one transformer with some transformation ratio, and then a leakage-- quote unquote, "leakage--" to each other internal node in the circuit. And I can expand out. And I'll always have exactly the right number of parameters to match the n by n inductance matrix. Does that make sense to everybody?

So I'm running out of time. But what I'll tell you about this model-- and the details are all in the text. The nice thing about this model is, firstly, it turns out there's always a set of measurements you can do on your circuit that will reveal directly what these ratios are.

So for example, I can find L11 just by open circuiting everything and measuring the inductance at this port. If I put a voltage here and I measure all the voltages at the other terminals, that gives N2, N3, and so forth. There's a set of measurements I can do that will always reveal these parameters. So if I want to create a model for a physical device, I can do that.

It also turns out that all these parameters can be directly related back to the inductance matrix values. And I put the equations in there. And it's exactly how you do it. But the nice thing is, if I have an inductance matrix description of the thing I wanted, I can create a circuit model that has that. This is a non-physical circuit model, but it matches the terminal relationships.

So I'm out of time. We've expanded from inductance to n-winding magnetic structures. Are there any questions before we wrap up? OK, have a great day. And we'll see you tomorrow.