

[SQUEAKING]

[RUSTLING]

[CLICKING]

**DAVID PERREAULT:** OK, why don't we get started? So last class, we started talking about magnetic components. And this is a pretty important topic in power electronics because most power converters have magnetic components at the heart of them. And we introduced the notion of just having an inductor. So maybe we have some core where we have some permeability of the core, some cross-sectional area of the core, and some length of the core.

And then we put some windings on it, maybe  $N$  turns of winding. And we have some voltage,  $V$ , and current  $I$ , defined with the associated variables convention.

And we analyze this thing, and what we found is that we would, first of all, if I had a flux density  $B$  in the core, I might have  $B$  in the core is equal to  $\mu_{\text{core}}$  times  $H$  in the core, the magnetic flux density and the magnetic field strength.

And we said, OK, on each turn of the winding, I'm going to have some  $B$  field. And the  $B$  field times the cross-sectional area is the magnetic flux in the core. And we said, OK, by Faraday's law, the voltage, say, on one turn-- I'll call it  $V_1$  turn-- would equal  $d\phi/dt$ , with the right polarity associated with Lenz's law.

And then if I have  $N$  turns, this would-- say, I have a voltage on this turn, a voltage on this turn, a voltage on this turn-- the total voltage would be equal to  $N$  times  $d\phi_{\text{core}}/dt$ . Or we could write this as  $d/dt$  of  $N$  times the flux through the core. And we would often write this as  $d\lambda/dt$ , where  $\lambda$  is the flux linkage.

That's its definition. So it's the flux linked by the net internal windings or  $N$  times the flux linked by each turn. And we also said that we derived the fact that  $\lambda$ , the flux linkage, is equal to some constant  $L$ , the inductance times the current, or  $d\lambda/dt$  is equal to the voltage is equal to  $L di/dt$ .

Just to show you that's true, we set up a demo. We actually-- what we have here is a toroidal inductor with a bunch of turns on it. And what we're going to do is we're going to apply a voltage to it. And what you can see right now is essentially the voltage being applied across this whole inductor. But what I did was we wound the inductor with uninsulated wires so that we could tap individual turns going around.

And what you can see is if I look just on one turn, for example-- come on. It would help if I looked at the right end. If I looked on just on one turn, I'll see one voltage. OK, you know what? I think we're going to try this another time. We'll bring this demo back later.

If it had worked, what you would have seen is the voltage building up around the core, which it did do in the lab 20 minutes ago. But that's the way it goes sometimes. I think we have some problem with what signal is being picked up.

What we did, starting from this, is saying, well, we used Maxwell's equations to get these results. And that was fine, but it was very complicated. So then we introduced this notion of a magnetic circuit model, where we said, OK, in an electric circuit, we have KVL and KCL. We have voltage sources, sometimes called an EMF.

We have current, which is our flow variable or our through variable. And we have resistances, which we talked about the resistance of a material to current flow through it when a voltage is applied. And we said, OK, we can build a magnetic circuit that describes magnetic structures, where we have an MMF, which is usually the ampere turns that's trying to drive flux.

Flux is our through variable. So MMF is our across variable. Flux is our through variable. And reluctance to magnetic flux depends upon the material and the geometry. And we said just as an electrical resistance is the length of the resistor divided by the area resistor and the conductivity of the material, permeability is just conductivity to magnetic flux.

And so this equivalent circuit model is very useful to very quickly making all the calculations that we would otherwise have to start applying Maxwell's equations to do.

So, for example, if we sat and said let's take a core structure like this-- and this would be called a C core, where I put a gap in the core. And we saw that we would do that in part because we can make the magnetic inductance more stable to material variations, permeability variations. We would also do it for reasons that we tend to store energy in the gap instead of in the core material, preferably.

And we go put in turns on this. So we'd have  $Ni$ ,  $N$  turns of current  $i$ , and we get voltage  $V$ . And we said my magnetic circuit model would be  $Ni$  is trying to drive flux in the core. And a positive current, by the right-hand rule, is trying to throw a flux clockwise around this core.

And then I would have the reluctance of the core, which is equal to  $l_c$  over  $\mu_c A_c$ . And the reluctance of the gap is equal to  $l_g$  over  $\mu_0 A_g$ . And then I can calculate what the net flux traveling around this path is. And that would just be this flux,  $\phi$ .

So what could we say? Well, we can calculate  $\phi$ .  $\phi$  is just simply equal to  $Ni$  divided by the reluctance of the core plus the reluctance of the gap. It's pretty simple resistive circuit there. And then we can say, well,  $\lambda$  is the flux linkage of this MMF source. So  $\lambda$ -- if I call that  $\phi$  core,  $\lambda$  is just equal to  $N$  times  $\phi$  core, which makes it  $N^2$  over the reluctance of the core plus the reluctance of the gap times the current.

And this thing here, this proportionality factor, is just the inductance  $L$ . So I can immediately from this structure get the inductance. I can also come back and say, OK, well, what's the voltage? The voltage is equal to  $d\lambda/dt$ , which is  $N^2$  over the reluctance of the core plus the reluctance of the gap times  $di/dt$ .

So I can come back and figure out what the voltage on this winding is. And when I do that, I have to be a little bit careful about the signs. And if you notice, the flux I'm talking about is the flux that's coming out of the positive terminal of the MMF source and the positive circuit-- relates to the voltage with the associated variables of the current going into that winding.

So if I define a current and I get the MMF source, the flux coming out of the MMF source gives me the voltage with this polarity. So it's the  $VNi$ --  $V$  times  $i$  is the power into the component. Any questions about that?

So that's just a very, very quick review of last class. The other thing we may want to pay a little bit attention to and what we often use these magnetic circuits for is that we assumed in all this that  $B$  was  $\mu H$ . I'm assuming that the relationship in the core material--  $B$  is equal to  $\mu_{\text{core}}$  times  $H$  inside this magnetic core.

Well, in a real core material,  $B$  is equal to  $\mu H$ -- if this is  $H$ , and this is  $B$  in the core-- that's true or approximately true for rather low flux densities. Above some level that we sometimes call  $B_{SAT}$ , or the saturation flux density, this tapers off. All the magnetic domains align. And I get something that does this. I'm simplifying a bit, but this would end up being  $\mu_0$ .

So in order to use this model, we know that we want to be operating down here, below the saturation flux density of the material. So I might also use this kind of model to say, at what current does this thing start to saturate given its geometry? Because  $B$  is related to flux. Any questions about that? So that was a lightning review of last class.

What I'd like to do today is start talking about transformers. And what's a transformer? You might think of a transformer, or at least a two-winding transformer, as a magnetic circuit where instead of having a winding and a gap to store the energy, I have a winding and another winding to transfer energy.

But a simple transformer-- maybe I'll draw it like this. Suppose I just have some core. And again, I'm going to assume it has some area of the core, permeability of the core, and some length of the core. And I'll put a first winding on it like this. And I'll say that this is  $N_1$ ,  $i_1$ , and  $V_1$ .

And let me put a second winding on it that I'll call  $N_2$ ,  $i_2$ , and  $V_2$ . And notice that I'm keeping associated variables, my voltages, and currents here. Now, if I thought about this thing, what would be the magnetic circuit model. Well, I said here's an MMF source. If I have a current into this winding, I have some MMF. So my magnetic circuit would look like this,  $N_1$ ,  $i_1$ .

Then I have the reluctance of the core that I have to worry about, so let me put that in. Here's the reluctance of the core. And now I have another MMF source,  $N_2$   $i_2$ . But now I've got to be careful because what's the relative polarity of what this MMF source is going to do to drive flux and what this MMF source is going to do to drive flux?

And in this case, if I have a positive current into the winding, by the right-hand rule, it's trying to push flux clockwise around the core, right? So I could imagine some core flux being driven this way,  $\phi_{core}$ , from this positive current,  $i_1$ , here.

Notice that the way I drew this, if I put a positive current  $i_2$  here, by the right-hand rule, it's also trying to drive flux in the same direction. That means in my magnetic circuit model here, I would have  $N_2$   $i_2$ . And notice that  $N_2$  and  $i_2$  and  $N_1$  and  $i_1$  are trying to push flux in the same direction around the core.

And you could say that's correct. It is correct. Why did it work out this way? Because you could look at it either one way or another. Because of the relative way I drew the polarity of this winding, which way it was wrapped on the core, or equivalently, which way I defined voltage and current. I could define it some other way. I could define voltage  $V_2$  the other way, in which case-- and  $i_2$  the other way, in which case I'd have to flip that MMF source. So the polarities matter a lot in this, so you ought to think about it.

The other thing I will say is when we talk about a transformer, at least a two-winding transformer, you'll often see something called the dot convention. And so somewhere in the structure, maybe you'll see this, and you'll see it in the schematic symbol. There's going to be a dot here and a dot here. What that means is in practice current into the dot throw flux-- try to throw flux the same direction around the core.

So that just is saying something about the relative definitions of the winding polarities just is denoted by the dots. So just think, currents into the dot throw flux the same way around the core.

OK, so this is my simple circuit model, and I have reluctance to the core as  $l_{\text{core}} / \mu_{\text{core}} A_{\text{core}}$ . What could I say about this magnetic structure as to what it would do? Well, let me start by considering the idealized case where  $\mu_{\text{core}}$  is really big. It's a really big number. If  $\mu_{\text{core}}$  is really big or if the area of core is really big, the reluctance of the core is really small. And for many transformer designs, that's what you would do. You'd try to approximate it that way.

So let's see what we would get. What would I get for voltage  $V_1$ ? Well,  $V_1$  is equal to  $d/dt$  of  $N_1$  times the flux in the core because  $V$  is equal to  $d\lambda/dt$ .  $V_2$  is simply equal to  $d/dt$  of  $N_2$  times the flux in the core.

And that's because notice in both cases, the flux in the core-- I've defined the flux clockwise here. So the flux of the core is the flux out of the top of this MMF source. It's also out of the positive terminal of this MMF source. So that's why the signs are the same here. That make sense to everybody? So what does this say overall? Well, I could pull the  $N_1$  and  $N_2$  out and divide the two.

And what this relationship tells me is that  $V_1$  and  $V_2$  are proportional because  $V_1$  is proportional to the flux in the core by the number of turns, and  $V_2$  is proportional flux in the core by the number of turns. What I get is  $V_2$  over  $V_1$  is equal to  $N_2$  over  $N_1$ . So this is one characteristic that I get in this system. Make sense to everybody?

Well, what else can I say about this system?  $N_1 i_1$  plus  $N_2 i_2$  is the total MMF here, and that's got to be equal to  $\phi_{\text{core}}$  times the reluctance of the core. That's just magnetic KVL here.

Well, all right, what can I say? Well, if I said-- if reluctance of the core was really, really small, goes to 0, then this right hand ought to go to 0. So what I have is in the limit, as  $\mu$  of the core goes to infinity, what I get is  $N_1 i_1$  plus  $N_2 i_2$  is equal to 0. Or equivalently,  $i_2$  over  $i_1$  is equal to minus  $N_1$  over  $N_2$ .

So what I have is two sets of relations here, one between the voltages and the sets of terminals and one between the currents of the sets of terminals. These two sets of equations that I've starred are known as the ideal transformer relations. This is what an ideal transformer would do. So we would often draw that like this.

Usually, they often put bars here. That's to symbolize the fact that there's a core there. We will have our dots, which are telling us about voltage polarities. And I would indicate the number of turns  $N_1$  to  $N_2$ . And then I would have  $V_1$ ,  $i_1$ , and  $V_2$   $i_2$ .

The other way you can look at the dots is the voltages have the same polarity if the positive terminals at the dot. So this is saying which way is the voltage? Is it-- positive terminal here and positive terminal here means the sign of  $V_1$  and  $V_2$  is the same.

So if you see this symbol, very often that's used to designate an ideal transformer that has these relations. And if you look at this, this is saying that the voltage scale is one way. So the voltage with its positive terminal at the dot this way is proportional to the voltage here. The current, whatever current comes into this dot, a scaled version of that current must come out of this dot because  $i_2$  over  $i_1$  has a minus sign here. So current goes into one dot, comes out the other dot is the way people think about it-- with the right scaling.

If you look at this carefully, this also means  $V_1 i_1$  equals minus  $V_2 i_2$  at all times. So an ideal transformer stores no energy. Whatever power flows in one side, flows out the other, but the voltages and currents are scaled. So that's often the way you use-- if you need to scale up and down a voltage-- if you scale up the voltage, it scales down the current, gives you this transformation that you want. Any questions about that?

So the first thing you're going to want to think about when you see this symbol is that you've got an ideal transformer. And these equations, which I think are in chapter 1 of *Principles of Power Electronics*, are just worth memorizing if you don't already have them in your head.

Let's now start talking about what real transformers do because it's not quite as simple and pretty as that. In the real world-- the reason this is-- and if I draw this symbol, usually what I'll mean is an ideal transformer. Well, real transformers aren't ideal. Well, how are they not ideal? They're not ideal in a lot of ways.

But let's think about, first of all, some of the principal parasitics that show up in these things. Well, where would I get the first one? The first principal parasitic kind of comes from this equation here. What did I do to make this assumption? What did I do to get to this current relation? I assumed that  $\phi_c$  times the reluctance of the core was 0 or the core reluctance was 0.

Well, the core reluctance isn't really 0. This is actually some non-zero value. That means that the current relations are kind of mismatched. I don't really get  $i_2$  over  $i_1$  being some perfect scaling, but I get a mismatch that's exactly by this amount.

So what does that look like in circuit terms? Well, suppose I took my transformer and I just put a winding on one side-- I have winding on both sides, but I open circuit the second winding. I just don't connect it to anything. Then what do I know about  $i_2$  if it's disconnected?  $i_2$  is 0 if it's disconnected.

The ideal transformer relations here would tell you that because  $i_2$  is 0, hence  $i_1$  must be 0. But if we just thought about this thing, well, if I open circuit the second one, I can ignore it. For practical purposes, it's not really there. But I do have a winding in the core. That looks kind of like an inductor, doesn't it? Well, so what would I expect--

What would I expect in a real system? Well, it's exactly-- I already have the model. I have already have the model right here. All I've done is made  $i_2$  0, which means this MMF source is 0. And I have this flux source,  $N_1 i_1$ , and a reluctance of the core. So what I would really get if  $i_2$  is 0--

What would I get? I would get  $\phi$  in the core would really be equal to  $N_1 i_1$  divided by the reluctance of the core, which would give me  $\lambda_1$  would be  $N_1^2$  over the reluctance of the core times  $i_1$ . This is just an inductance that we're going to call  $L_{\mu 1}$ . It's the magnetizing inductance of the core reference to the first winding.

So essentially, if I have nothing connected to the second winding, this thing just looks like an inductor. And what I'm going to get is, if I just differentiate this, I'm going to get something that's  $V_1$  is equal to  $L_{\mu 1}$  times  $di_1/dt$ . I'm going to get a current into the first winding that essentially is the current into an inductor.

And why is there a current into the inductor? Because I have flux that's being generated in the core. That flux is storing energy in magnetic form, and that has to come out being represented somehow electrically. So what would I do to represent that-- I have finite flux in the core. The flux would have been 0 or negligible if the reluctance of the core was 0, but it's not.

So what I could say then is that maybe I could model this thing this way. I could model the transformer as an ideal transformer,  $N_1$  to  $N_2$ . Here's  $V_1$  and  $i_1$ . And here's  $V_2$  and  $i_2$ .

But maybe I could just say, all right, let me model this as an extra inductor. This is my ideal transformer here. Let me model this behavior as another inductor. And I'm going to call this inductor  $L_{\mu 1}$ . And I'm going to call this current  $i_{\mu 1}$ .

So if the secondary side is disconnected, the ideal transformer has no current going into it, but I still get a magnetizing current. And the way I think about that magnetizing current is the energy stored in this so-called magnetizing inductance, this fictitious inductance that I'm using to model the behavior of an actual device,  $\frac{1}{2} L_{\mu 1} i_{\mu 1}^2$  models the energy stored in the core.

So the fact that I have non-zero flux to generate my induced voltage on the other winding means that I'm going to end up with some energy stored in the core.  $L_{\mu 1}$  is not infinite, and I get some finite current here that's storing energy. Any questions about that?

So if I have a real transformer, I often think of it as an ideal transformer plus some magnetizing inductance that I can often think of as a completely separate inductor to model the behavior of the actual device.

By the way, I put this on the primary side, in the  $V_1 i_1$  side, as opposed to the so-called secondary side, the  $V_2 i_2$  side. I could have done it the other way. I could have modeled the transformer equally like this.  $N_1$  to  $N_2$  and had  $V_1, i_1, V_2, i_2$ .

And I could have placed my magnetizing element on this side. This would have been  $L_{\mu 2}$ . The magnetizing inductance referred to the secondary side, which would have been equal to  $N_2^2$  or  $N_2$  over  $N_1$  quantity squared  $L_{\mu 1}$ , or equal to  $N_2^2$  over the reluctance of the core.

So I could place this fictitious component on either side of the core because whichever place I place it, it's modeling the same phenomena, the energy stored in the core-- the energy associated with the flux being thrown around the core. Yeah.

**AUDIENCE:** So but when you have both sides conducting current, wouldn't you have a different value in total? Wouldn't you have to do both?

**DAVID PERREAULT:** No. No, because think about it. Suppose I still have the secondary disconnected. If I apply a voltage to the primary side, I get a scaled voltage applied to the secondary. I get some current,  $i_{\mu 2}$ , which is going to be  $N_2$  over  $N_1$  times  $V_1$ .

And then the  $d/dt$  of that-- or the integral of that is going to give me this current, sorry. But there's nowhere for this current to flow except out of this dot, which means then there's a current flowing into this dot. And if I look at what this current coming in is, it looks exactly the same as if I had placed it on this side and had nothing on the secondary side.

So ultimately, this magnetizing inductance really just reflects the current mismatch that's  $\phi$  core times reluctance of the core. And I can place it either place, but I don't need to place it both. It's one or the other. If I apply a voltage here, I'll get a certain current into  $L_{\mu 1}$ . And if I scale-- instead I put it on the other side, and I scale it right, it's just coming out of the ideal transformer on this dot and into the ideal transformer on that dot.

**AUDIENCE:** So does that mean that the inductance of the transformer doesn't depend on the magnitude of the currents?

**DAVID PERREAULT:** The magnetizing inductance has nothing to do with the magnitude of the currents, that's right. And the same way-- I mean, ideally speaking-- the same way in the inductor, the inductance value I get doesn't relate to the currents. It's just the scaling between flux linkage and current. Yeah, so it doesn't depend on the currents. And if I had some other load out here, I could just keep modeling everything I'm doing. I just need one extra element to model that parasitic behavior. Great question. Any other questions?

OK, so that's one practical nonideality. There's a second practical nonideality that we also want to think about very often. And what is that?

Well, if I had, again, an infinitely permeable core or a permeability that was so much higher than the surrounding universe that I could ignore any flux that goes anywhere except in the core, that would be great. But the problem is, as we said, permeabilities are 1,000 or 10,000  $\mu_0$  or something like that. Magnetic circuits are kind of leaky. The flux pipes are very leaky pipes.

What that means is if I applied some Ni over here, I will indeed get some flux going around the core, but I will get an additional component of flux that might link winding 1. But you know what? It can come out of the core and link back this way. So I'm going to get some leakage flux I'm going to call  $\phi_{\text{leakage 1}}$ , which links the primary turn, primary winding but eventually either here or here or somewhere else jumps out of the core and doesn't make it over to the second winding.

Likewise, you could imagine that I could have some flux that would link the second winding but then jump out of the core-- and I'll call that  $\phi_{\text{leakage 2}}$ -- and never links the primary winding. And this parasitic leakage flux from one winding to the other imposes additional nonidealities on the behavior of the system.

And it becomes particularly worse in a percentage basis if the core permeability is high. But appreciate, even if I made the core permeability infinite, there is still some flux that's going to jump out. It just may not be a big fraction of it. Does that make sense to everybody?

So the question is, how would I model that kind of behavior? Ah, something I forgot to tell you. So we're going to model this behavior in one second. There's something I forgot to tell you about the magnetizing-- inductance of the magnetizing flux, which I better not forget to tell you.

One of the things-- one of the aspects of this magnetizing behavior is that it's a reason you can't use transformers at DC. So we said we want to convert voltage. Why did Westinghouse and Tesla win over Edison? It's because Edison didn't have a great way to convert voltage from one level to another. He couldn't step down voltage very easily.

You can do it with AC, but it's hard to do with DC. Why is that? What would happen if I applied a constant voltage across this primary of this transformer?

**AUDIENCE:** It would short?

**DAVID PERREAULT:** Well, what would happen is I'd have a continuously increasing magnetizing current forever.  $V$  is equal to  $L \, di/dt$ . That would mean if I have a constant voltage here, I'd have a constant  $di/dt$  in this inductor.

Or looked at another way,  $V$  is equal to  $L \mu \frac{di}{dt}$ . That means  $i_1$  is equal to  $\frac{1}{L \mu}$ , the integral of  $V dt$ . Keep in mind that the flux linkage-- another way to look at it is, what's the flux linkage in the core?

$\lambda$  is equal to  $L i$ . But  $\lambda$  is equal to  $N \phi$ , or say  $N_1 \phi$ --  $\lambda$  is equal to  $N_1 \phi$ , which is equal to  $N_1$ , the area of the core times the flux density in the core. And this is equal to the integral of  $V_1 dt$ . That's what  $\lambda$  is.

This means that if I have a limit, a saturation flux density limit on my core,  $B_{core}$  has to be less than  $B_{sat}$  or it saturates, and as somebody said, shorts out, what I need is the integral of  $V_1 dt$  has to be less than or equal to  $N_1 B_{core}$  times  $A_c$ , area of the core.

And what this means is if I have a sinusoidal voltage, that means this integral is going to go up, and then it's going to go down and then go negative. There's a limit of the volt seconds I can apply to the core that's limited by this saturation flux density.

So one important consequence of this parasitic-- and I apologize for not mentioning it earlier-- is the fact that there's a relationship between the number of turns and the saturation flux density and the cross-sectional area and the maximum volt seconds you can apply on the winding before you get it out of the saturation range.

Once you get it out of the saturation range, the permeability goes away. And then more or less the transformer shorts out, and you get huge magnetizing currents, and bad stuff happens. So that's the other consequence of this particular parasitic.

The second parasitic-- first of all, any questions about that? Yeah.

**AUDIENCE:** Are there any applications where you could use this like a switch supply, where you're applying a DC voltage for a little bit and then you switch it off and so that way you're-- like, the volt seconds that you have the transformer on is few enough that you don't reach that saturation field but you're still using it for your DC voltage? Is there any use for that?

**DAVID PERREAULT:** Sort of. The notion of applying a voltage for a little while and storing some energy in this magnetizing inductance, which is sort of what you just said, is exactly what you do in certain kinds of converters. So in some kinds of converters, we will actually use this parasitic component to our advantage, and we'll talk about how you design for that. But that's a limited time. The integral of  $V_1 dt$  is limited by the volt seconds. You still can't apply it forever.

And yes, we will use it that way. So sometimes parasitic is a bad thing. Sometimes it's a good thing. And we'll see using it both ways. Great question. Any other questions? OK, so I apologize for forgetting to mention the other impact of this parasitic.

Let's come back to the fact that in addition to this parasitic, I have some flux that links winding 1 that doesn't link winding 2 and some flux that may link winding 2 that doesn't link winding 1. Well, how would I model that? Well, what does the magnetic circuit for that look like once I add things in?

Well, I still have  $N_1 i_1$ . And I still have my reluctance of my core. And I have  $N_2 i_2$ . But then what I'm doing when I'm saying, oh, there's some flux that's going to come up here and jump out of the core, I'm sort of saying there's some path for flux to flow that doesn't go through the other winding.



In what I've drawn so far, anything that goes through my circuit must go around the loop here. And I have phi core I'm imagining doing this. What I'm saying now is, oh, well, I can have some flux that comes up here through this MMF source but returns through some other path that I'm going to call phi leakage 1.

And I'm going to have a second flux path that I'll call reluctance leakage 2 that's going to go through-- flux may come down here and return up here. And I'm going to call that phi leakage 2. So this is my extended magnetic circuit model, accounting for the fact that there's other paths for flux to flow and not link the two windings together. Any questions about that?

So if I add this on, what would I get? Well, that's the magnetic circuit model. What is phi leakage 1? Phi leakage 1 is simply equal to  $N_1 i_1$  over whatever this reluctance is. So I'll call this reluctance leakage 1. Phi leakage 2 is equal to  $N_2 i_2$  over reluctance leakage 2.

And by the way, sometimes since these have big air path components, sometimes it's tricky to figure out what the values of leakage reluctance 1 and leakage reluctance 2 are. But nonetheless, there's some value that you can figure out one way or another. And then we have phi core, which is unchanged. It's  $N_1 i_2$  plus  $N_2 i_2$  divided by the reluctance of the core. So that's my magnetic circuit model for the whole thing. Any questions about that?

So what do I get? Well, I can basically figure out what the-- I can basically figure out what the net flux linkage on winding 1 is. What's lambda 1? Lambda 1 is equal to  $N_1$  times the total flux coming out of the positive terminal of the MMF source,  $N_1 i_1$ .

So that should be  $N_1$  times phi leakage 1 plus phi core, which would be equal to  $N_1$  squared over reluctance leakage 1 times  $i_1$  plus  $N_1$  times phi core, which is  $N_1$  squared over the reluctance of the core  $i_1$  plus  $N_1 N_2$  over the reluctance of the core,  $i_2$ .

Lambda 2, which is the flux linking the second MMF source, is equal to  $N_2$  times phi leakage 2 plus phi core. It's the same story all over again. I'll just substitute in my values here. And what I get is this is going to be equal to  $N_2$  squared over reluctance leakage 2 plus  $N_2$  times phi core, which is  $N_2$  squared over the reluctance of the core. This is times  $i_2$ . This is times  $i_2$ . Plus  $N_1 N_2$  over the reluctance of the core,  $i_2$ .

So that's two equations expressing the two flux linkages in terms of  $i_1$  and  $i_2$ . I can put that in a matrix form. And that would just be lambda 1, lambda 2, which is equal to essentially the integral of  $V_1$  and the integral of  $V_2$  dt.

And that would be equal to  $N_1$  squared over reluctance leakage 1 plus  $N_1$  squared over the reluctance of the core plus  $N_1 N_2$  over the reluctance of the core times  $i_1$   $i_2$ , so matrix equation. I get the same thing. I get  $N_1 N_2$  over the reluctance of the core times  $N_2$  squared over the reluctance of the core plus  $N_2$  squared over reluctance leakage 2. That's the whole matrix equation.

So instead of the integral of-- or instead of  $V_1$  is equal to something times  $di_1/dt$ , I have a matrix equation between the voltage on the-- the current into the two windings and the voltages on the two windings.

If I differentiated this equation to get  $V_1$  and  $V_2$ , I could write this as  $V_1$   $V_2$  is equal to  $L_{11}$ ,  $L_M$ ,  $L_M$ , and  $L_{22}$  times  $i_1$ ,  $i_2$ , dot. Or maybe I should just get rid of the dot, and I'll just write this as lambda 1 and lambda 2.

So there's now a matrix equation relating flux linkages and currents. Any questions about that? What do I want to come back to? Let me come back to a equivalent circuit model associated with this. I gave you an equivalent circuit model. What happens to my-- what happens to my voltage and current relationships?

And here's what you can do. You can take this model or this model here and come up with an equivalent circuit that has the same relationship between voltages and currents. And it looks like this. I have an ideal transformer  $N_1$  to  $N_2$ . And then I have a magnetizing inductance.  $L_{m1}$  is equal to  $N_1^2$  over reluctance of the core.

Then I have a leakage inductance that we're going to call  $L_{leakage 1}$ , which is equal to  $N_1^2$  over reluctance leakage 1. And then I'm going to have a second parasitic inductance,  $L_{leakage 2}$ , which is equal to  $N_2^2$  over reluctance leakage 2.

And I put these together, and then I have  $V_2 = i_2$  and  $V_1 = i_1$ . So all that happens, if I include the fact that in this drawing there's some additional leakage flux paths that go out of the area that I ideally would have, is that I get two independent inductors here that get added on to my full transformer model.

The energy stored in this kind of purple inductor here relates to the magnetic energy that's stored in the field associated with this flux path. Likewise, the energy stored in this inductance is exactly the energy stored in the flux that links winding 2 but not winding 1.

And so what this does is-- whereas the magnetizing inductance created a mismatch in the current relationships between the primary and secondary currents deviating from my ideal current relationships, the leakage inductances, so-called, that I would add on mess up the ideal voltage transformation.

Because the voltage, if I call this  $V_1'$  and  $V_2'$ , these follow the ideal voltage relations. But the voltage, the actual voltages of the transformer terminals do not. So this is what we would consider a transformer model that has an ideal transformer at its heart but then incorporates three additional inductances to model the energy stored in the core and the energy storage and the leakage fields around the transformer.

So I apologize. I've run over there. I'll take any last quick question before we wrap up, or I'll happily answer questions after class. OK, have a great day. We'll pick this up on Wednesday.