[SQUEAKING]

[RUSTLING]

[CLICKING]

DAVID PERREAULT: So just to remind you, from the last lecture, we talked about the notion, the difference between a linear power supply and a switching power supply. And we're talking about, in this case, DC to DC conversion, but the same thing is true actually of RF amplifiers and other areas where you can have switching amplifiers and linear amplifiers.

And we said the notion of a linear amplifier was simply that I have some input source, V in. I'll use a transistor. And here's my load resistance. So here's V out. And what we'll do is we'll have a reference voltage.

And we'll feed back the output voltage and control the base or the gate of our transistor so that we get some drop-- I'll call it Vd here-- so that this essentially looks like some variable resistor that we do a resistor divider, and we get our output voltage from our input voltage.

And we said the real problem with this was, OK, I have my voltage drop, Vd, and whatever current is coming from the source-- and that's the same thing as the load current. It's coming out of the source, coming through this past transistor, and going to the load. And so if I ask what the power dissipation was, the instantaneous power dissipation in this transistor is equal to Vd times IL.

If I said, What's the average power dissipation? it's the average of that, and it's not equal to 0. So essentially, if I have a variable resistor, I'm dumping energy into that variable resistor. And especially when I have large conversion ratios, that gets really bad.

So what we said is, OK, we can be way more clever than that, and we can go build a switching converter. And the idea would be this. We'd have V in. We'll have a switch, and this will be built out of semiconductor switches-- q of t is equal to 1, q of t is equal to 0.

And then we will take this output, and we'll run it through some lossless LC filter, and then I'll have my output here. And the notion is by chopping this back and forth like this-- so I have some switching function, q of t is equal to 1 and 0, and I'll control this with some switching period and some duty ratio. That's a pulse width modulation. This is q of t.

Then I can have this voltage that I'll call Vx, which is sort of a copy of that. When q of t is 1 Vx is V in. When q of t is 0, Vx is 0. And I get a pulsed waveform that looks like this. This is Vx. And so forth.

And when I filter Vx, the idea is this acts as a low pass filter, and it basically extracts the DC component of that. And so then I get an output voltage, V out, that is basically approximately equal to-- V out is approximately equal to d times V in.

So by controlling timing, by controlling the width of the pulse and the cycle, I control the average voltage at the output. And the beauty is, as we argued, switches are ideally lossless, inductors are ideally lossless, capacitors are ideally lossless. So the efficiency of this whole box can in theory approach 100%. So that's the fundamental concept.

We're going to have switching. We're going to have energy storage. And we're going to take energy from an input to an output. And we're going to operate cyclically. Now, things are a lot more-- there's a lot more to it than just that. There's many design considerations in all of this. First of all, how do you design these inductors? How do you pick a topology? What can it do? How do you control it? What are the dynamics? All these kinds of questions we're going to answer in this term.

But that's the basic concept. So before I get on to the material today, are there any questions just about the basic idea here? OK, so what I'd like to start doing today is giving us some of the tools to analyze these kinds of systems.

And what we're going to see is we're going to do a lot of our analysis in the time domain because it turns out that our circuits tend to be either nonlinear or time varying, depending upon how you want to look at it. And time domain analysis is a really good way to look at these things. And what we would like is some tools to figure out what's going on. I'm going to introduce you to two concepts today that we're going to come back to again and again.

And the notion is that the first thing I'd like to talk about relates to the switches themselves, I haven't said how we're going to implement this single-pull double-throw switch. And what the switching configuration looks like varies from circuit to circuit. But usually, we're going to implement those with semiconductor switches.

Now, you can imagine semiconductor switches that are fully controlled. I tell them when to turn on and off. But more often than not, the switch-- some aspects of your switch may be either semicontrolled or uncontrolled. That is, external circuit events will dictate when the device turns on and off. And so one of the things you'll need to do if you're going to analyze some circuit is figure out when do these uncontrolled devices turn on and turn off as part of figuring out what, for example, some circuit topology will do or can do, for example.

So let me give you the basic approach that we use to do that. And I'm going to start with this. Some people might think-- this is known as a-- operated this way, it's known as a buck converter. It's a relatively simple power converter. I'm going to go to perhaps the simplest power converter you can think of. And it just simply looks like this. And this is just by way of illustration.

So suppose I have Vs sine omega t, some AC source. And I'm going to put a diode here, which is my canonical, completely uncontrolled switch. A diode turns on and off when the external circuit tells it to turn on and off. We can have also devices that are semicontrolled. Maybe we only turn them-- tell them when to turn on. Or maybe we only control when they're turning on and off under some conditions. And we'll see that in much more detail.

But suppose I have this circuit. And maybe I have some-- the interesting thing I'm interested in getting here is V out. And maybe this is just some value R. And we have some current that's going to come here. And let me call this ID. And we will also have some voltage VD.

Now, let's talk about diodes for a second. People who have had semiconductor classes, you know a diode has an exponential characteristic. But for basic analysis, if we're going to treat things as switches, we can really simplify that. So we often think about what we would call an ideal diode. What is an ideal diode do?

An ideal diode has the following IV characteristic. OK, so here's ID and VD. And I might have a situation where the diode is on. So this is when the diode is on.

That is, it has no forward drop, but it can carry as much current as you want. I might also have a case where I have the diode being off. And so here is the off state of the diode. So if I'm off, basically I'm thinking of VD and ID like this. Here's VD and ID.

VD is whatever it wants to be as long as it's negative. And ID is 0 because this is an open circuit. And I should just be clear I'm talking about an ideal diode looking like this. And if I'm on, what I'm assuming is that essentially VD is 0, and I have some positive ID.

So essentially, in treating it like a switch, I'm either treating it like a perfectly closed switch, which happens in this part of the IV plane, or I think of it as a perfectly open switch, in which case it happens in this part of the IV plane. So that's how I will think of it as a diode. And yes, a real diode would have some exponential characteristic. But for analysis, it's much simpler to treat it this way and then add in nonidealities later.

So a diode is an uncontrolled switch. What controls its switching? Well, the right way to think about this is based on what a diode cannot do. Notice that there is no characteristic down here with ID negative.

So a diode cannot support a negative current and no positive voltage. This should be ID and VD. So it cannot have a positive voltage, because it'll turn on. It cannot have a negative current, because it'll turn off.

And so in figuring out in our circuit what a diode is doing, we often figure it out based on what it can't do rather than what it can do. And I'll get to what I mean by that in a moment.

So let's think about this circuit. How will I analyze it? Suppose I have a circuit like this, for example, with one or more diodes in it. And this circuit is trivial. It has one diode. But you can actually find power converter circuits for high power that have 48 diodes in them. And so then part of your job is they're turning on and off, and you've got to figure out what's going on.

What would I do here? Well, what we would tend to use is something known as the method of assumed states. And here's the Method of Assumed States, or MAS, as I call it. One, assume a state for each switch.

So some switches you know. If you fully control the switch, you know when it's on and off because you're telling it to turn on and off. But if you don't know, if you have diodes, for example, in your circuit, guess. Assume it's on or off at a given point in time.

What you would then do is say, OK, take that diode and replace it either with a short circuit if I've assumed it's on or an open circuit if I've assumed it's off. And now I've replaced things with shortened opens, so now my circuit's simpler. So I've assumed a state is substituted in. Then I analyze the V's and the I's in the circuit.

Now, if what I have is switches, inductors, capacitors, resistors, that kind of thing, then that just becomes a linear circuit problem that's straightforward to solve. So analyzing the V's and I's, in that case, is pretty easy, whereas if I want exponentials in there, then it gets a little ugly.

The next thing I do, and this is the key thing, check if switch conditions are violated. So what I'm really going to do is I'm going to go say, oh, I've assumed my switch is on, so I've replaced it with a short circuit. Did the diode current end up being negative? If it did, I know it can't do that because diodes can't do that.

If I've assumed it's off, the question I have to ask myself is, did the diode voltage end up being positive? So what I'm going to do is I'm going to assume something. I'm going to assume it's a shorter and open and then check if the other condition is violated. And the reason I have to do it that way is because if I have assumed it's a short, then I can't check its voltage because I've already assumed its voltage is 0. I've got to check the other terminal condition.

All right, four, if not, we're happy, keep going in time. If violated, I have to make a new set of assumptions and come up here. So guess again. What I might do is if I found a device condition violated, maybe I'll switch the condition state of that, assume state of that diode, and go back and do it over again.

So I can start to guess at a given point in time what the state of the uncontrolled switches is and then check if I'm wrong and keep iterating that until I figure it out I'm right.

So let me just illustrate that for a very simple condition. In this particular circuit, I have two possibilities. If the diode is on-- and this is why the world's simplest circuit-- if I assume the diodes on, then I've got V out here is just equal to the input voltage. And I ought to look at this current to check whether I'm making an error.

And if I've assumed it's off, then what I would need to look at, I would have V out like this, and I would need to look at the diode voltage to see if it's gone positive.

So let's just apply this. And yes, I grant you, you've long since figured out what the circuit's doing in your head, but it's not so simple if you have a whole pile of uncontrolled devices or a more complicated circuit, as we'll see. So let's just look at this example. So here we go.

I've got my input source Vs sine omega t, so it looks like some sine wave. Here's omega t. And I want to know, what, for example, is the output voltage doing there? Well, let's assume-- let me just start at t equals 0 plus, like right here. Or omega t equal to 0 plus.

Well, if I assumed the diode was off, I'd have this picture here. If I assume the diode is off, I'd have this picture here. But if this is open, the diode current is 0, so there's no current through the resistor, so there's no voltage here-- so in that case, VD would equal Vs sine omega t, right? And if I just told you that Vs sine omega t is positive, well, that would mean that the diode voltage is positive. And hence, this violates this assumption. And so it can't be true. This cannot be the state if Vs sine omega t is positive.

So what that would tell me is that I must have at least in this range-- the diode has to be on for at least this range. And so I would get V out looking exactly like VD because if I look at the top assumed circuit, Vs sine omega t is applied directly across V out. And it looks like this. Here's V out.

And what would the diode current look like, ID? Well, ID is just Vs over R, so it would do something like this. Well, what happens when I cross this boundary? If I continue to assume I was in the same state where the diode is on, then I would have ID here go negative right here.

But we know it can't do that. So what do I know? I know my assumption that the diode was on must be off. And so in this part of the cycle, the diode must be off. So pretty simple. If the diode is off, then ID is 0, and VD is 0, and V out is 0. And this is the waveform I get. And it would look like this.

So we've analyzed the circuit. It took a lot of time to do it, which is just by way of illustration. But that's the notion of assumed states. I assume a state for the device. I calculate along. And then I see if I'm wrong at any point about what the diode can and can't do. Let me stop there and just ask if there's any questions about the concept. Yeah.

AUDIENCE: Are you always-- are you going to get a unique answer when you do this?

DAVID PERREAULT: Very good question. Are you going to get a unique answer? And the answer to that is so long as-- it gets more complicated when you have controlled and uncontrolled devices. But if I know an initial state, it will do a unique thing.

I could give you something where I didn't tell you the state variables in the circuit, and then you wouldn't know what the state of the devices were. But generally, yes. And I should say, by the way, that this notion of making some assumption and calculating, there's a lot of circuit simulators-- fast simulators, say, switching circuits-- that's exactly how they operate. And they go through, and then they change states, and they keep going. So you can do this, and you will get an answer.

Those who are old timers in the field, like myself, at one point or another have written a simulator that works that way to simulate some circuit. These days, these circuit simulators, the general purpose ones are so good that you'd never bother writing your own. But nonetheless, it turns out to be very useful to analyze a circuit. So yes, generally you'll get a unique solution, assuming you know all the state variables. And you can think of that as if you built the thing, it had to do something, right? And it can only do one thing.

So that's the method of assumed states, and we'll come back to this with much more complicated circuits. But I wanted to give you the notion of, when you have an uncontrolled device, what do you do?

There is another topic I'd like to cover today that relates to how we generally analyze power converter circuits. And we said, well, I'm going to do something-- and there's all kinds of power converter circuits, but generally, I'm going to do something like this, where typically I'm going to operate something periodically. And then I want to analyze what its behavior is.

And because we're operating things periodically, we can often end up with an operating regime that we will call periodic steady state. Or we abbreviate Periodic Steady State as PSS.

Now, the notion of periodic steady state is a little bit like the notion of sinusoidal steady state, which you would have studied in linear circuits, except that we don't typically have all our waveforms being sinusoidal, because we have some kind of nonlinear circuit going on. But it's a related concept.

And the idea is this. We're operating cyclically. Eventually what we expect is for every cycle of the circuit to do the same thing. So periodic steady state operation is something like this. So suppose I have some operating period T-- 2T, 3T, et cetera. I would be in periodic steady state operation if every operating cycle was some carbon copy. So if I looked at all the waveforms, maybe it would do-- if it did something in this cycle, it would do the precise same thing in the next cycle. The third cycle would be a carbon copy of that. That's periodic steady state operation.

And what I mean is generally about all the waveforms in the circuit-- what I'm saying is this, if I just shifted it back a cycle, it's just repeating. Do we have to check all the waveforms in the circuit? Well, if we check the state variables, then we sort of know what the rest of the variables in the circuit are going to do. So I only really have to think about state variables. And generally, we could often just say look at the things of interest.

And maybe if it takes this waveform x, whatever it is, a voltage or current in the circuit-- takes on some value at the beginning of the cycle and it returns to the exact same value at the end of the cycle, then probably it's also periodic steady state operation. So sometimes we don't really think about looking at every point, but we just look at the beginning and the end of each cycle.

So that's periodic steady state, and we're going to see that it's really useful to think about circuits in periodic steady state operation. They don't have to be. Even a cyclically operated circuit may not be in periodic steady state. You could imagine some operating regime where you're going through a transient. We'll see one-- or maybe in this cycle it did this, in this cycle did this, in this cycle it did this. So we're switching things cyclically, but the waveforms are not repeating. And this would be clearly not PSS.

So we often want to think about what will our circuits do in a steady state, when they're doing the same thing every cycle? We'll also think about these conditions. But we often like to start with this condition. Why is that? Because we can say a lot of things about what a circuit will do once it reaches periodic steady state. Let's just give an example of that.

Suppose I have an inductor, and I have some V sub L voltage across the inductor and I sub L current into the inductor. What I know is that V sub L is equal to L dIL/dt OK, well, suppose I took the average of each side of this equation, the time average. And maybe I'll average over all time, or maybe I'll average over a cycle. But let's talk about averaging over all time.

Then I could say, OK, the average V sub L is equal to L times-- is equal to the average of L dIL/dt, which is-- I could pull this L out of the average, which is the average of V sub L is equal to L times the average of dIL/dt. So what does that tell me?

Well, if I look at the periodic steady state waveform, suppose, for example, I said this was some random variable x. Suppose x was equal to I sub L. What is dIL/dt on average of this waveform? Well, the current went up, the current came down, went up, came down, right? On average, the current is not going anywhere. On average, dIL/dt is 0. I'm just repeating every cycle.

So what I can say is that in this case, in PSS, this is equal to 0. If I have an inductor and periodic steady state, the average voltage across it has to be 0 because, on average, the current through is not changing. It might be doing something cyclically, but it's just sitting there on average.

So what we can say is that for inductors, the average value of the voltage across an inductor is equal to 0 in PSS. I could have made the same argument about capacitors. The average value of a current through a capacitor is equal to 0 in PSS.

So suddenly I can start having an understanding of requirements on the circuit operation if I'm sitting there in periodic steady state operation, which is very often where we start our analysis. So if I want to know what some cyclically operating circuit like this is doing, I might just start there. And in fact, I did that subtly when I analyzed the circuit.

We said this is a low pass filter. The output voltage extracts the DC component of Vx. That's precisely the same thing as saying the average voltage across the inductor is 0. So we actually already used that subtly, but we just recognized that it was a low pass filter. But it applies to any element in your circuit.

This is very much like saying-- the average voltage across an inductor as being 0 is like saying at DC, or in steady state when nothing's changing, an inductor looks like a short. Or in DC or its steady state, when nothing's changing, a capacitor looks like an open, has no current on average. So that's the basic idea.

But what we're going to do going forward is we're going to start to figure out how to analyze all kinds of circuits. And we're going to-- this is a very useful tool to figure out what they do in the periodic steady state, which is a very good first step to figure out what they're going to do in the nonperiodic steady state condition. But we're often operating in periodic steady state, so that's why we look at it.

And I should say, it's not uncommon-- as a consultant, for example, there's some problem with their power converter, they ship you up their documents. Here's this circuit, and they tell you nothing else. And what do you have to do? The first thing you have to do is figure out, what will it do? How will it operate? And the first place to start with that is, how does it operate in steady state? And of course, sometimes they'll tell you, well, let me tell you about how my circuit operates. But of course, they wouldn't be calling you if it did what they thought it did.

So very often you have to tell them why. And tools like this are very good at figuring out what is it going to do and why is it going to do it. So let me just give you a super simple example of that and how it can inform your thinking.

And here's the circuit I'm going to do. Suppose we came back to this circuit. And what I got when I looked at that output voltage, this voltage here is a periodic steady state voltage. It actually has a DC component. So that thing is actually the world's simplest rectifier. It takes an AC voltage and generates an output voltage that has a DC component.

That's why I would use a switch. I've now taken AC and turned it into DC. You can't do that with a linear circuit element, but a diode you can use to do that. So I might use this if I had some AC voltage and I wanted an output with a DC component to it. Or what I merely might want is a constant DC output. Well, what did we do last time we wanted a constant DC output?

Last class, when we analyzed our converter, we kind of generated a pulsing waveform, and then we threw a filter on it. Maybe I should just throw an inductive filter on this thing and I can get now constant output, which I might want if I'm going to run some DC load with it. OK, so let's try that just as an example and see where we get. So here's the circuit I'm going to propose. And we'll take a look at this.

Here is Vs sine omega t. Here's my diode. I'm just going to now put an inductor. And here's my output. So I can look at some voltage that I'll call, for example-- I'll call this ID.

Let me now-- I have some distinction now between the voltage here and the voltage at the output. But let me do this. Let me call this voltage Vx, and I'll call this voltage V out. And let's think about analyzing this circuit.

Well, my goal was to use this inductor so that ID, which is also IL here, is smooth. So maybe the assumption I could make is let me just start by assuming this diode is just on all the time. I'll make that assumption. We said we're going to analyze the circuit. We'll make some assumption about the state. Let me just assume the diode is on. So in that case, I got Vs sine omega t-- I'll assume that the diode is always on. I'll replace it with a short. I have my inductor, and I have my output.

And here's my diode current, ID. And here's my output voltage, V out. Let me analyze this circuit. So I'll just do KVL around the loop here. So I've got Vs sine omega t minus VL-- I'll call that VL-- minus V out is equal to 0.

Well, this is a legitimate equation. I could take the average of each side of this equation, and it's still a legitimate equation. So maybe what I'll do is I'll take this KVL equation-- I'll take the average of the whole thing, and that gives me the average of each individual component. What's the average of Vs sine omega t?

AUDIENCE: 0?

DAVID PERREAULT: 0. That goes away. What is the average of the inductor voltage? Well, I just told you that if I'm in periodic steady state and maybe I'm interested in what this circuit does in the periodic steady state, this is 0 in PSS, which just gives me then the average voltage of V out would be 0 in that case because the other two terms are 0, and this is the only thing left.

Well, something's funny about that, right? I just told you we were building a rectifier, and now I've told you I've got a 0 output voltage, which isn't exactly what a rectifier is supposed to do. Something's funny here. What's funny is my assumption's stupid. I assumed that this diode was on all the time. Well, if the diode was on all the time, I suppose I could have just replaced it with a short circuit, right?

But what this voltage being 0 in periodic steady state told me is I've made a mistake. I've done something, perhaps not stupid, but I've gotten over aggressive with my assumption that this diode stays on all the time. And in fact, I can tell that because if the average voltage was 0, and presumably some of the time this current is positive, so I have a positive voltage-- and if it's average is going to be 0, then some of the voltage must be negative some of the time.

And if the voltage across the resistor is negative some of the time, the current through the resistor is negative some of the time. But we already said this diode can never have a negative current. So clearly my assumption was bad. Or when I assumed that this diode was turned on, that might be true for some time, but it can't be true all the time.

So my quest to use this inductor to filter the waveforms maybe can't work out or at least as I've done it so far. What would I get for waveforms if I did this? What you would see is something like this. With this circuit what I would get is this.

Here's my Vs sine omega t. I know my diode has to be off some of the time. Well, let me assume that right where Vs sine omega t is 0, the diode is off. All right, so if I assume this diode is off and I replace this with an open circuit-- so I'll just go do that. Let me just assume-- I just went through 0. It's an open circuit.

What do I know? Well, I know that if there's no current-- if this is an open, there's no current this way. That means there's also no dI/dt, Which means that the voltage across the inductor is 0 because V is LDI/dt. Voltage across the resistor is 0 because the current is 0. And that means that this VD-- since this is 0 voltage and this is 0 voltage, VD is the same as Vs sine omega t.

So what I can tell is that, geez, if the diode was off here-- and I know it's off some time, so probably maybe up to some point here the diode is off. But as soon as I cross 0, that assumption is bad because VD would no longer be negative. It would be positive. So the diode must turn on right at t equals 0-- or omega t equals 0.

So the first part of what I did was correct. The diode turns on. Vx when the diode is on is exactly going to follow Vs sine omega t for t greater than 0. So it's going to follow the same argument. What's going to happen here, though? Is the diode going to turn off here? Well, let's think about what this current would be.

I could analyze, and I could say, look, at t equals 0 plus, inductor current 0, and I've replaced-- at that point, I've replaced this switch with a short circuit. I can analyze that circuit. That circuit's just this. I can analyze this circuit for the inductor current, knowing that IL of 0 is 0.

And if I plotted that, what I would get is something that did, I don't know, something like this. It would start off very small, and it would get bigger. And then it would-- eventually, it would cross through 0 because the voltage would get negative and negative-- more and more negative. And then it would keep going, et cetera.

But what do I know? I know that IL can never go negative because the diode current would go negative. So what would happen is if this was ID, the diode would stay on as long as ID is positive. And when ID tried to go negative, this would go negative, and the diode would just turn off, and ID would go to 0. It'd do something like this.

Now, what does exactly that solution look like? We can calculate that. It depends on the parameter values. But what we can tell from all this is that at some point, this diode is going to turn off. And it's going to limit the current. And the current is not going to be a smooth, constant current like I might want if I wanted a purely DC output current and voltage.

Just to illustrate that, I've got a simulation here to show you. And all I did was go into LTspice, which I encourage you to all download, and simulate it. So here we have a sinusoidal voltage source, an almost ideal diode. We gave it a small forward drop, but if we gave it a 0 forward drop, it would be ideal. We put in a big inductor and a resistor.

And here you can see in red, on the left axis, we have voltage. It's just a sinusoidal voltage, and the 0 is here. In the right-hand axis, you can see the diode current. And what you can see is right when-- just after the voltage crosses 0 positive, the diode current starts to ramp up. It goes up and up and up. It hits about just under an amp. But then the voltage gets negative and, boom, turns off.

If I plotted this voltage as well-- oops, I'm not sure it gave me-- see, we can see this thing where the diode-- the voltage here, which I plotted in yellow, jumps back to 0. This particular parameter is just letting go much more of the cycle. Does that make sense to everybody? Any questions about that?

So what would I do to fix this? The real problem I have is what's going on here? This voltage went positive. Diode turns on. I start shoving current through the inductor. Voltage goes negative-- what I wanted to do and what happened over here was that when the voltage went negative, the diode shut off, and then I got 0, which is what I wanted. So I had the positive part of the sine wave and then 0 for the other half, and I got an average value.

Here, what happens is the inductor forces the diode to stay on, and I get this chunk of negative voltage until the diode voltage averages to 0-- I'm sorry, until the inductor current averages-- until the inductor voltage averages to 0, in which case the circuit switches. What could I do if I wanted to do something more like this waveform but smooth the output?

Well, if I got clever, I might say, hey, why don't I build instead this circuit? I will build this, but instead of just having D, let me call this D1. And I'll call this then ID1. My problem is that Vx is going negative. And then that means that instead of getting this positive bump and then 0, I'm getting this negative part because the inductor current is holding the diode on.

Well, maybe I can just get clever, come in, and add a second diode here, D2. This diode D2 can take the inductor current. So if I have I sub L, now I sub D1 is going to be different than I sub L, so I guess I'll have to give it a different color. Let me call this I sub L, if anybody can see that.

So I sub L can now be big. It doesn't have to go through D1. Instead, it can go through this diode D2. Well, how would I analyze this circuit? Well, I can do the method of assumed states. But now I have two switches, which could give me, in theory, up to four possible states-- both off; one off, one on; the other off, the other on; both on.

But if I'm clever, maybe I assume that they're not both on at the same time, typically, or off at the same time, hopefully. So what would I do if I analyze this circuit? And let me just assume the inductor is pretty big so that IL is going to be approximately constant. What would I get in that case? I get something like this. Here's my sinusoidal voltage.

If my inductor current is positive-- suppose D2 was on-- as soon as Vs sine omega t crosses 0 positive, if D2 is on, this voltage would then get to be positive. So I know as soon as I cross 0 positive, D1 is going to be on in this range. And I'm going to get a voltage Vx that looks like this. It's going to follow this.

Now, what happens when x goes negative? If I assume that D1 one is on and Vs sine omega t becomes negative and D2 is off, I'd have this configuration. I'm assuming he's on, and I'm assuming he's off. And my inductor current is doing something here.

This diode 2 voltage-- if Vs sine omega t becomes negative, that means this diode voltage would become positive. Can I have a positive diode voltage? No. That means that this diode must turn on when this voltage goes to 0. So at the minimum, I know that D2 two has to be on here. Now, if I assume that both D1 and D2 are on here, I'd be shorting out the source. So that can't happen.

So what happens is that when I cross this point, this diode turns on, and this diode turns off. And what I will get is when the bottom diode is on, Vx is going to be 0. And I'm going to get something that looks like this. And I get the waveform I had before. But now what happens? I'm applying this yellow waveform, Vx, to this big LR filter.

Well, if the L is really big, it extracts this, and I get V out is equal to-- is approximately equal to the average of Vx. So the average of V out-- well, what's the average of voltage Vx? Well, I would say that the average value of Vx is simply 1 over 2 pi, the integral from 0 to pi of Vs sine omega t, d omega t, because I did everything in electrical angle here instead of time.

If I take this integral, this just ends up being Vs. This means being cosine of 0 minus cosine of pi. This just ends up being Vs over pi. So what I can say is the average value of Vx here is just going to be Vs over pi. And because the average value across the inductor is 0, that means V out has to be approximately Vs over pi. So basically, V out is going to be basically Vs over pi.

And now I've got a DC output current and a DC output voltage, which is what I wanted out of this circuit. Let me just show you that really quickly. If I put down this simulation, I'll put up my other simulation. All we've done here is I've added a second diode. This is what's known as a half wave rectifier. Let me show you two things.

First of all, you can see Vs sine omega t here is this red, which is on the left axis. I'm sorry, this is-- this is V sub x. I'm sorry, this is my yellow waveform over here. But look at the inductor current. What do you think is happening here? This is the inductor current building up over time. The blue waveform is I sub L starting at some time equals 0. It's not constant.

If I look at this, would you say that this blue waveform-- you can see all these cycles going on-- is the blue waveform in periodic steady state? No, it's ramping up over time. So anything I might say about steady state operation is not true down here right after I've started this circuit up and it's coming up. Real circuits have dynamics. It's doing something dynamic. But the interesting thing is, eventually, I get way over here, and look at this. If I come over here-- and let me just zoom in on this right hand portion of the circuit after I've waited for a while.

Everything's repeating every cycle. That's periodic steady state operation. And the interesting thing is, if I look at this-- if I look at-- this inductor current is almost constant. Vx is exactly this half wave rectified waveform. If I looked at the voltage here, and I will now look at it, it is almost constant. And it's just about 1/3 or 1 over pi times the peak value of the sine wave.

So we've been able to use the fact that I understand the average value across the inductor to tell us what the output voltage is this thing is going to be in periodic steady state. So to conclude, what we've got is method of assumed states to figure out when our diodes are turning on and off. And periodic steady state operation, we can use some of those relationships to start figuring out what voltages and currents will get in my circuit.

That's all for today. I'm like a minute over, but if there's any last-minute question, I'll happily take it. OK, have a great day. I'll see you next class.