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**DAVID PERREAULT:** OK, why don't we get started. So we've been talking a bit about magnetics and particularly how you model magnetics. Today, I'd like to talk a bit about magnetics loss. And we can only really scratch the surface of this topic. I mean, it actually takes up a whole chapter in *Principles of Power Electronics* beyond what's in the basic magnetics chapter.

So it's a pretty deep topic if you really want to design very high-performance things like how you predict the losses or calculate the losses. And also, magnetics design, which is the focus of *Principles of Power Electronics* chapter 20, is also a pretty extensive topic. Nonetheless, you can get pretty far with some basic information. And we've put up on the web just some basic inductor design tips, and that should be enough to get you going.

In today's lecture, I'm going to go into magnetics loss, and I'm going to wave my hands enough that you might start to feel a slight breeze, in the sense that I'm not going to justify or prove anything I say. My goal is just to give you enough information so that you know some things that you might look out for when you're designing magnetics.

And the fact that I'm doing that is not going to prevent you from designing perfectly good inductors for your design project or design a perfectly good transformer. But when you start to really want to push the envelope, suddenly you've got to start thinking about these effects I'm going to talk about pretty carefully.

So if I'm going to design a magnetic component, I really have to think about loss, and loss is important for two reasons. If you have bad magnetic components, they tend to-- the magnetic components tend to be the dominant source of loss in a converter, for the most part. So in a typical design, if you don't do a good job on the magnetics, your efficiency is going to stink.

The second piece of it is that what sets the size of the magnetic component is often the law. So what prevents you from making things smaller, often, is that if you make it too small, it'll get too hot, and you'll be out of luck. So thinking about, What are the loss mechanisms? and what to watch out for is what we want to pay attention to.

Now, if I have a typical magnetic component, what do I have? I have a winding and a core, usually. So the two subsets of losses I've got to worry about are, one, the winding loss-- and sometimes this is called copper loss because most windings are made of copper. Typically copper wire is used, although aluminum wire is not infrequently used, as well, especially if you want something that's going to be very light because aluminum has better conductivity per unit mass than copper does.

The second thing is, of course, core loss. And so we're going to consider each of these things today and just give you some things to watch out for. Now, at the most basic level, if we're going to think about your winding loss, you're going to wind-- so suppose we're designing an inductor. You're going to wind a bunch of turns, typically on a bobbin that's going to go on a core.

So just to give you an example, if you look at this in the top right here, this is a half of a core. And then you put two of those half of a cores together, and you might grind the center post to give you a gap to store the energy in, and you would tend to wind your windings around the center post.

So if you're going to have an inductor, you're going to wind it with some length of wire. So a DC-- the resistance of the wire is simply going to be  $\rho$  times the length of the wire divided by the area of the wire. So you would often pick your wire gauge to give you a certain area. You'll have some length. You wind it up. And resistivity times length over area is your wire resistance.

One thing I'll just give you a heads up just to pay attention to is that, if you think about the resistivity, which is the resistivity of copper, it's actually a pretty strong function to temperature. If you raise the temperature by 100 degrees C, which is a lot, but not necessarily unreasonable, the resistivity of the wire tends to go up by about 40% because that's just the characteristic of copper.

So usually, you want to use the resistivity at whatever the maximum temperature you think your wire is going to get to because that's sort of the worst case, and that's going to give you the largest resistance. So then you'd calculate your loss. If you're only considering-- this is the DC resistance. And we'll come back to that.

Power is just going to be equal to  $I_{RMS}$  quantity squared times  $R_{wire}$ . OK if the RMS is dominated by the DC component, you have almost a current component, then RMS is close to the IDC, and you might get  $IDC^2$  times  $R_{wire}$ .

So it's pretty straightforward to calculate. And so then the trade-off you're making is, how fat do you want the wire to be? And how much length of it do you need to get your loss low enough?

Something that does come up is-- and the reason why I'm talking about this wire area is because we're going to fit our turns in this window. So there's some cross-sectional area, the area of the window, that you can put your turns of wire in. And so eventually, you can only put so many turns of a certain gauge wire before you run out of window room.

But then you say, well, I can use a small wire and put a lot of current through it. And yeah, maybe you can, but you've got to worry about overheating things. So something that often comes up when you're doing this is, exactly how much current can I put through that piece of wire?

So if I wanted to say, what's the allowable RMS current, maybe, that my wire can carry? And that's an interesting question because there's not a single answer to that question. It's usually dictated by how hot things are going to get. And that, in turn, makes it dependent on heat transfer.

Now, we haven't really talked about thermal models and heat transfer yet. And we will. But generally, people often will work by some kind of rule of thumb to start. In the end, you're going to end up with a design. You're going to find out if it gets too hot or not. But as a place to start, you might take a guess at the current density,  $J$ . That's in amps per centimeter squared or something.

And the question is, how much current density, how many amps per meter squared can I put through my wire? And that number depends on size. If I have something really big, there's a long distance to get heat out. It has a very high volume-to-surface-area ratio because volume goes as a cube, and surface area only goes as the square of linear dimension.

So if you have something small, it's easy to get heat out of it. If you have something big, it's hard to get heat out of it. So typically, the current densities you can use depend upon how big the thing you're building is. Smaller, you can use higher current densities. A typical number might be on the order of 500 amps per centimeter squared. And we're talking about RMS values.

But again, it depends on scale. This chart, which I've pulled out of KPVS-- what it shows is, if I took this kind of core, which is known as an RM core, or Rectangular Modular core, RM 4 core is really small. RM 14 core is the biggest. It's maybe getting close to the size of my fist.

And this shows, if I have only winding loss, how hot will the thing get based on measured data? And what we can see is maybe you have a few-hundred watts per centimeter squared for a big element and maybe much higher for a small element. And this is for a 25-degree C rise.

On the one hand, maybe you can take more than a 25-degree C rise. It's not unusual to have 50. On the other hand, you also have other losses besides the winding loss. So maybe this is kind of representative of what might be allowable for current densities. And so you can see my number of 500 amps per centimeter squared is right here.

So typically, you might start your design off and say, if it's big, maybe I'll use 100 or 200 amps per centimeter squared as my guess. If it's smaller, maybe I'll go to 500 amps per centimeter squared or even a little bit higher. And then what's really going to limit me is, how hot does the thing get? Any questions about that? OK.

That's all well and good. And that's a basic notion. Here's how I calculate my winding loss. I want to fit a lot of turns in a small cross-sectional area, so I've got to pick an A wire, and I'm going to be limited by how much current I can carry in that wire by some number that I make an estimate at and then go figure out how hot it gets afterwards.

There are, however, other effects. And chief among these is what happens at high frequency. So all of this calculation of the resistance is the DC resistance. This is what happens if I put a constant current through the wire. If I put a current that varies very quickly in time through a big wire, what happens is I don't get to use the full wire cross-section.

There are two effects. One is known as skin effect that I'm going to talk about now. And here's the idea. Suppose I had some wire. Here's some length of wire. And I put some current through it.

Now, a DC-- what I expect is that current, if it's going down the wire, to distribute itself evenly across the volume of the wire, evenly across the cross-section of the wire. And that's what you get. The current will just spread out and be carried evenly everywhere in the wire.

But now I have this AC current going into the wire. And that AC current develops a magnetic field. So the current that's in the middle of the wire is going to generate an H field in the wire and, hence, a B field in the wire. And that represents magnetic flux being carried around inside the wire.

But what do we know about something that generates a  $dB/dt$ , a change in magnetic flux density? We know by Faraday's law that that generates a voltage. And that voltage that you generate is going to cause, essentially, eddy currents to circulate within the wire. And they're going to try to reject the change in magnetic flux density in the wire.

So what you're going to get is induced eddy currents that do something like this. They're going to come up the wire in the middle and go down the wire on the outside. And so what's going to happen is, in net, the current in the middle of the wire will start to cancel.

And so what you will get if you go to a high enough frequency is that the net currents will only be carried, essentially-- if I just said, where is the net current-- will be carried only in a band around the surface of the wire. So instead of getting the full wire cross-section at high frequency, you will only get to use the surface of the wire. That's what's known as skin effect.

Why does this happen? It happens exactly for these eddy current reasons. To actually solve things, you generate what's known as a magnetic diffusion equation. You solve it, and you can solve for, what is the current density profile going in from the surface of the wire?

This is kind of annoying because I got a certain amount of wire, and I wanted to use it to carry current. And now what I'm telling you is, if the frequency is high enough, I don't get to use all the wire. And I've just got this dead space in the middle of my wire, and I'm not getting to use it. Well, I said that's at high frequency and low frequency. What do I mean by that?

There's something called the skin depth, which falls out of this analysis I mentioned. And the formula for skin depth, which is written as  $\delta$ , is equal to the square root of the conductivity, in this case of the copper, divided by  $\pi$  times the permeability, in this case of the copper, and the frequency. This skin depth is a characteristic length that says, how quickly do the fields and currents fall off inside the wire?

So if you take an electromagnetic analysis-- you've probably done this analysis before. But what it says is, notice that this is a function of frequency. The skin depth decreases as  $1/\sqrt{f}$ . So essentially, if the skin depth is on the order of the diameter of my wire, I pretty much carry it approximately constant across the surface of the wire. If my skin depth gets small compared to the wire, that's what I'm only going to carry it in the surface.

And it doesn't just get carried-- here, I'm giving you a circular cross-section. Let's just think of a rectangular cross-section. And I would say, OK, at DC, if I looked at the current density versus the wire dimension between, say, 0 and the width of the wire, I had single-sided conduction, which I'll talk about in a minute, at DC, what you'd get is a uniform current density. This is  $J$  versus this depth into the wire. This is at DC.

At high frequency AC, what happens is you get a lot of current on the surface. And it falls off in an exponential. And this goes as sort of  $e^{-x/\delta}$ . It falls off exponentially with a length constant that's the skin depth.

So when do you have to worry about this? When your wire is on the dimension-- or larger, significantly larger, than dimension of the skin depth. So you just calculate the skin depth, and you'll find out if you're going to have a lot of skin effect.

Interestingly, if you have a case where the width of the wire-- I'm having a foil where I'm carrying current into the board now. So this is the notion there's a foil going into the board. Or you can do something similar here. Interestingly, the loss is exactly the same, in this case, as if I carried all the current in one skin depth. And it's just an interesting thing you can derive.

So you can think about it from a cross-sectional area carrying capability. You can imagine, even though I had  $w$  of width of wire, I can only use perhaps  $\delta$  of it. And I can just calculate the AC resistance as if I had the skin depth. So I would go back, and instead of carrying-- having the area of the wire, which might be the height of the wire times the width of the wire, I'd have the height of the wire times  $\delta$ .

Or another way to express that is, if  $w$  is much greater than  $\delta$ , if the width of my wire is much greater than the skin depth, then I can write the AC resistance as being equal to the DC resistance, which is what I calculated over there, times  $w$  over  $\delta$ .

So if my width of my wire is 10 skin depths, my AC resistance is 10x the DC resistance. And from this, my AC resistance goes up as the square root of the frequency of what I'm carrying for when I get into this regime. Any questions about that?

This chart here just tends to show, this is the field versus the width. So basically, we've got a current source coming here and carrying current. And then the question is, where in this conductor does the current return? This shows the field inside the conductor, and then this shows the associated current.

So if I have a width of this conductor that's thin compared to a skin depth or on the order of a skin depth, yeah, the current is almost evenly distributed across. But once I turn up the frequency, and the width is much bigger than the skin depth, the current falls off exponentially from the edge of the conductor where there's high field.

So when I say, oh, Why is the current sticking to the surface of the wire? the question is, does it stick to this side, or does it stick to this side? It sticks to whatever side of the conductor has high magnetic field imposed on it. The way you would analyze this is you'd say, OK, let me analyze this. Where are the magnetic fields around the conductor? Where there's high magnetic fields, that's the surface of the conductor to which the current is going to get compressed. Any questions about that?

Now, I don't know about you, but that kind of bends my brain a little bit. So I thought the nicest thing is to actually see this for real. And you can see that here, the  $H$  field-- this is the  $H$  field, and this is the current density. We can't see the current density, but maybe we can look at how the  $H$  fields drop inside the conductor, and that will relate to how the currents distribute inside the

**PROFESSOR 1:** So to demonstrate this, we have this giant block of aluminum here, and the aluminum forms a one-turn secondary on this transformer. So here, you might be able to see there's a core here. We have a winding over here that we're going to drive. But this is one turn around the inside of this transformer. And so we're going to have current flowing in there.

And it's hard for us to measure the current, but the current generates a magnetic field. And so we have this magnetic field probe over here, and we're going to move the magnetic field probe. Right now, the end of the probe is right on the inside edge here. And we're going to move it about halfway across the block, and then we're going to move it back again.

Now, when we move it from the edge to the middle, we're going to excite our transformer at 60 hertz, so relatively low frequency. And so the skin depth will be quite long, and so we should get good current in the block. And we'll see that it goes quite deep into the block.

Then when we return, we're going to turn up the frequency to 240 hertz. And then we're going to see how that compares to the 60 hertz result. So the first time, I'm just going to run it so that you can just see how the apparatus is working.

So I'm using a stepper motor here to move the probe. And so you can see that it's slowly pulling out. And then when we get about halfway, then it's going to go back. It's changed the frequency at this point, and then it's going to go back in again. So now I'll change the display so you could look at the scope.

It takes a second for VJ to lock. So now this is the-- it's the strength of the magnetic field, which is proportional to the strength of the current. So as it's going in-- or as it's going from the edge to the middle, you can see that the magnitude is going down. It has kind of an exponential look to it. And now it's changed frequency, so you can see that the amplitude is smaller right away.

So I've tried to adjust it so that the two amplitudes are the same at the outside edge and so you can see what the relative distances are for the two different frequencies. The green one is 60 hertz, and the yellow one is 240 hertz. OK, I'll give you back your screen.

**DAVID PERREAULT:** Thanks, Steve. So you could see that the skin depth-- that's the exponential length constant with which things decayed-- was about a factor of 2 different in those two examples because the frequencies were about a factor of 4 different. So what do I hope that you're going to take away from that?

What I hope you're going to take away is you're going to think, when you design something, oh, if I'm trying to carry high-frequency currents, I better think about how big my conductor is relative to the skin depth. And maybe I can use the DC resistance, or maybe I ought to figure out what the skin depth is and calculate the AC resistance of my winding.

And by the way, I put in the notes-- this approximation is for if  $w$  is much bigger than the skin depth. In the lecture notes, I put what the approximation looks like if it's in some intermediate value. So it's not like there's a frequency-dependent AC resistance, but it's important because sometimes what you can find is, you're trying to carry large currents at high frequencies, you find out that your AC resistance is significantly bigger than you would have gotten if you just calculated this value. Any questions about that? Yeah.

**AUDIENCE:** Is this [INAUDIBLE] of having one big conductor with many parallel conductors?

**DAVID PERREAULT:** Yes and no. Yes, you can. But you just can't have parallel conductors because, indeed, what is one big conductor except a bunch of parallel conductors? But what you can use is something called litz wire. And here's a picture of litz wire.

So what you do is-- if you want to carry a lot of current, so you need a lot of cross-section, but your skin depth is small, what do you do? You get a bunch of insulated strands. So each of these strands is insulated from one another. But then you can't just parallel them because of the way the magnetic field will behave. Basically, the current will still be carried in the outer ones and not in the inner ones.

But what they do is they basically weave these windings together. In the simplest thing, you can twist it. But the key thing is you want-- sometimes the conductor is on the inside, and sometimes it's on the outside. So if you imagine that you weave your windings in a pattern called litz transposition such that each piece of conductor spends an equal amount of time on the surface, and in the middle, and the center, then if you want to think about it, the current doesn't know which way to go. They all look equal, so it goes through equal through everybody, and everybody's happy.

So you'll see a lot of high-frequency designs where you're carrying high currents that will use litz wire as the conductor for precisely that purpose. And when you want to make really big wire, you make smaller litz bundles. And then, as you see down here, you weave those together or twist those together in order to get this effect.

And there are papers that show you, well, how small your litz bundles have to be, but the first thing you might think of is, all you're dealing with is skin effect, is you would like your litz wire to have a diameter that's on the order of a skin depth is a good place to start, if all you're thinking about is skin effect. Excellent question. Any other questions?

So skin effect is a pain. But it turns out that there's something even worse than skin effect that can occur, and that's known as proximity effect. What is proximity effect? Well, proximity effect is when we get eddy currents in a wire due to fields from some other piece of conductor nearby. And I'm still generating loss.

So imagine that-- and here's an example. Imagine I have a piece of conductor, and maybe this is a foil conductor carrying some current in my-- I'm carrying current in this direction. And because of where this conductor is sitting, it's sitting inside some H field, like here, some external H field on each surface.

And this conductor could be carrying no net current. It's just sitting there. But because of some other current nearby, it has a field on it. Well, what should I effect. I'm trying to punch each field through the conductor. And again, that's going to generate some eddy current trying to reject the flux from the center.

So what that's going to do is that's essentially going to generate a current. It's going to generate an eddy current that goes in on this surface and comes out on this surface. Why? Because that's going to generate-- if this is an AC current, the  $dB/dt$  generates these circulating currents inside the conductor that try to reject the field from the center.

Here in this plot you can see, these are solutions. Here's where I have a piece of conductor, and there's vertical fields on each surface. So the conductor is not carrying any net current. And what we can see is we get eddy currents that decay away into the middle of the conductor with the same skin depth relationship.

Why is proximity effect particularly bad? Because I'm inducing loss in this conductor, even though I'm not getting to use that conductor to really carry any current. Well, why should I care about this proximity effect? Because imagine I have some core, like a pot core, where I have some center post, and then I have this window where I'm winding currents.

So here is my core. So I'm looking at a cross-section of the core, like this. And suppose this is a transformer. Then maybe I wrap my conductor in and back out. So maybe I have some foil conductor or a set of wire conductors that's going in here and carrying current in this way and then is wrapped around, and it's coming out here. And I have a layer of that. [INAUDIBLE]

And maybe I have a second layer because I'm winding out, and I have another second layer. Going in, going in, going in, going in, and coming out. It's doing that. And maybe over here I have some secondaries, so maybe the secondary carries an equal and opposite current. Maybe it carries twice as much current. It's a 2-to-1 transformer.

So here, I have two dots, two dots coming out, two dots coming out, two dots coming out. And same thing on this side-- it's two dots going in and so forth. So I'm plotting the current density, and these are the net currents that are being carried.

So what happens? Inside my core-- if this is my core, if this is high-permeability material, there is no H field. So if I execute Ampere's law, integral of  $H \cdot dl$  around, say, a loop here, the only field that appears is in the window. And what that means is, in this picture, if I plotted the H field, I ought to have some H field here that's associated with this current from the middle.

And if I looped around two layers of winding, I get two H fields and so forth. So what's happening is I go across the window, my field builds up. In the first layer, I've got one unit of H. In the second layer, I've got two units of H. If I had three layers, I'd have three units of H.

This layer is only carrying one unit of current, but you can see that it has an average of 1.5, or it has-- I'm sorry. It has a field that's imposed on it that's bigger than the net current it's carrying. So this is an example where fields owing to this layer are hitting this layer. Fields owing from these two layers would hit the next layer.

And what happens is then I've got this effect where I'm imposing fields on my conductor, and I'm going to get eddy currents in the second layer that's going to cause loss. And the problem with that is, especially if I have very many layers of winding, and I have very high AC current components, that induced loss gets reflected-- the way it looks electrically is there's a much higher AC resistance to the winding.

So you could calculate your wire resistance, your theoretical wire or DC wire resistance and say it's 10 milliohms. And then you could think, oh, but if I include skin effect, it's 20 or 30 milliohms because I have a bunch of skin effect. And then you figure out what the proximity effect is. If you have a bunch of layers, and the loss can go as the square of number of layers, you could have 300 milliohms.

So whereas your DC resistance might only have been 10 milliohms, suddenly you've got 300 milliohms. That's a factor of 30 times the loss you were expecting. So when does this occur? This occurs when you have conductors that are large compared to a skin depth or on the order of a skin depth and when you have many layers of winding such that fields from one winding are hitting the other.

If you're designing something that's carrying mostly DC, eh, don't worry about it. If you're designing a really high-frequency transformer, you might want to start thinking about it because it can be a big effect.

And in fact, what does that say you should do? Typically, if I said, OK, because of skin effect, I wanted my wire to be on the order of a skin depth or thicker, or at least my strand diameter for my litz, if you include proximity effect, very often you want your strand diameter or your wire diameter to be maybe 0.3 skin depths.



Or you get into this weird situation-- and so sometimes that forces you to use really fine wire or really small strands, even smaller than a skin depth. So think 0.3 skin depths might be what you'd get if you had a lot of layers. So that's just a warning because you can get into this weird situation-- I've been there-- where you think, oh, my conduction loss is too high. I'm measuring it. I've got too much resistance.

So what's my natural inclination? Put in more copper. Use fatter wire. And you put in fatter wire, and the resistance goes up. And the reason the resistance goes up is because this loss, as you make this conductor wider, there's more room for the eddy currents to circulate because of proximity effect. If you make the current conductor thinner, this current going this way and this current coming back kind of overlap and cancel, and you get less proximity effect loss.

So sometimes it can be better to use thinner wire instead of thicker wire to get less loss in net. So that's just a heads up about that. Any questions? Don't worry about it if you're carrying mostly DC. It's not a problem. It's just something for you to know about when you start to think about designing really high-frequency things.

**AUDIENCE:** [INAUDIBLE] at 56 [INAUDIBLE]?

**DAVID PERREAULT:** Well, OK, so the skin depth at 60 hertz, and this depends again on temperature, is on the order of 3/4 of a centimeter. So if you were talking a really mondo-- like, you wanted to carry 10,000 amps, yeah, well, then you're going to have to strand and insulate your wire and litz wire. And so if you look at things like steam turbine generators, they do use litz. Their bundles are like this. But it's big, compressed chunks of copper.

So this applies at any frequency if you're carrying enough current. If I'm building a 60 hertz transformer that's carrying an amp, my conductors are naturally going to be much less than the skin depth, and I wouldn't even think about it. OK, so that's winding loss. And these are just some things to think about. Watch out for skin effect, and really watch out for proximity effect if you're carrying high AC currents.

The other piece, of course, of any inductor or any transformer is the core. And ideally, a core material is lossless. We have always idealized, if you will, our BH characteristic of our core to look something like this.

So I might have plotted B versus H to do something like, I have some ideal-- this is  $\mu$  of my core. And if I get above saturation, it starts to do something like this, and it goes back to  $\mu = 0$ . So that's my idealization.

But it actually doesn't look like that quite on a real core. So if I just thought, suppose I built a core, and it had some area of the core. It had some  $\mu$  of the core, area of the core, length of the core. And I put a winding on it, and let me imagine that my winding has no resistance.

What would the net power going into the core look like or the net energy? Let's calculate the energy that I'm putting into the core. So the input energy would be the integral over a period of  $v$  of  $i$  of  $dt$ . It's the voltage applied times the current.

And if there's any loss in this system in the core, I will have an integral of  $v_i$  that would be not 0. Ideal inductor, of course, that ought to be 0. But if I have loss, it won't be. And what does this look like?

Well, let me rewrite this in magnetic terms. What is  $v$ ?  $v$  is equal to  $N$  times the area of the core times  $dB/dt$ . Why? Because  $v$  is  $d\lambda/dt$ , and  $\lambda$  is  $N\Phi$  so then I have  $N\Phi$  times  $dB/dt$ .

What is the current? Well, keep in mind, if I integrated the  $H$  of the core around the length, I should have  $Ni$ . So what I get is  $H$  in the core times the length of the core is equal to  $Ni$ . So I ought to be able to write  $i$  as being equal to  $H_c$  times  $l_c$  over  $N$ .

So then I could write  $i$  of  $t$  as being equal to  $H_c$  times  $l_c$  divided by  $N$ ,  $dt$ . So let me cancel out the  $N$ 's here, and let me isolate-- I could write this as-- let me take out  $l_c$  and  $A_c$ -- the length of the core times the area of the core-- this is just the volume of the core here-- times the integral of  $H$   $dB$ .

And I'm going over a cycle, so presumably,  $B$  is looping. So this is around a closed cycle. So this is the energy I lose in going and cycling around. It's the integral of  $H$   $dB$  in the core. And what we will see is that, in a real core material,  $B$  and  $H$  don't quite go up and down proportional to one another the way this illustrates.

What you really see is something that does this. It goes up, and then it comes down with some slight spacing, like that. And if I think about the integral of  $H$   $dB$ , that's really this area. So  $H$  and  $B$  really form a hysteresis loop, and the area inside that hysteresis loop is the energy I lose in a cycle. Any questions about that?

This would sort of suggest that I lose so much energy per cycle for a given  $B$  or  $H$  swing. And if I do it twice as fast, I lose twice as much and so forth. And at low frequencies, that's kind of true. At low enough frequencies, that's kind of true. When that's true, they call it hysteresis loss.

Unfortunately, when you get higher in frequency-- where does this loss come from, I should say. It's because you're doing work to move magnetic domains around. They're mechanically moving around. And by the way, when the magnetic domains jump, they introduce little eddy currents in the material. And so that generates loss.

So you do have a bunch of loss moving the magnetic domains around. And hence, that appears as this net loss that-- even if you ignored winding loss or some additional loss by swinging AC flux in the core. This loss is sometimes called classical hysteresis loss.

Unfortunately, while at low frequencies, it's sort of proportional to frequency, when you get up to high frequencies, it turns out the shape of the loop changes. So it turns out that actual loss in many magnetic materials is not proportional to frequency. It's proportional to something higher than frequency. It just depends on the magnetic material. So that's one thing you've got to keep in mind.

So people sometimes talk about hysteresis loss, but what this hysteresis loop looks like is frequency-dependent. The other thing I would say is that for this kind of loss, you can calculate it as loss per unit volume. If I told you the shape of the hysteresis loop and what  $B$  you were swinging over, you could say, this area times the volume of the core times the frequency would give me my power dissipation in the core material. Make sense, everybody?

I should say that this kind of loss, owing to the magnetic domains, is one form of loss you can get. There are other forms of loss you can get, however, in cores that are not so-called hysteresis or related loss. And one of those is, in fact, eddy current loss in the core.

Think about this. What is this core doing? If I put a current in here, I'm putting a flux and a flux density of  $B$  through the core itself. Well, at low frequencies, especially, we often use conductive magnetic materials. We use steel. Steel is pretty conductive.

And so if I looked at this cross-section that I drew here in white, what I would get is I would have some flux going in like this. And what would that do? If I have an AC flux,  $\frac{dB}{dt}$ , I'm going to generate a field inside the conductor that does this.

And that AC field is then going to generate eddy currents which try to counteract that and reject the flux in the core. So in other words, the fact that I'm carrying AC flux through a conductive material means I can get eddy currents circulating in the core.

That's kind of unfortunate. I'm sorry. If I put the flux in the core, the eddy currents are going to circulate this way. So this is i eddy. When do I start to care about this eddy current loss? I care about it when the dimension of the core is on the order of the skin depth.

And if I get a cross-section that's too big, two things happen. First, I get a lot of eddy current loss, and that just manifests itself as additional loss. And by the way, it scales up with area because I have more room for eddy currents to run around in, so it's not even lost per unit volume. It gets worse as the cross-sectional area gets bigger.

But the other thing is, if it's bad enough, it sends a reject flux from the center of the core, and I don't even-- I might drive up the concentration of flux at the edges and maybe even saturate it. So what do people do about this? If you have high frequency materials like ferrite, it turns out, they're not very conductive, so you don't usually worry about eddy currents in your core, unless you're at really high frequencies.

But if you're at low-frequency materials like steels, you do have to worry about this. How do they fix this? Instead of making one big, giant core cross-sectional area, they sort of take the same trick. What they do-- they don't need to litz transpose it because it's not like they're carrying current in it. What they'll do is instead of making a big cross section, they'll make this out of little sheets like this that are insulated from one another called laminations.

So instead of building up a big, fat piece of steel, they'll build it usually out of stampings and insulate them and then stack them up. And the nice thing about this is, instead of this big eddy current being allowed to run around this huge loop and cause lots of loss, it's being forced to circulate in these tiny little areas. And as long as these laminations are thin compared to a skin depth, the eddy currents more or less cancel out, and you don't get much eddy current loss.

So you'll see a lot of 60 hertz transformers or electric machines are all built out of laminations. How do they size the skin depth of those laminations-- how do they size the lamination thickness? They ask what the skin depth in the steel is, and they make the lamination sort of thin compared to that. Any questions?

**AUDIENCE:** What characterizes [INAUDIBLE] core as being higher [INAUDIBLE]? Is it related to permeability, or is it just a property?

**DAVID PERREAULT:** Well, so what's the-- yeah, it comes down to, what's the skin depth? So if you have high permeability of the skin depth, it gets small. So you just kind of plunk this in and find out, what is the dimension that I would care about?

I should say, I can fix eddy currents just by laminating. So then I'm left with this hysteresis kind of behavior. But I said that hysteresis, in reality, frequency-dependent. And the truth of the matter is, it's not proportional to frequency.

How do people treat it? They treat it rather empirically. What they really do is they plot loss per unit volume versus flux density at different frequencies. And you might get curves like this. And you can see that the loss goes up some. This is a log log plot. So it comes up, goes up at some power of the flux density, and it also increases somehow in frequency.

And they model it, and they say, OK, the power per unit volume--  $P_{\text{sub } V}$  is power per unit volume is equal to some constant  $K$  times some frequency to a first power and some AC flux density amplitude. And this is usually calculated-- this is only usually measured for sinusoids, unfortunately-- so some other power,  $\beta$ .

And basically, what they do is they'll collect data like that. They'll figure out the core loss from the kind of measurements we're going to do. I'll show you right now. And they'll say,  $K$  alpha and  $\beta$ , or they'll give you a plot.

And you look at this and say, OK, this is a high-frequency material. I'm going to put 10 millitesla through it, and I'm going to do it at 5 megahertz, and I have 500 milliwatts per centimeter cubed, and that's how you calculate it. So it's very, very empirical.

Unlike winding loss, where you can write lots of equations and predict it very beautifully, core loss is very much determined by measurement. It's very hard to link the microscopic behavior to the macroscopic behavior except in limited circumstances. Now, I'm almost out of time here, but what we wanted to show you-- and maybe I will hand this over to Monsi.

**PROFESSOR 2:** So we have our inductor here, and I'm plotting the current through the inductor that's proportional to  $H$ , and it's on the x-axis. And we are also plotting the voltage through the inductor, and it's integrating it, and it's on the y-axis. So we are trying to get the BH core for the core And I'll slowly increase the current that's being pushed through the inductor. So let's see. Yeah.

So this is where we can see it's slightly reaching the saturation of the core. And this is the BH core. Do you want to add something to that?

**DAVID PERREAULT:** No. So there you can see, just based on the earlier calculation we did, there's this hysteresis loop between  $B$  and  $H$ . You would see that it changed shape versus frequency. And from this, that's what a real core material does look like, even though we pretend it doesn't do that. So all you've got to do is be careful. If you're going to have a lot of AC flux density, figure out your loss per unit volume and use it.

So that's my very hand-wavy introduction to loss in magnetics. I think it'll be enough to get you through to designing some pretty reasonable inductors and transformers. And we will pick a new topic up next class. So have a great day.