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**DAVID  
PERREAULT:**

OK, why don't we get started? So last class, we talked about some things that happened with rectifier, on the DC side. Today, I'd like to talk about some measures that we use when we're looking at the AC side, so for example, if I had a rectifier connected to the grid. But what we're going to talk about applies to all kinds of energy transfers mediated by time varying waveforms.

Before I get into it, however, I just wanted to remind you of a few definitions because we'll come back and use them. And the first of these is simply the RMS, or the Root Mean Square, of a signal. So if I have a signal  $x$  of  $t$ , the root mean square is the square root of the mean of  $x$  squared of  $t$ . And I'd call that  $x$  RMS.

So there's a variety of reasons why you might want to calculate the RMS of a signal. But it's particularly useful in thinking about energy. Why? Because suppose I had, for example, a current waveform,  $i$  of  $t$ , into a resistor. And I said, how much power is being transferred into that resistor on average? Well,  $p$  of  $t$  is simply equal to  $v$  times  $i$ , which would be equal to  $i$  of  $t$  squared times  $R$ .

And then if I wanted the average power, I would calculate that as  $\frac{1}{T}$ , the integral over  $t$  of  $i$  squared of  $t$   $dt$  times  $R$ . So I can take out the  $R$ . And this gives me  $i$  RMS squared times  $R$ .

So calculating the RMS of a current means it's the DC quantity-- it's the fixed quantity-- that will transfer the same amount of power into a resistor as a current. Or I could have done this with the voltage, if it's a resistor. So I don't need to know everything about the waveform. All I need to know is this RMS, and  $i$  RMS squared times  $R$  or  $v$  RMS squared divided by  $R$  gives me the power dissipation or resistor.

Now if I don't have joule heating, if I don't have resistive loss, maybe the RMS isn't the right quantity. But very often it is. So that's why we tend to care about it. The second thing-- first of all, any questions about the meaning or use of root mean square?

The second thing I'd like to remind you of is the notion of orthogonality. Now, I think it's particularly obvious what orthogonality means when you have a pair of vectors, that they're perpendicular in some sense. But what does it mean for continuous time waveforms?

Well, two waveforms,  $x$  of  $t$  and  $y$  of  $t$ , are orthogonal on an interval if the following-- the integral from  $a$  to  $b$  of  $x$  of  $t$  times  $y$  of  $t$   $dt$  is equal to 0. This is the natural assumption of the inner product for discrete sets or vectors or something onto continuous time waveforms. So if I take the product of the two,  $x$  of  $t$  and  $y$  of  $t$ , and I integrate them over the interval I'm interested in and I get 0, that means they're orthogonal.

What that implies, among many other things, for example, is if I had some element and I had a voltage. Say the voltage was  $x$  of  $t$  and the current was  $y$  of  $t$ . That would suggest that over that interval of time,  $x$  of  $t$  and  $y$  of  $t$  would conspire to deliver 0 energy into the element. So we have a general definition for orthogonality and it has some implications about voltages and currents if I use it for that.

Just as a reminder-- come back to some trigonometry. If I have the integral over  $2\pi$  of sine of  $n\omega t$  times sine of  $m\omega t$  plus some angle,  $\phi$ ,  $d\omega t$  is equal to 0 if  $n \neq m$ , meaning if I have two sinusoids at different frequencies-- and here I'm thinking about on some commensurate interval. So  $n$  and  $m$  are just different. They're orthogonal. If I multiply sine at one frequency by cosine or sine of at another frequency and I integrate them, I'm going to get 0, as long as I'm over a commensurate interval.

So sine at one frequency is orthogonal to sine waves at any other frequency. And this is the fundamental trick that we use to develop Fourier series. That's how we extract out Fourier coefficients, is using the orthogonality of different frequencies.

Another thing I'll remind you of is simply that if I have the integral of 0 over  $2\pi$  of sine of  $\omega t$  times cosine of  $\omega t$   $d\omega t$ , I will get 0. So not only are sinusoids at different frequencies orthogonal, but sine is orthogonal to cosine even at the same frequency. And that's just a straightforward trig identity, easy to prove.

In general, more generally, we can write that the  $1/2\pi$  the integral over 0 to  $2\pi$  of sine  $\omega t$  times sine of  $\omega t$  plus some angle  $\phi$   $d\omega t$  is simply equal to  $1/2$  cosine of  $\phi$ .

So if  $\phi$  is 90 degrees, that becomes a cosine, and I get 0. If  $\phi$  is 0 and I have sine squared, this is really saying that the average value of sine squared over an interval is  $1/2$ . So this is just basic trigonometry again, but it's a handy result to remember. And we'll see why we care about this shortly. Any questions about that?

So we'll come back and use that. But let's start to think about AC waveforms. And let's talk about plugging something into the wall. So here's my wall. I have  $V_s$  sine  $\omega t$ .

And if I go back to the source, somewhere in there I'll have a breaker or a fuse. And then I'll have some wall wiring, hopefully very low resistance. I'll call this  $R$  wire. And then I'll come somewhere eventually to my plug, and I can plug something in. And here's my load resistor that I'm going to plug into the wall,  $R_L$ .

Maybe this is my space heater to keep me nice and toasty warm in the winter because it's a little bit cold. So I'm going to go plug this into the wall. First of all, just as an aside, why do we have this fuse or breaker here? What's it designed to do?

**AUDIENCE:** To prevent overcurrents.

**DAVID** Well, that's true. But for what reason, specifically? What's it preventing an overcurrent of?

**PERREAULT:**

**AUDIENCE:** That the source is not spiked consistent. And if it spikes, it'll break.

**DAVID** That's true. I think those are exactly correct answers. The thing I was actually looking for is a little bit more subtle. Why would the-- certainly if the source goes crazy and gets too big, we would want it to pop out. But what it's really designed to protect--

**AUDIENCE:** Load.

**DAVID**  
**PERREAULT:**

Actually, that's a very good thought. Actually, it turns out not to be the load. I mean, one thing is if something goes wrong with the load and the load draws too much current, you don't really rely on the breaker or the fuse to protect it. By then, this thing is toast. What the breaker is specifically designed to do-- I mean, if it's the fuse in the equipment, maybe it's trying to help you prevent the equipment from burning down.

But the breaker back in your home or in the wall is designed to protect the wiring because you could, in principle, plug something in that would be happy enough, wouldn't die, but would draw too much current. And then what happens when your wall wires overheat? They start a fire and then really bad things happen. So the fuse of the breaker is really designed to make sure that the wiring doesn't overheat.

You can't count on that helping you protect your equipment. Your equipment, you might put your own fuse in there to help you keep the equipment from burning down. But it's a good thought. So it is to protect things. But really, it's designed to protect the wire.

Now, for most purposes, we can usually ignore the wire resistance. So let's for the moment pretend the wire resistance is 0 because if you're burning lots of power in the wire, you're probably unhappy anyways. So we'll just ignore that for our calculations. So what is the power going into my space heater here?

Well,  $P$  of  $t$ -- here is  $V$  sub  $L$ . Here is  $i$ . So  $P$  of  $t$  would equal-- as we calculated, it would just be equal to  $i$  squared of  $t$  times  $R$ , which would be-- if  $V_L$  is approximately equal to  $V_s$  because I'm ignoring the wiring resistance, I could calculate it that way. And I could say that the average power-- I'll just call that  $P$  or  $P_{AV}$  is simply  $I_{RMS}$  squared times  $R_L$ .

So if, for example-- if  $V_{RMS}$  is equal to 115 volts-- that means a peak sinusoid more like 170 volts-- and the fuse or the breaker is 15 amps, now this plug here-- if you're two plugs-- you have a long one and a short one-- if your plug looks like this, that's a 15-amp circuit.

So it's intended that you're plugging that back into a 15-amp line. If you looked at this one in particular, it actually has another sideways pin that you could actually plug something in up to 20 amps, and you'd have a slightly different plug to do that. And that means the wall wiring in this case is a little bit heavier and can carry more current and the breaker's designed to carry that.

So this is by the by. But suppose I had a 115-volt RMS, which is about what it's supposed to be, and 15 amps. That would give me-- if I plug this into here would give me about 1.7 kilowatts. So in fact, if you go to the store and you look, you'll see a lot of space heaters rated for just about 1.7 kilowatts or a lot of things you're going to plug in at home. Why? Because you can put them on a 15-amp circuit, and that's what a lot of people have in their homes.

So all right, that's good. I can stay nice and warm. Suppose I wanted to plug in a different load. So suppose I come back, and here's  $V_s$  sine  $\omega t$ . And here's my fuse and my wire. And now I'll come and plug-- suppose I plug in an inductor into the wall here,  $L$ .

It's not that cold these days. Maybe I just want to, while I'm working, bathe in the luxuriant magnetic fields from my inductor. So I plug an inductor into the wall.

Well, again, ignoring our wire-- so I'm assuming that this voltage is the same as the source voltage. Then what would I get? Well, I would get  $i$  of  $t$  would be what? Well,  $V$  is equal to  $L di/dt$ .

So this should be  $\frac{1}{L}$ , the integral of  $V_s \sin(\omega t) dt$ . Well, this should be equal to  $-\frac{V_s}{\omega L} \cos(\omega t)$ . All right, so that's what I get for a result for my current in steady state into my inductor.

What is the power that's being transferred into the inductor on average? Why don't we calculate that? We said  $P_{av}$  is equal to  $\frac{1}{2\pi}$  the integral from 0 to  $2\pi$  of the voltage, which is  $V_s \sin(\omega t)$ , times the current  $-\frac{V_s}{\omega L} \cos(\omega t)$ .

I just did my average in angle instead of in time. Well, what does this work out to be? Well, this is  $-\frac{V_s^2}{2\pi\omega L}$  times the integral over  $2\pi$  of  $\sin(\omega t) \cos(\omega t)$ .

Can anybody tell me what that is?

**AUDIENCE:** 0.

**DAVID PERREAULT:** 0. Why is it 0? Because the voltage is a sine, and the current is a cosine. And we just decided that sines and cosines were orthogonal. So I'm going to draw no average power into my inductor. That kind of makes sense because the inductor is ideally a lossless element. So it shouldn't be able to absorb average power.

What I am doing, however, I'm not absorbing power on average, but I do have current. There's current going into the inductor, and there's voltage on the inductor. So I do have some energy transfer instantaneously. Or what I have is slosh of energy from the wall into the inductor and from the inductor back into the wall but no loss as long as long as I can ignore our wire. That's what's known as reactive power transfer because it's a reactive component.

So that makes sense. But notice that while I'm drawing no average power, I still have current. And that means I am still-- I have some current here, so I am still dissipating some energy in that wiring resistance. And I'm still limited in the current I can draw before the breaker goes or before the fuse pops because the fuse is designed to essentially pop-- or the breaker is designed to pop based on a long-term RMS current.

So what does that look like? Well, if I could figure out the RMS current,  $I_{RMS}$  here is equal to the peak value-- it's a sinusoid. It's the peak value over the square root of 2. So it would be  $\frac{V_s}{\sqrt{2}\omega L}$ .

So if I again have  $\omega$  is equal to 377 radians per second in the United States-- that's 60 hertz, essentially--  $L$  of about 20 millihenries gives  $I_{RMS}$  of about 15 amps at 115 volts AC.

So what does that mean? If I go out and get myself a 21-millihenry inductor and I plug it in, life's good. I enjoy the magnetic fields. If I instead brought my 19-millihenry inductor and plugged it in, well, I'm going to lose the breaker, and I'm going to be sitting in the dark. So while the inductor doesn't consume average power, it does draw a mean square current, and it does degrade the ability of me to utilize my source.

So I have two outlets here. Suppose I'm putting in my 20-millihenry inductor. And I can do that. Life's good. Nothing's going wrong. I haven't exceeded the breaker. But now if I want to plug in my space heater as well because I'm feeling cold, I'm out of luck because I'm already drawing 15 amps RMS. I draw any more current, the breaker goes.

So plugging in the inductor hasn't used any average energy, but it has degraded my ability to draw power from the source. I've utilized the capability of the source to deliver me energy, even though I haven't used the energy. Any questions about that?

Now, as you might imagine, the utility hates that. Their job is to sell you energy. And you're using up the capacity without using up the energy. So what people have done is they've come up with a measure of energy utilization. How well are you utilizing the capability of the source to deliver you energy? And this measure is known as power factor.

So power factor is defined as follows. It's the average power you're drawing divided by the RMS voltage times the RMS current. Now, when I talk about power factor, we mean the power factor at a port. So I have some pair of wires.

And I look at this port, this interface's two-wire pair, and I'm talking about some voltage  $v$  and some current  $i$  here. So it's basically relating to the power that's going across this dashed line. And why do I choose this measure? Well, what would be--

If I had a resistor, the average power is equal to  $V_{RMS} I_{RMS}$ . So the power factor is one. So I basically best utilize that source as I can. I'm drawing energy out of the source, and it's all going to me. All the RMS current is contributing to energy transfer from the source to the load or across this boundary.

If I had an inductor  $L$  or a capacitor  $C$ , for that matter, I'd have average power is equal to 0, and I'd have a power factor of 0. That means I'm utilizing up the capability of the source by drawing RMS current but delivering no energy. So the power factor has a magnitude that's less than 1. The best case from the utility's perspective, if your power factor is 1, that means all the current I'm drawing is going to energy transfer.

The power factor can go down to 0, which means I'm drawing current, but I'm not drawing any power on average. Any questions about that general idea? Yeah.

**AUDIENCE:** If you have a power factor of 1, then wouldn't all the electricity we burned up in the transmission lines?

**DAVID**  
**PERREAU:** Power factor of 1-- OK, first of all, the transmission line-- when I talk about power factor, I'm talking about it at a place. So I'm talking about the voltage here and the current here. So power factor doesn't concern itself with what energy is being dissipated elsewhere.

All it's saying is that the current I'm using here is doing the best possible job of transferring energy across this boundary without wasting any of it. That's not to say that there is no dissipation back over here somewhere. And with a resistive load, a power factor of 1 means you're doing as little badness as you can.

In other words, in order to draw energy in this example, yeah, I am dissipating something in the wire; there is some loss for the utility that they're paying for. But I'm minimizing the amount that's getting burned in the wiring resistance to transfer energy to my load. Whereas in the inductive case, I was doing the opposite thing. I was sort of not taking any energy but still causing them to have to pay for the loss in the transmission line. Does that make sense?

**AUDIENCE:** Thank you.

**DAVID** Other questions or comments? Yeah.

**PERREAULT:**

**AUDIENCE:** So when we say the average power out of port, is the idea then that it would be-- if you had some resistive element at that port--

**DAVID** Say it again.

**PERREAULT:**

**AUDIENCE:** If we say average power out of port, then it says if we were to have some resistive element at that port, that's what it would be, right?

**DAVID** Yes. If I had some resistive element on the-- yeah, the question is, what is the power at the port? It's the power that's going across this dashed line on average. So if I have a resistor over here, that's where it's going. Yes

**PERREAULT:**

OK, so we have this notion of power factor. And by the way, power factor can technically be between 1 and minus 1. Minus 1-- a negative number would just suggest that actual power is flowing that way. But the utility is quite concerned with power factor and not to be confused with actual power dissipation.

I should say, by the way, that this average power is sometimes known as a quote, unquote, "real power." So if you talk, hear somebody talk about real power, that's what they're talking about. VRMS IRMS is what's known as apparent power, which is greater than or equal to the real power.

And this is just saying, what effective power would I be drawing considering that I have that much current and that much voltage if it was going into a resistive load?

So let's expand our thoughts a little bit because I've been so far talking about resistors and inductors and capacitors, which have the characteristic that they all draw sinusoidal waveforms. So nothing wrong with anything we've done so far, but the world is bigger than that. And let's come back and think about, for example, the rectifier we had from last time.

So last time, we said, OK, I'm going to hook up to some AC voltage source. So I have a sinusoidal voltage source, and then that's coming into some rectifier. And I'm getting some output current, which is ideally DC. And maybe I would say, OK, let's talk about this current drawn from the rectifier,  $i$  of  $t$ .

So what does that look like? In the example we had last time, if I have a sinusoidal voltage source, for a power converter circuit, the best-case scenario is it looks like a resistor because that's when you get power factor of 1. Unfortunately, a lot of power converters don't look like that.

And in fact, if we took the half wave rectifier from last time and I said, What did the line current look like? it would look like this. It would look like-- in positive half of the line cycle, it would give me some current  $I_d$ . And in the second half of the line cycle, it would give me 0. So I'd be drawing a square wave. And in fact, in that half wave rectifier waveform, it's actually a square wave with an offset, particularly horrible. And here's  $\omega t$ . This is  $\pi$ . And this is  $2\pi$ .

So the point is, when we start introducing power conversion, we can also often have nonlinear loads, meaning that even when I'm driving it with a sinusoidal voltage, I don't get a sinusoidal current. So let's start to think about how might I represent that case.

And this is particularly-- when I did this half wave rectifier, it was a half wave rectifier with an inductive filter, which draws that kind of square, wavy waveform. Well, let's think about how I might represent this current  $i$  of  $t$ .

One way to think about that is in periodic steady state, I get a periodic current, right? So in PSS, what I'm talking about right now, I can represent  $i$  of  $t$  with a Fourier series. So  $i$  of  $t$ , we might write this as the summation from  $n$  equals 0 to infinity of  $I$  sub  $n$  sine of  $n$  omega  $t$  plus phi sub  $n$ .

This is one way I could write a Fourier series. We can find the Fourier coefficients. And just to be clear, I'm defining phi 0 to be equal to pi over 2 or 90 degrees so that  $I$  sub 0 is just the DC component.

OK, well, I can represent this pink waveform or any other AC waveform that's not sinusoidal as long as it's periodic with some Fourier series. You can relatively easily show what the RMS of that waveform is. If I have-- if I break it down with this Fourier series, we could write RMS as follows.

IRMS is simply equal to the square root of  $I_0$  squared plus  $1/2 I_1$  squared plus  $1/2$  half  $I_n$  squared and so forth. So it becomes the sum of the squares. The halves are because of the mean square of the individual components, and  $I_0$  is constant, so it doesn't have  $1/2$ .

How do you find this? Well, you could just apply the definition of RMS with this waveform and then use orthogonality, and it'll pop out.

So fine, I can express my RMS current. How would I write the power associated with that waveform given this? Well, what I could do is I could say this. The average power,  $P_{AV}$ , I could write as  $v$  of  $t$  times the average of  $v$  of  $t$  times  $i$  of  $t$ .

So this would be  $1/2 \pi$  integral from 0 to  $2\pi$  of  $V_s$  sine omega  $t$  times the summation  $n$  equals 0 to infinity  $I$  sub  $n$  sine of  $n$  omega  $t$  plus phi sub  $n$  d omega  $t$ .

So I've got this integral of a product. I can interchange my summation and integration here. So what would this give me?

This would give me  $1/2 \pi V_s I$  sub 1-- I'm sorry,  $1/2 \pi V$  sub  $s$ , sorry-- summation  $n$  equals 0 to infinity of  $I$  sub  $n$  sine of omega  $t$  sine of  $n$  omega  $t$  d omega  $t$ . Does that makes sense to everybody?

**AUDIENCE:** Shouldn't there be an integral as well?

**DAVID** There should be an integral in here, shouldn't there/ Thank you. So all I've done is I've pulled out the summation.

**PERREAULT:** But look at this. What would you tell me-- and this should be sine omega  $t$  plus phi sub  $n$ .

What would you tell me about this integral here? Well, if  $n$  is not equal to 1, what is this integral?

**AUDIENCE:** 0.

**DAVID** 0 because sinusoids at different frequencies are orthogonal. So I can throw every other term but the  $n$  equal 1

**PERREAULT:** term away. And so what I'm going to get is-- and I can put this  $1/2 \pi$  here. And if we come back to my original equation here, I have that  $1/2 \pi$  here.

So what I get is  $V_s$  times  $I_1$  over 2 cosine of  $\phi_1$ . So the nice thing is that everything drops out. So what does this tell me? This says, if I have this distorted waveform, it's no longer a sinusoid, this pink current that has a whole bunch of harmonic junk in it.

The only part of that that contributes to average power, to real power, is the fundamental of the current because every other current component at all other frequencies, DC and all harmonics, is orthogonal to the voltage.

Moreover, it's not just the fundamental, but it depends upon the cosine of that fundamental. In other words, it's only the component of the fundamental that's in phase with the voltage. If  $\phi_1$  was 0, cosine of  $\phi_1$  would be 1. So if I decomposed the current, the fundamental current into something that was in phase with the voltage and something that was 90 degrees out of phase of the voltage-- I can split it up that way-- only the component that's in phase with the voltage contributes to energy transfer.

So if I want to transfer energy, the only part of that pink waveform that's helping me is the part at the frequency of the sine wave of voltage and only the piece of that fundamental component that's in phase with the voltage. I could rewrite this--

Remember that for a sinusoid, the RMS is  $1/\sqrt{2}$  times the peak value. So I could have rewritten this as  $V_{RMS} I_1 \text{ RMS} \cos$  of  $\phi_1$ . That's just to say that this  $1/2$  comes from the RMS component. So I can write it this way too.

So what does this say about power factor? Well, I can write this average power factor. So now that I can write it, I can say, what is the power factor? And when I'm saying what is the power factor, in this particular example, I'm saying, what's the power factor at this port right here, at the rectifier input port? Because I have this current and  $V_s \sin \omega t$ .

The power factor is simply going to be the average power, which is  $V_{RMS}$  times  $I_1 \text{ RMS}$ , the RMS of just the fundamental component, times the cosine of the fundamental phase of the current divided by  $V_{RMS} I_{RMS}$ .

Or I could cancel out  $V_{RMS}$  because they're both just the RMS of a sine wave. And this would give us  $I_1 \text{ RMS}$  over  $I_{RMS}$  times the cosine of  $\phi_1$ .

This decomposition is useful. This piece here is sometimes called  $k_d$ , the distortion factor. What's it a measure of? It's saying, look, this is the  $I_{RMS}$  of the whole thing.  $I_1 \text{ RMS}$  is simply equal to the square root of  $1/2 I_1^2$ .

So it's saying this is a measure-- this factor, this distortion factor is a measure of how much the harmonic content is degrading my power factor. I just told you that all the harmonic content doesn't help you transfer energy. That means it's degrading your power factor. So this number  $k_d$  is clearly some number of magnitude less than 1.

If it's a pure sinusoid,  $I_1$  of RMS is equal to  $I_{RMS}$ . And so that becomes 1. But when I start to add in any harmonic components or DC components,  $I_{RMS}$  becomes bigger than  $I_1$  of RMS, and this distortion factor starts degrading my power factor.

This here is sometimes called  $k_\theta$  or the displacement factor. And  $\phi_1$  is sometimes called the power factor angle.



This is saying, suppose I had a-- suppose my distortion factor was just 1-- there's no distortion. Well, if my phase angle  $\phi$  of the current with respect to the voltage is 0-- they're in phase-- then this becomes 1. As the current starts to get out of phase with the voltage, this cosine of  $\phi$  has a magnitude less than 1.

So we can hurt our power factor, we can hurt our ability to utilize the source to transfer energy. It's not how much energy we're dissipating. It's how much utilization do we have. I can damage it in two ways. I can damage it by drawing a bunch of harmonic current that doesn't deliver power or by delivering components of current that are out of phase with the voltage, which also doesn't deliver power.

So this is kind of a very useful way sometimes to decompose what's going on. Because if I really cared about the power factor, I might need to know that. And you say, well, why is that true? Because suppose I connected up a rectifier like this with drawing the square wave current.

First of all, this would be horrible because it has a DC value, which a lot of transformers doesn't let you do. But suppose I put that aside. Within my 15-amp limit, I have a limited amount of power I can draw with this rectifier just because of the harmonic currents. The fundamental of this pink waveform is actually in phase with the voltage. So I'm not hurting myself by having phase shifts like I did with an inductor or capacitor.

But I am hurting myself or limiting the amount of power I can draw by the fact that there's a certain amount of nonfundamental component in the current waveform. Well, if I got rid of the DC component, for example, suddenly my RMS would go down and my power transfer would stay the same. And I'd be a happier camper. That might be one thing I could do, for example.

But these kind of calculations can tell you, how are you-- what should you do to do better utilization of your source? Whereas the best thing you can do is look like a resistor. Any questions about any of this so far? Yeah.

**AUDIENCE:** You made a couple of comments about utilities caring a lot about this. It seems like all of the stuff that this involves is on the user's end. Can the utility do anything about this? Or is it just they wish we could do better?

**DAVID PERREAULT:** Yeah, that's a very, very good question. The answer is if you are an industrial user of power, if you're running a big factory that's drawing lots of electricity for whatever your industrial process is, yes, the utility can come and say, you've got to clean this up. Or what they really do is they start charging if your power factor is not good enough. And then you go clean it up because you don't want to pay them. So that's one way it's limited.

But you're absolutely right. There are some perverse incentives. So increasingly, for different kinds of loads, requirements are being placed on the quality of the waveforms drawn. And that's a very complex thing. In some cases, they actually quote power factor. In some cases, they quote other measures. Like, they assume that your phase angle is going to be happy, but how much distortion do you put in?

And that becomes-- if you want your Energy Star rating, for example, on your appliance, then suddenly you've got to meet that. Or if you want to sell a computer power supply, there are certain standards you have to meet. But those are a little bit haphazard because people were doing all kinds of nasty stuff to the grid back in the day and nobody paid attention. And now they're trying to clean it up after the fact.

So it's mostly if you're a big industrial user of energy that they're really regulating your power factor. But increasingly with other kinds of loads, especially as new kinds of power supplies and things come up, there are more and more strict regulations. But excellent question. Other questions or thoughts?

OK, I should point out one thing about this breakdown. This whole breakdown into distortion factor and displacement factor worked because I assumed that the voltage was a sinusoid and that the current might have harmonics. I could have done the same kind of breakdown if the current was a sinusoid and the voltage had harmonics. In that case, it would just be  $V_1$  RMS over  $V_{RMS}$  and then cosine of the phase shift.

If you have both the voltage is not sinusoidal and the current is not sinusoidal, you can't break things down this way. You can, however, still calculate power factor. And it turns out that the way you get a power factor of 1, which is the best you can do, is you make your current waveform look exactly like your voltage waveform. It has the same harmonic content and the same phase shift at every frequency. And that's the best you can do to transfer energy. And you can think about it as transfer the least-- the most energy per unit loss back in the source somewhere is the way you might think about that.

Any other questions before we show you some of these things? Yeah.

**AUDIENCE:** Is there any intuitive way to understand where the parent power is going from harmonic components? Like, what is actually causing that distortion?

**DAVID PERREAULT:** Well, what's causing the distortion has something to do with the circuit. So this is all posited on I have a voltage waveform and I have a current waveform. And suppose the current is distorted. So what's causing all that has to do with the circuit? The way you might think about the impact is very much in thinking about how does the harmonics content-- this comes back to the fact that sinusoids at different frequencies are orthogonal.

And so you degrade your energy transport because of that orthogonality. And sine and cosine at a given frequency is orthogonal, and that causes this degradation. So that's the way I would look at it. In terms of what's causing that or how do you fix that, that's more of a circuit thing or a system design issue.

OK, let's do a demo. [INAUDIBLE] has very kindly set this up. And what this is, this is a piece of equipment that's designed to look at line waveforms and measure power factor and other things of signals. So basically, things that get plugged into this power strip, this box analyzes the voltage and the current waveform and then computes things like power factor for you.

And I don't know what this particular Yokogawa power analyzer cost. I think it was under \$10k. So for you, if you want one at home, you can go buy one. Actually, you can get one cheaper these days. But that's about what this cost. So what you can see here is she has the voltage waveform running. It's 110 volts, but there's no current because she has nothing plugged in.

Now, [INAUDIBLE] is going to plug in a traditional incandescent light bulb. Here we go. Lambda 4 up there, for whatever reason, is the symbol they're using for power factor. But what you can see is the current, which is in purple, is basically sinusoidal and in phase with our nice voltage. And so we get a power factor of 1. And you can see she does a nice job of lighting up this kind of very Edisonian bulb. So thanks.

And one thing I didn't point out, that was about 50 watts there. So that kind of makes sense for that kind of brightness. What [INAUDIBLE] is going to plug in right now is a fluorescent light bulb. And there we're actually getting a lot brighter light here. This is brighter than the incandescent light bulb. Notice that we're only drawing about 12 watts. So from an energy consumption, just purely how much power we're drawing from the grid, this is a way better deal, even though the light's kind of not as nice.

And if we did an LED bulb, it would be even better. But look at the current waveform. That current waveform, the purple, is absolutely horrible. It's massively dominated by its harmonics. You can visually see that, yeah, it has a fundamental component, but it's pretty bad, right?

The reason, this has a diode rectifier in it at the front end. But instead of running it with an inductive filter, they used a capacitive filter. So you get this kind of big pulse of current and then nothing. And then that internal capacitor discharges. So this leads to terrible power factor. And you can see that the power factor is only about 0.53.

So what that says is if you use lots of these bulbs, you're kind of utilizing up the capability of your source to deliver power. What that means is while we're not measuring it here, you're dissipating more power back into the wall wiring than you need to. And in fact, if you went and bought one of the really new LED-- one of the really new LED power supplies, they might have a much better power factor because they're being incentivized if you want the Energy Star stamp to do that.

Last example we'll show you is one that doesn't have a power converter. This is a fan. So this is a typical motor load. And here you see the current is reasonably sinusoidal, but there's a phase shift between the voltage and the current. The current lags the voltage why? Because it has to magnetize the motor, and the motor doesn't look like a perfect resistive load.

So we get energy transfer, and there's a little bit of distortion owing to the machine, to the electric machine. But nonetheless, it's pretty close to sinusoidal. But we only get about 0.7 power factor because of this lag. So people are asking about, well, what about if you're a big industrial user? A lot of big industrial companies have lots of motor loads.

They're using the pumps and whatever have you they're driving. And that tends to generate inductively loaded power factor. So this isn't so much the distortion factor. This is displacement factor. The interesting thing, you can fix that one. If I put some capacitor in parallel with the line, I could source. I could kind of cancel out the inductive element and clean up my power factor and make it very close to 1.

So if your problem is phase shift, you can fix it by adding some reactants to the line or susceptance to the line. If your problem is because you've got a crummy power converter distorting things, that's much harder to fix, and you've got to fix your power converter typically, although there are other tricks as well.

So that's about it for today on power factor. Are there any final questions before we wrap up? Yeah.

**AUDIENCE:** Is the power supply coming from the outlet?

**DAVID** I'm sorry?

**PERREAULT:**

**AUDIENCE:** Is the power supply coming from the outlet?

**DAVID** Oh, is the power that she was showing coming from the outlet? Yes, it was.

**PERREAULT:**

**AUDIENCE:** Is it commonplace to not be beyond 20 volts?

**DAVID** Ah, so that's a very good question. In this particular case, we were running this through a variac, which lets us modulate the voltage a little bit. But the answer is, more broadly, typical range of voltage is minus 15% to plus 10% of the nominal.

And in fact, if you're going to make a laptop power supply, you've got to make it go down way low because the line can be all over the place. The thing is, that's pretty sinusoidal actually. You'll often see it often clips at the top with very old-fashioned peak clipping loads. So you can get a lot of distortion on your voltage too, even though this is pretty sinusoidal. So yeah, it's not as accurate as you might wish it to be. All right, have a great day.