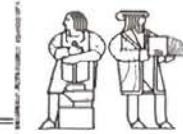


MASSACHUSETTS INSTITUTE OF TECHNOLOGY



DEPARTMENT OF ELECTRICAL ENGINEERING AND COMPUTER SCIENCE

6.622 Power Electronics Assessment #10

Due: Thursday May 11, 2023 at 11:00 pm (Cambridge time)

YOUR NAME

Solutions

YOUR KERBEROS ID

General Instructions:

1. You must complete this assessment on your own with no consultation or discussion with any other person, excepting 6.622 staff, of whom you may ask clarifying questions. Do not discuss your solutions with anyone until the solutions have been released.
2. You may use a calculator and review the course lectures, handouts, notes, textbook (Principles of Power Electronics) and other materials provided for the course on Canvas when completing this assessment. Please do not use other computational tools or reference materials.
3. Please do all of your work in the space provided. In particular, try to do your work for each question within the boundaries of the question, or on the additional pages at the end of the uploaded document, clearly marking those pages to indicate what problem they relate to. Place the answer to each question within the appropriate answer box.
4. The assessment must be completed and uploaded by the indicated date/time to receive credit.
5. Please make sure to show all of your work. This is important both for you to receive credit for a correct answer and to receive partial credit when an answer is wrong or incomplete.

Name: _____

Problem 1

Figure 1 shows an input filter for a switching power converter. The current drawn by the converter is represented as i_x , and the voltage supplying the converter is v_Y . The converter switching frequency $1/T$ is 1 MHz. You may assume the filter components are ideal.

- a. Select component values L , C , and R for the filter such that
 - i. The maximum output impedance of the filter, Z_O , is 1Ω or less at all frequencies
 - ii. The filter achieves an attenuation of approximately 40 dB (a factor of 100) in current at the switching frequency. That is, $|i_Y / i_X| \approx 0.01$ at the switching frequency.
 - iii. The filter is well damped, such that it has less than 10 dB of peaking in $|i_Y / i_X|$ near the undamped natural frequency of the filter.
- b. Suppose that the load on the filter (slowly) adjusts the local average current $\langle i_X \rangle$ drawn based on the local average voltage $\langle v_X \rangle$ to maintain a constant average power draw P_O . Please find a (low-frequency) equivalent small-signal resistance r_E for this load for the operating point $P_O = 10 \text{ W}$ and $V_X = 20 \text{ V}$. Will this load greatly affect the filter damping? Justify your answer.

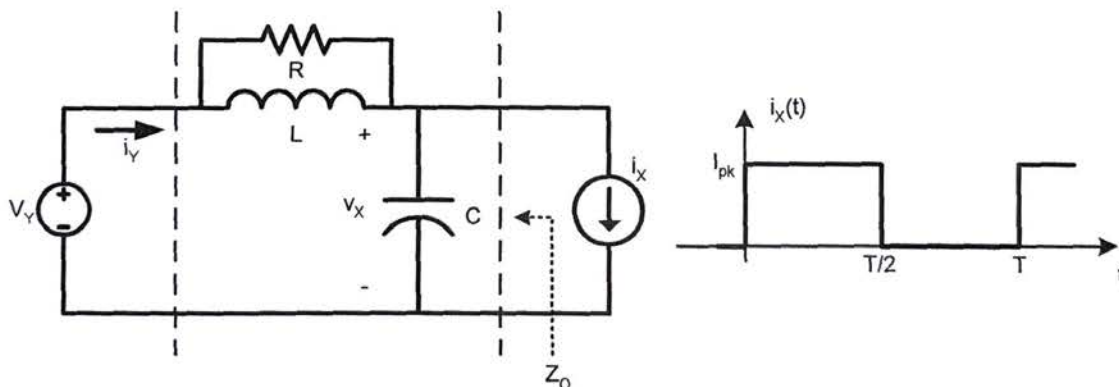


Figure 1

First, some calculations:

$$H_i(s) = \frac{I_y}{I_x} = \frac{\frac{1}{R} + \frac{1}{sL}}{\frac{1}{R} + \frac{1}{sL} + sC}$$

$$H_i(s) = \frac{s + R/L}{s^2 RC + s + R/L}$$

Pole locations are at

$$s^2 RC + s + R/L = 0$$

$$\Rightarrow s = -\frac{1}{2RC} \pm j \sqrt{\frac{1}{LC} - \left(\frac{1}{2RC}\right)^2}$$

Defining characteristic impedance

$$Z_n = \sqrt{\frac{L}{C}}$$

undamped natural frequency

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

network quality factor

$$Q_0 = \frac{R}{Z_n}$$

we set: $RC = Q_0/\omega_0$

$$R/L = \omega_0 Q_0$$

$$R = Q_0 Z_n$$

$$s = -\frac{\omega_0}{2Q_0} \pm j \sqrt{\omega_0^2 - \left(\frac{\omega_0}{2Q_0}\right)^2}$$

damped nat. freq. ω_d

$$\text{or } \omega_d = \omega_0 \sqrt{1 - \left(\frac{1}{2Q_0}\right)^2}$$

$H_i(j\omega)$ will peak (if it does) near $\omega = \omega_d$ which is close to ω_0 for $Q_0 \uparrow$

We can also calculate:

$$|H_i(j\omega)| = \frac{\sqrt{\omega^2 + (R/L)^2}}{\sqrt{\omega^2 + (R/L - \omega^2 RC)^2}}$$

which may be rewritten as

$$|H_i(j\omega)| = \sqrt{\frac{\omega^2 + (\omega_0 Q_0)^2}{\omega^2 + (\omega_0 Q_0)^2 (1 - (\frac{\omega}{\omega_0})^2)^2}}$$

$$\text{Note that } |H_i(j\omega_0)| = \sqrt{1 + Q_0^2}$$

filter output impedance

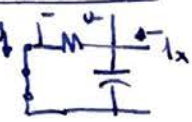
$$Z_{out} = \frac{1}{sC + \frac{1}{R} + \frac{1}{sL}} = R \cdot \frac{s}{RCs^2 + s + R/L}$$

$$|Z_{out}(j\omega)| = R \sqrt{\frac{\omega^2}{\omega^2 + \left(\frac{R}{L} - \omega^2 RC\right)^2}}$$

$$L \left\{ \begin{array}{c} \downarrow \\ R \\ \uparrow \\ C \end{array} \right\} \leftarrow Z_{out} \quad \left[\begin{array}{c} \max |Z_{out}| = R \\ @ \omega = \omega_0 \end{array} \right]$$

We can also express

$$|Z_{out}(j\omega)| = Q_0 Z_n \sqrt{\frac{\omega^2}{\omega^2 + \left(\omega_0 Q_0 - \left(\frac{\omega_0}{\omega}\right)^2\right)^2}}$$

@ $\omega \gg \omega_0$ $H_i \approx \frac{1/R}{\frac{1}{R} + sC} = \frac{I_y}{I_x}$ 

(neglect current through inductor)

$$H_i(j\omega) \approx \frac{1}{1 + j\omega RC} \approx \frac{1}{j\omega RC}$$

$$|H_i(j\omega)| \approx \frac{1}{\sqrt{1 + (\omega RC)^2}} \approx \frac{1}{\omega RC}$$

Filter Design: $\omega_{sw} = 2\pi \times 10^6$ rad/sec

- (i) to achieve $z_{out,max} = 1\Omega$ we set $R = 1\Omega$
 (ii) To get an attenuation of 40db (0.01) in $H_i(\omega_{sw})$:

$$|H_i(j\omega_{sw})| \approx \frac{1}{\omega_{sw}RC} \Rightarrow RC \approx \frac{100}{\omega_{sw}}$$

this gives $C = \frac{100}{R\omega_{sw}} \approx 15.91\mu\text{F}$ (idealized)

- (iii) If we wanted $|H_i(j\omega)| = 1.414 (\sqrt{2})$ or 3dB

$$\sqrt{1+\omega_0^2} = \sqrt{2} \Rightarrow Q_0 = 1.$$

However, filter gain peak is actually @ $\omega = \omega_d$ and filter gain will be larger than @ $\omega = \omega_0$.

For margin, set $Q_0 = 1/2$ (critically-damped).

[detailed analysis shows $Q < 0.92$ to get

$$|H_i(j\omega)|_{max} < \sqrt{2}.]$$

$$z_n = \sqrt{\frac{L}{C}} = \frac{R}{Q_0} \Rightarrow L = \left(\frac{R}{Q_0}\right)^2 C$$

$$L = 4C = 63.7\mu\text{H} \text{ (idealized)}$$

For these choices we get:

$$z_n = \sqrt{\frac{L}{C}} = 2$$

$$R = 1\Omega$$

$$Q_0 = \frac{R}{z_n} = \frac{1}{2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = 3.14 \times 10^4 \frac{\text{rad}}{\text{sec}} = \frac{1}{200} \omega_{sw}$$

(It can be shown that

$$\omega_0 \approx \frac{Q_0 \cdot \omega_{sw}}{100}$$

to get desired attenuation)

(a) Specify filter L, C, R

Idealized design for $Q_0 = 1/2$ and $z_{out,max} = 1\Omega$ gives:

$R = 1\Omega$, $L = 63.7\mu\text{H}$, $C = 15.91\mu\text{F}$. In reality we would want design margin, accounting for parasitics, etc.

Name: Solutions

$$\langle v_x \rangle = \frac{P_0}{\langle i_x \rangle}$$

$$P_0 = 10\text{W}, V_x = 20\text{V} \Rightarrow I_x = 0.5\text{A}$$

$$\frac{\partial \langle v_x \rangle}{\partial \langle i_x \rangle} \bigg|_{V_x, I_x} = - \frac{P_0}{I_x^2} = - \frac{10}{0.25} = -40 \Rightarrow r_e = -40\Omega$$

(b)

$$r_E = -40\Omega$$

Will the power draw characteristic of the converter significantly affect filter damping?
Justify your answer.

Since the maximum filter output impedance is $R = 1\Omega$
the equivalent resistance for damping is $R \parallel r_e = \frac{1 \cdot (-40)}{1 - 40} = 1.02\Omega$

So filter damping is NOT significantly changed.

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