6.6220: Power Electronics *Prof. David Perreault*

Lecture 3 — Load Regulation

Scribe: MIT Student

1 Review

1.1 Method of Assumed States

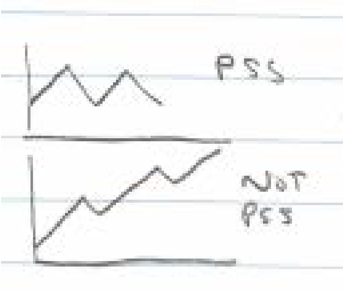
- Assume a state for all switches
- Calculate voltages and currents in the system
- See if any switch conditions are violated
- If not, done. If so, assume a different set of states and try again

1.2 Periodic Steady State

- Converter waveforms repeat every cycle
- In P.S.S.

$$< I_C > = < C\frac{dV_C}{dt} > = 0 \tag{1}$$

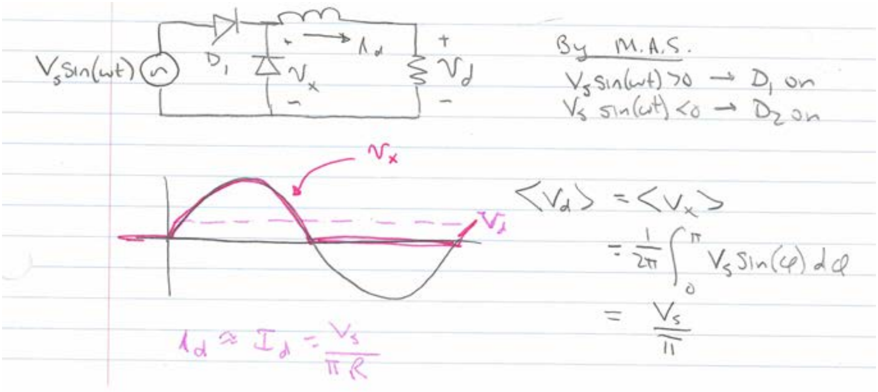
$$< V_L > = < L\frac{dI_C}{dt} > = 0 \tag{2}$$



2 Rectifiers and DC-Side Characteristics

⇒ Have as show and tell: Alternator, Isolated DC/DC conv.

Last time we introduced half a wave rectifier.



By M.A.S.

$V_s \sin(wt) > 0 \rightarrow D_1$ on

$V_s \sin(wt) < 0 \rightarrow D_2$ on

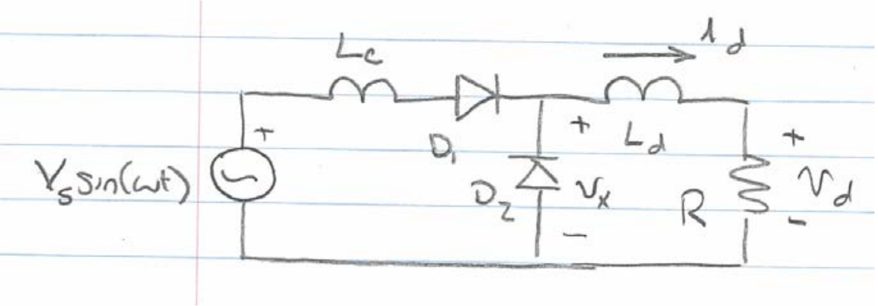
$$\langle V_d \rangle = \langle V_x \rangle = \frac{1}{2\pi}\int_0^{\pi} V_s \sin(\varphi)\, d\varphi = \frac{V_s}{\pi}$$

$$i_d \approx I_d = \frac{V_s}{\pi R}$$

Now consider adding some AC-side inductance L_C (reactance $X_C \triangleq wL_C$)

This is a relevant issue. L_C can be . . .

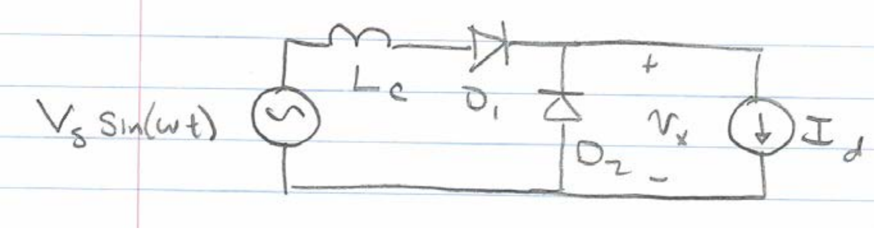
1. Transformer Leakage Inductance (AC Power or DC-DC)
2. Generator Machine Leakage
3. Line Inductance



Typically, $L_d >> L_C$ (filter much bigger than leakage). For simplicity, assume $L_D \to \infty$ so load looks a current-source.
It is a "special" current source since $< V_L >= 0$ in P.S.S.

$$\therefore I_d =< \frac{V_x}{R} > \tag{3}$$

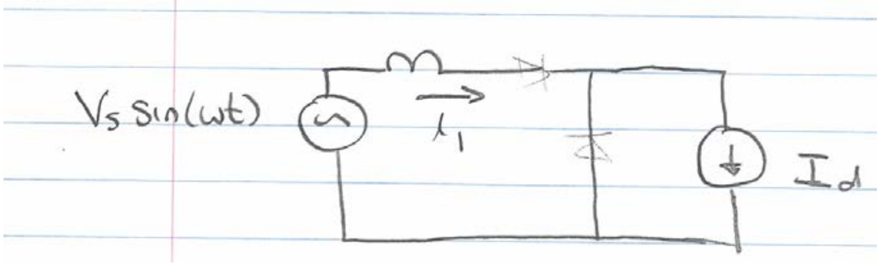
2.1 Simplified Model



Assume we start with D_2 conducting, D_1 off, $V_s \sin \omega t < 0$. What happens when $V_s \sin \omega t$ crosses zero?

- D_1 off is not longer valid ($V_d > 0$) $\therefore$D$_1$ turns on.
- After turn on I_1 still is zero (no instant I change in L_C) $\therefore$D$_2$ remains on.

During **commutation**, both devices are on. (NB: commutation means to switch or travel between points.)



$$L_C\frac{dI_1}{dt} = V_s\sin(\omega t) \Rightarrow I_1 = \frac{V_S}{\omega L_C}\int_0^{\omega t}\sin(\omega t)\,d(\omega t) \tag{4}$$

$$I_1 = \frac{V_S}{\omega L_C}[\cos(0) - \cos(\omega t)] = \frac{V_S}{\omega L_C}[1 - \cos(\omega t)] \tag{5}$$

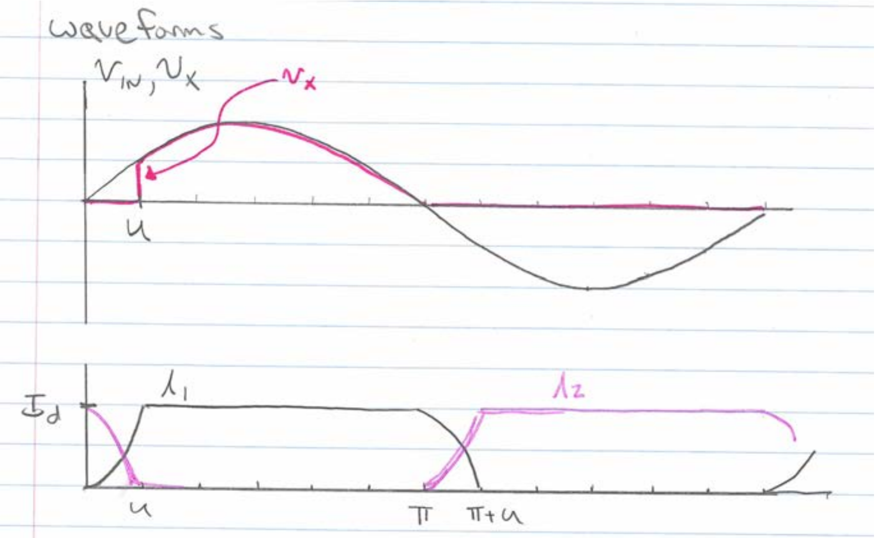
This is valid until some electrical angle $\omega t = u$, when the current through D_2 reaches zero ($I_1 = I_d$)

$$I_d = \frac{V_s}{\omega L_C}[1 - cos(u)] \tag{6}$$

Commutation interval u such that

$$\cos u = 1 - \frac{\omega L_C I_d}{V_S}$$

The dimensionless term $\frac{\omega L_C I_d}{V_S} = \frac{X_C I_d}{V_S}$ comes up a lot in rectifiers and is called **Reactance Factor**



We lose a piece of V_X (and average output voltage due to commutation (during which V_X is zero). This in turn causes a reduction in filtered output voltage.

$$V_O =< V_x >= \frac{1}{2\pi}\int_u^{\pi} V_s sin(\varphi)d(\varphi)$$

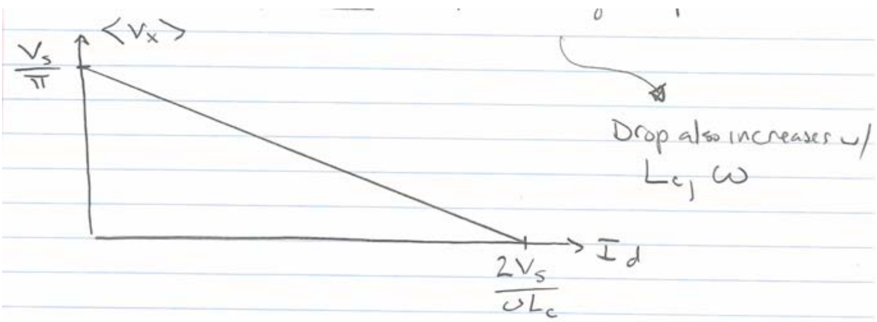
$$= \frac{V_s}{2\pi}[cos(u) - cos(\pi)]$$

Plugging in for $\cos u$ from our previous calculation.

$$V_O =< V_x >= \frac{V_s}{2\pi}\left[2 - \frac{\omega L_c I_d}{V_s}\right]$$

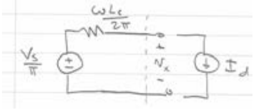
$$V_O = \frac{V_s}{\pi}\left[1 - \frac{1}{2}\frac{\omega L_{cId}}{V_s}\right]$$

From this we can see:
As load current increases output voltage drops!



At open ckt, we get the same $< V_0 >$ as with no L_C. As the current increases, V_0 drops. This is known as **load regulation**, and is typically disliked (we often want constant output voltage independent of load.)

We could model this (in an average sense) as



Note that this is **only** valid in terms of average V-I characteristics. No real dissipation occurs!

Notes: Load regulation is important in many applications.

1. It dominates the performance of automotive alternators
2. It impacts the behavior of DC/DC converters
3. It determines how circuits behave under short- circuit conditions

3 Main Points

1. AC-side reactance in rectifier circuits introduces a **commutation interval** during which multiple devices are on so current can switch between them. This period is u long. For HW rectifier ...

$$\cos u = 1 - \frac{\mathrm{X}_C I_d}{V_S}$$

2. The commutation period causes the output voltage to be held low during commutation. The longer the commutation, the lower the output voltage.

⇒ This introduces load regulation of the output.

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