6.334 Lecture Notes Load Regulation

Review

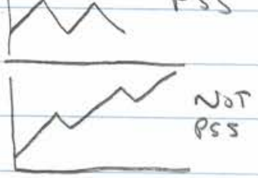
METH. ASSUMED STATES

1. Assume a state (on/off) for all switches
2. Calculate voltages/currents in system
3. Check if any switch condition violated
4. If not → ok

   If so, assume a different set of states

PERIODIC STEADY STATE:

Converter waveforms repeat every cycle



In P.S.S. $$\langle i_c \rangle \left\{ = C \left\langle \frac{dv_c}{dt} \right\rangle \right\} = 0$$

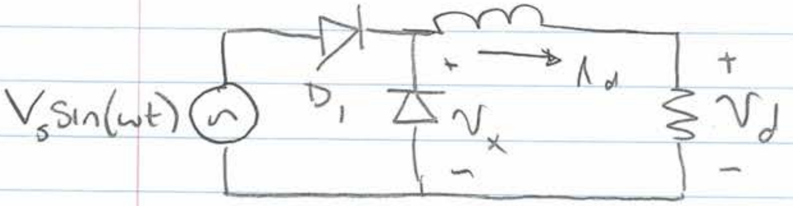
$$\langle v_L \rangle \left\{ = L \left\langle \frac{di_L}{dt} \right\rangle \right\} = 0$$

Today: Rectifiers + dc-side characteristics.

⇒ Have as show + tell: Alternator, isolated dc/dc conv.

Not usually used for line or generator appl., but often appears in dc/dc conv.

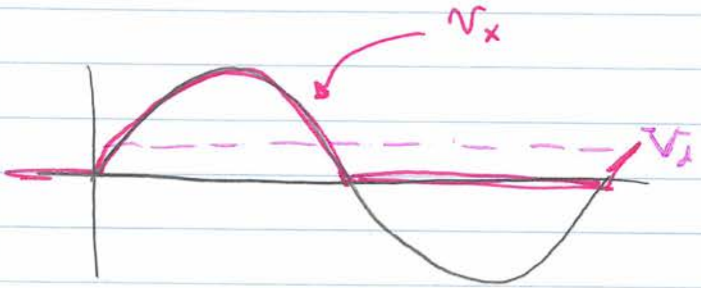
LAST time we introduced a half-wave rectifier



By M.A.S.

$V_s \sin(\omega t) > 0 \rightarrow D_1$ on

$V_s \sin(\omega t) < 0 \rightarrow D_2$ on



$$\langle v_d \rangle = \langle v_x \rangle = \frac{1}{2\pi}\int_0^{\pi} V_s \sin(\varphi)\, d\varphi = \frac{V_s}{\pi}$$

$$i_d \approx I_d = \frac{V_s}{\pi R}$$

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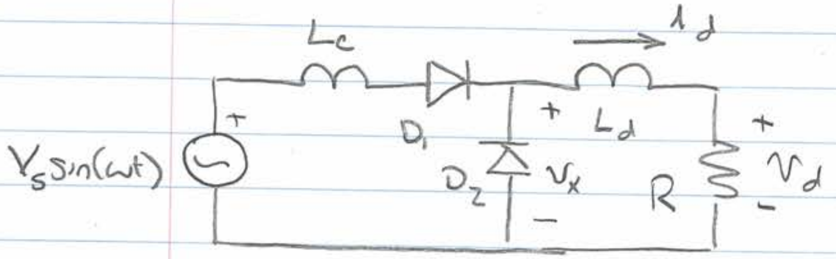
Now consider adding some ac-side inductance L_c (reactance $X_c \triangleq \omega L_c$)

This is a relevant issue. Lc can be
1. transformer leakage inductance (ac power or dc-dc)
2. generator machine leakage
3. line inductance.

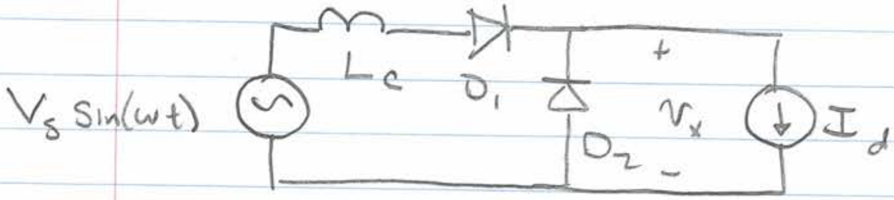


Typically $L_d >> L_c$ (filter much bigger than leakage)
⇒ for simplicity, assume $L_d \to \infty$ so load looks like a current-source.

It is a "special" current source since $\langle V_L \rangle = 0$ in P.S.S.

$$\therefore \quad I_d = \langle V_x / R \rangle$$

Simplified model



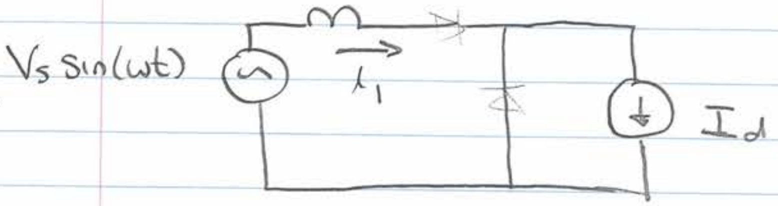
Visual presentation:
Draw voltage cycle on board & then fill in as we calculate.

Assume we start with D_2 conducting, D_1 off, $V_s \sin(\omega t) < 0$, what happens when $V_s \sin(\omega t)$ crosses zero?

→ D_1 off is no longer valid ($V_d > 0$) ∴ D_1 turns on

→ after turn on λ_1 still $= 0$ (no instant λ change in L_C) ∴ D_2 remains on

During <u>Commutation</u>, both devices are on ← (NB: commutation means to switch or trave between points)



$$L_C \frac{d\lambda_1}{dt} = V_s \sin(\omega t) \Rightarrow \lambda_1 = \frac{V_s}{\omega L_C} \int_0^{\omega t} \sin(\omega t)\, d(\omega t)$$

$$\lambda_1 = \frac{V_s}{\omega L_c}\left[\cos(0) - \cos(\omega t)\right] = \frac{V_s}{\omega L_c}\left[1 - \cos(\omega t)\right]$$

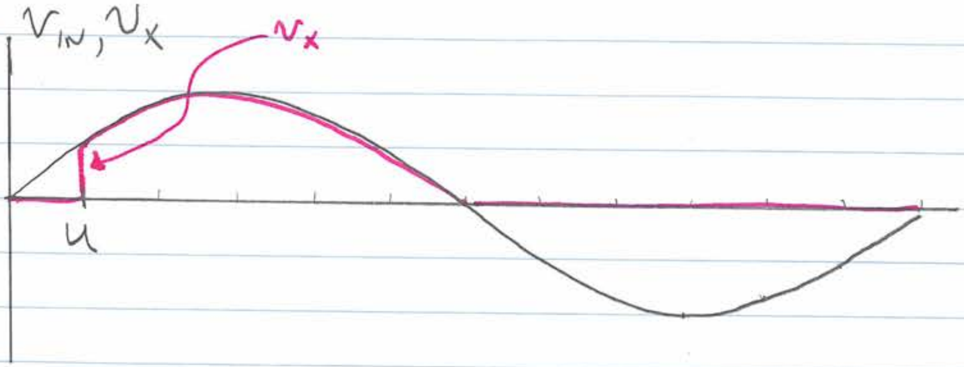
this is valid until some electrical angle $\omega t = u$, when the current through D_2 reaches zero ($\lambda_1 = I_d$)

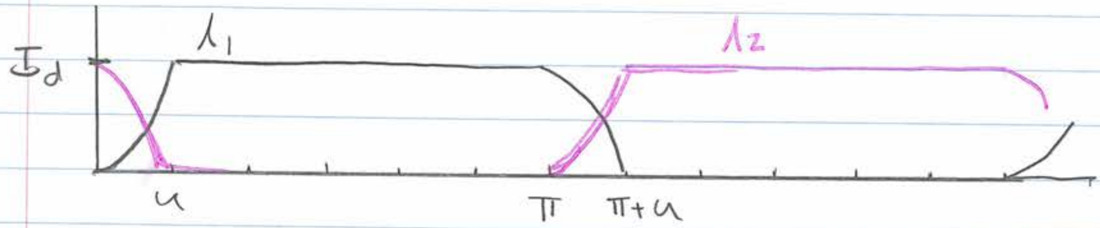
$$I_d = \frac{V_s}{\omega L_C}\left[1 - \cos(u)\right]$$

$$\boxed{\text{Commutation interval } u \text{ such that} \quad \cos u = 1 - \frac{\omega L_c I_d}{V_s}}$$

The dimensionless term $\frac{\omega L_c I_d}{V_s} = \frac{X_c I_d}{V_s}$ comes up a lot in rectifiers + is called <u>Reactance Factor</u>

waveforms





we lose a piece of v_X (and average output voltage) due to commutation (during which v_X is zero.) This in turn causes a reduction in filtered output Voltage

$$v_o = \langle v_X \rangle = \frac{1}{2\pi}\int_u^{\pi} V_s \sin(\varphi)\,d\varphi$$

$$= \frac{V_s}{2\pi}\left[\cos(u) - \cos(\pi)\right]$$

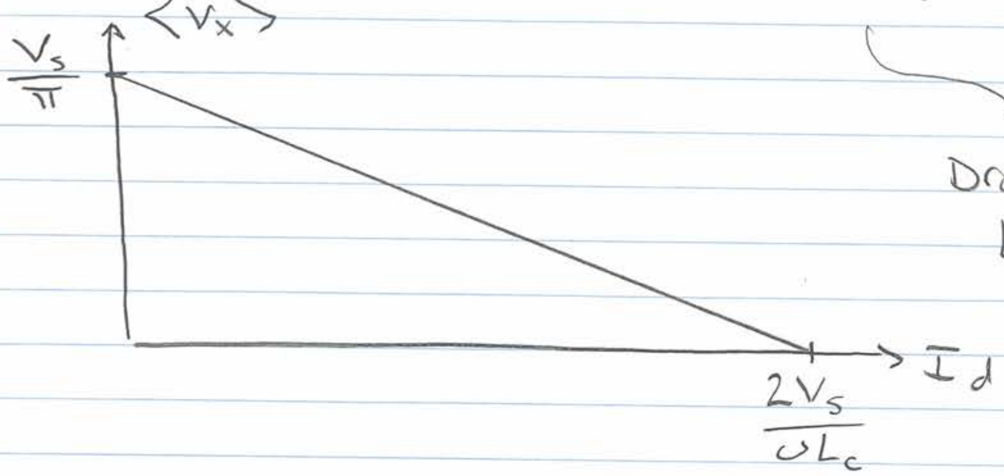
Plugging in for $\cos(u)$ from our previous calculation

$$v_o = \langle v_X \rangle = \frac{V_s}{2\pi}\left[2 - \frac{\omega L_c I_d}{V_s}\right]$$

$$★ \quad \boxed{v_o = \frac{V_s}{\pi}\left[1 - \frac{1}{2}\,\frac{\omega L_c I_d}{V_s}\right]}$$

From this we can see:

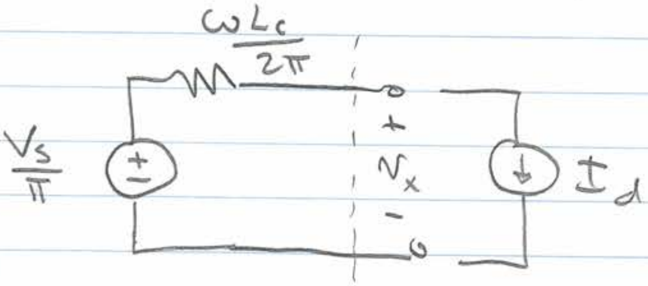
as load current increases output voltage drops!



Drop also increases w/ L_c, ω

@ open ckt we get same $\langle v_o \rangle$ as with no L_c. As current increases, v_o drops. This is known as <u>LOAD REGULATION</u>, and is typically disliked (we often want constant output voltage independent of load.)

we could model this (in an Average sense) as



Note that this is <u>only</u> valid in terms of average V-I characteristics. No Real dissipation occurs!

Note: Load regulation is important in many applications.

1. It dominates the performance of automotive alternators
2. It impacts the behavior of dc/dc converters
3. It determines how circuits behave under short-circuit conditions

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MAIN POINTS :

1. AC-side reactance in rectifier circuits introduces a commutation interval during which multiple devices are on so current can switch between them. This period is u long. For HW rectifier

$$\cos u = 1 - \frac{X_c I_d}{V_s}$$

2. The commutation period causes the output voltage to be held low during commutation. The longer the commutation, the lower the output voltage.

$\Rightarrow$ This introduces load regulation of the output.

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