6.6220: Power Electronics *Prof. David Perreault*

Lecture 4 — Power Factor

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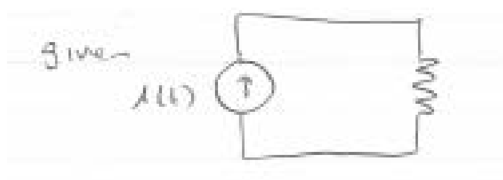
1 Definitions and Identities

1. The RMS (root mean square) of a waveform):

$$X_{RMS} \triangleq \sqrt{\frac{1}{T}\int_{<T>} X^2(t)\,dt]}$$

This is useful:



$$p(t) = I(t)^2 R$$
$$< P >= \frac{1}{T}\int I(t)^2 R\,dt = I_{RMS}^2 R$$

The RMS is the DC (current or voltage) that gives the same average power dissipation in a resistor as the time-varying waveform.

2. Two waveforms are orthogonal on $[a, b]$ iff

$$\int_a^b x(t)y(t)\,dt = 0$$

Now, as sinusoidal with different frequencies are orthogonal ...

$$\int_0^{2\pi} \sin(m\omega t) sin(n\omega t + \theta) d(\omega t) = 0 \text{ if } n \neq m$$

And as sin and cos are orthogonal ...

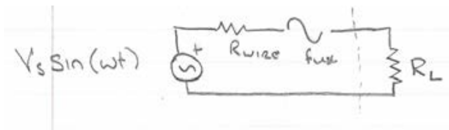
$$\int_0^{2\pi} \sin(\omega t)\cos(\omega t) = 0$$

In general ...

$$\boxed{\frac{1}{2\pi}\int_0^{2\pi} sin(\omega t) sin(\omega t + \phi)\,d\omega t = \frac{1}{2}\cos\phi}$$

2 Background

Suppose we plug a resistor into the wall



Neglectance R_{wire}

$$p =< V(t)I(t) >$$
$$=< I(t)^2 R_L >$$
$$= I_{RMS}^2 R_L$$

At 115 V_{RMS}, 15 A fuse, we can draw $p \simeq 1.7kW$ from wall (negatively R_{wire})

Suppose instead we plug an **inductor** into the wall

Neglectance R_{wire}

$$I = \int \frac{V}{L}\, dt = \frac{V_s}{\omega L} \cos \omega t$$

So

$$< P >= -\frac{1}{2\pi} \frac{V_s^2}{\omega L} \int_0^{2\pi} \sin{(\omega t)} \cos{(\omega t)}\, d(\omega t) = 0$$

Because sine and cosine are orthogonal.

This makes sense, inductors store energy not dissipate it. (We slosh energy back and forth with the source.)

However, while no **average** power is drawn, there is instantaneous power (slosh energy into and out of wall) and RMS current drawn.

$$I_{RMS} < \frac{V_S}{\sqrt{2}\omega L}$$

At 115 V_{RMS} and $60Hz$

$$L < 20mH$$
$$I_{RMS} > 15A$$

While we don't dissipate energy in the load, we **still** dissipate energy in R_{wire} ($I_{RMS} > 0$), and can blow the fuse.

In this case, we are still not using the power capability of the source well.

To provide a measure of the utilization of the source, we define **Power Factor**:

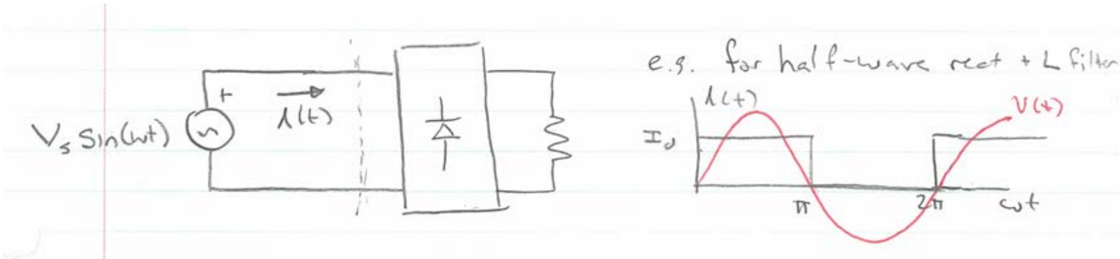
$$P.F. \triangleq \frac{< P >}{V_{RMS} I_{RMS}}$$

Where begin $< P >$ represents the real power and $V_{RMS} I_{RMS}$ represents the apparent power.

For a resistor, $< P >= V_{RMS} I_{RMS} \rightarrow P.F. = 1$ and an inductor $< P >= 0 \rightarrow P.F. = 0$. Note that **we define power factor at a port**.

3 Rectifiers

Consider a rectifier drawing some current waveform.



In P.S.S., we can express $I(t)$ as a Fourier Series.

$$I(t) < \sum_{n=0}^{\infty} I_n \sin(\, n\omega t + \phi_n \,)$$

$$\text{where } \phi_0 = 90^\circ$$

For this, it is easy to show:

$$I_{RMS} = \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2} + \ldots + \frac{I_n^2}{2} + \ldots}$$

We can calculate power as:

$$< P >= \frac{1}{2\pi} \int_0^{2\pi} V_s \sin \omega t \sum_{n=0}^{\infty} I_n \sin(\, n\omega t + \phi_n \,) d(\omega t)$$

$$= \sum_{n=0}^{\infty} V_s I_n \cdot \frac{1}{2\pi} \int_0^{2\pi} \sin{(\omega t)} \sin{(\omega t + \phi_n)}\, d(\omega t)$$

From our orthogonality theorem, all but $n = 1$ term $\rightarrow 0$.

$$< P >= V_s I_1 \cdot \frac{1}{2\pi} \int_0^{2\pi} \sin{(\omega t)} \sin{(\omega t + \phi_1)}\, d(\omega t)$$

From identity …

$$< P >= \frac{1}{2} V_s I_1 \cos \phi_1$$

or

$$< P >= V_{RMS} I_{1,RMS} \cos \phi_1$$

So the only current that contributes to real power (average power) is the portion of the **fundamental current in phase with the voltage**!

Now:

$$P.F. = \frac{< P >}{V_{RMS} I_{RMS}}$$

$$= \frac{V_{RMS} I_{1,RMS} \cos\phi_1}{V_{RMS} I_{RMS}}$$
$$= \frac{I_{1,RMS} \cos\phi_1}{I_{RMS}}$$

Therefore,

$$P.F. = \frac{I_{1,RMS}}{I_{RMS}} \cos\phi_1 = k_d k_\theta$$

where $k_d = \frac{I_{1,RMS}}{I_{RMS}}$ is the distortion factor and $k_\theta = \cos\phi_1$ is the displacement factor.

- k_d **Distortion factor** ($0 \leq k_d \leq 1$) tells us how much utilization is reduced because of harmonic currents that don't contribute real power.
- k_θ **Displacement factor** ($|k_\theta| \leq 1$) tells us how much utilization of the source is reduced because of phase shift between voltage and fundamental current.

Note:

1. Point out that since voltage is sinusoidal, due to orthogonality we only factor x for power from the fundamental current, and only from the component in phase with the voltage.
2. Breakdown and interpretation become more complex if both V and I are nonsinusodial. Less usual case but we can still define P.F.

If time, introduce TDH (Total Harmonic Distortion) $\quad \text{THD} \triangleq \sqrt{\frac{\sum_{n\neq 1} I_n^2}{I_1^2}}$

RMS of harmonic content normalized to RMS of fundamental. This assumes no DC content.