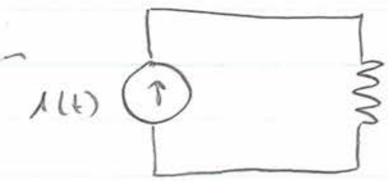
6.334 Lecture Notes Power Factor

DEFINITIONS + IDENTITIES

1. The RMS (root-mean-square) of a waveform:

$$X_{rms} \triangleq \sqrt{\frac{1}{T}\int_{\langle T \rangle} X^2(t)\,dt}$$

This is useful: given



$$p(t) = i(t)^2 R$$

$$\langle P \rangle = \frac{1}{T}\int i(t)^2 R\,dt = i_{Rms}^2 R$$

The rms is the dc (current or voltage) that gives the same avg. power dissipation in a resistor as the time-varying waveform.

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2. Two waveforms are <u>orthogonal</u> on $[a, b]$ iff

$$\int_a^b x(t)y(t)\,dt = \emptyset$$

Now $\int_{\emptyset}^{2\pi} \sin(m\omega t)\sin(n\omega t + \Theta)\,d(\omega t) = \emptyset$ if $n \neq m$

$\Rightarrow$ sinusoids w/ different frequencies are orthogonal

$$\int_0^{2\pi} \sin(\omega t)\cos(\omega t) = \emptyset$$

$\Rightarrow$ sin + cos are orthogonal

<u>In general</u>

$$\boxed{\frac{1}{2\pi}\int_0^{2\pi} \sin(\omega t)\sin(\omega t + \phi)\,d(\omega t) = \frac{1}{2}\cos\phi}$$

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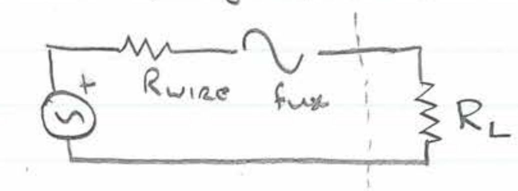
BACKGROUND:

Suppose we plug a resistor into the wall

$V_s \sin(\omega t)$



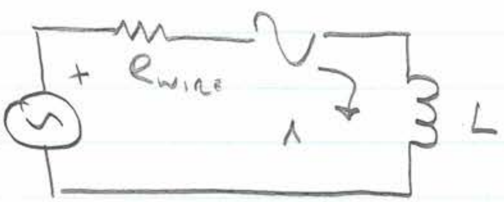
NEGLECTING R_{WIRE}:

$$P = \langle V(t)\, i(t) \rangle$$
$$= \langle i(t)^2 R_L \rangle$$
$$= i_{rms}^2 R_L$$

@ 115 V_{rms}, 15 A fuse, we can draw $P \simeq 1.7\,kW$ from wall (neglecting R_{WIRE})

Suppose instead we plug an <u>Inductor</u> into the wall

$V_s \sin(\omega t)$



NEGLECTING R_{WIRE}

$$i = \int \frac{V}{L}\, dt = -\frac{V_s}{\omega L} \cos(\omega t)$$

$$\text{so } \langle P \rangle = -\frac{1}{2\pi} \frac{V_s^2}{\omega L} \int_0^{2\pi} \sin(\omega t) \cos(\omega t)\, d(\omega t) = 0$$

BECAUSE SINE, COSINE ARE ORTHOGONAL.

$\Rightarrow$ THIS MAKES SENSE. INDUCTORS STORE ENERGY, NOT DISSIPATE IT. (we slosh energy back + forth with the source)

However, while no <u>Average</u> power is drawn, there is instantaneous power (slosh energy into + out of wall) and rms current draw

$$i_{rms} = \frac{V_s}{\sqrt{2}\,\omega L}$$

@ 115 V_{rms}, 60 Hz

$\Rightarrow L < 20\,mH \rightarrow i_{rms} > 15\,A$!

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while we don't dissipate energy in the load, we still dissipate energy in R_{wire} ($i_{rms} > 0$), and can blow the fuse.

$\Rightarrow$ In this case we are not using the power capability of the source well.

★ To provide a measure of the utilization of the source, we define POWER FACTOR:

$$\boxed{P.F. \triangleq \frac{\langle P \rangle}{V_{rms}\, i_{rms}}}$$

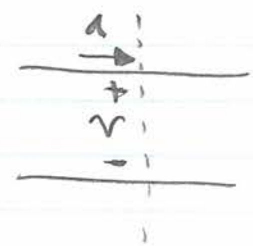
$\langle P \rangle$ ← Real power P

$V_{rms}\, i_{rms}$ ← Apparent power S

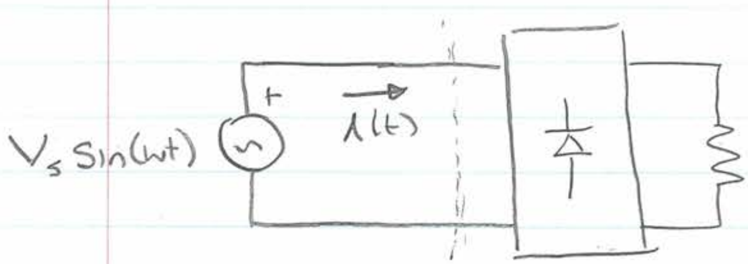
For a resistor, $\langle P \rangle = V_{rms}\, i_{rms} \rightarrow P.F. = 1$

an inductor, $\langle P \rangle = 0 \rightarrow P.F. = 0$

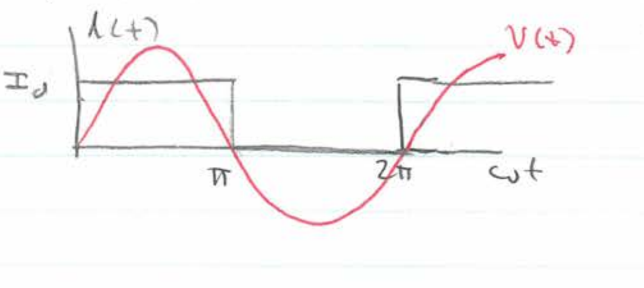
Note: we define power factor at a port:



★ Consider a rectifier drawing some current waveform



e.g. for half-wave rect + L filter



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In P.S.S. we can express $i(t)$ as a Fourier Series

$$i(t) = \sum_{n=0}^{\infty} I_n \sin(n\omega t + \phi_n) \qquad \text{where } \phi_0 = 90^\circ$$

for this it is easy to show:

$$i_{RMS} = \sqrt{I_0^2 + \frac{I_1^2}{2} + \frac{I_2^2}{2} + \dots + \frac{I_n^2}{2} + \dots}$$

we can calculate power as:

$$\langle P \rangle = \frac{1}{2\pi} \int_0^{2\pi} V_s \sin(\omega t) \sum_{n=0}^{\infty} I_n \sin(n\omega t + \phi_n)\, d(\omega t)$$

$$= \sum_{n=0}^{\infty} V_s I_n \cdot \frac{1}{2\pi} \int_0^{2\pi} \sin(\omega t) \sin(n\omega t + \phi_n)\, d\omega t$$

from our orthogonality theorem, all but $n=1$ term $\to 0$

$$\langle P \rangle = V_s I_1 \cdot \frac{1}{2\pi} \int_0^{2\pi} \sin(\omega t) \sin(\omega t + \phi_1)\, d(\omega t)$$

from identity

$$\langle P \rangle = \frac{1}{2} V_s I_1 \cos(\phi_1)$$

OR $$\langle P \rangle = V_{RMS} I_{1,RMS} \cos(\phi_1)$$

So the only current that contributes to real power (average power) is the portion of the fundamental current in phase with the voltage!

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Now: $$PF = \frac{\langle p \rangle}{V_{rms} I_{rms}}$$

$$= \frac{\cancel{V_{rms}}\, I_{1,rms} \cos(\phi_1)}{\cancel{V_{rms}}\, I_{rms}}$$

$$\therefore \quad P.F. = \underbrace{\left(\frac{I_{1,rms}}{I_{rms}}\right)}_{k_d \text{ distortion factor}} \cdot \underbrace{\cos(\phi_1)}_{k_\theta \text{ displacement factor}} = k_D \cdot k_\theta$$

K_d distortion factor ($0 \leq K_d \leq 1$) tells us how much utilization is reduced because of harmonic currents that don't contribute real power

K_θ = Displacement factor ($|K_\theta| < 1$) tells us how much utilization of the source is reduced because of phase shift between voltage + fundamental current

$\boxed{\text{★ SHOW DEMO HERE} \Rightarrow \text{SEE NEXT PAGE}}$

Note: (1) point out that since voltage is sinusoidal, due to orthogonality we only xfer power from fundamental current, and only from that component in-phase with the voltage.

(2) Breakdown + interpretation becomes more complex if both v + i are nonsinusoidal. Less usual case but we can still define P.F.

If time, introduce $$THD \triangleq \sqrt{\frac{\sum_{n \neq 1} I_n^2}{I_1^2}}$$ Total Harmonic Distortion

Rms of Harmonic content normalized to rms of fundamental

→ assumes no dc content

$$K_D = \frac{1}{\sqrt{1 + (THD)^2}} \qquad THD = \sqrt{\frac{1}{K_D^2} - 1}$$

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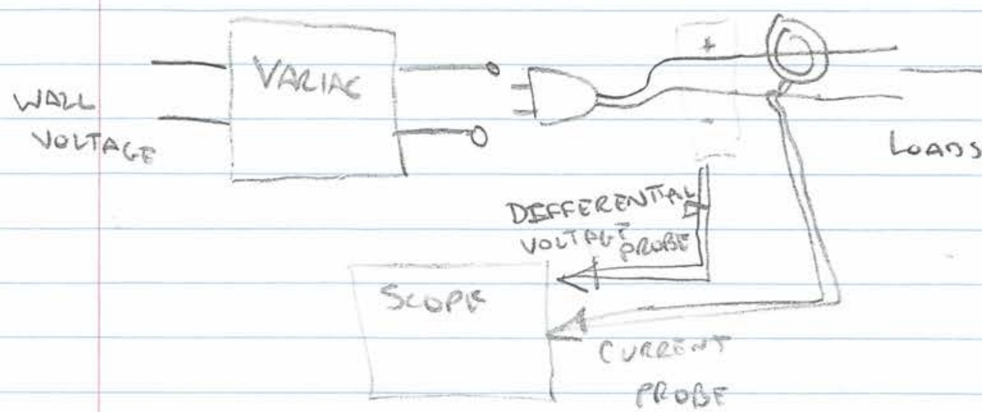
DEMO



1. DESK LAMP W/ AC CONTROLLER
2. PEAK DETECT RECTIFIER W/ LIGHTBULB

3. 1 ϕ INDUCTION MOTOR PF ~0.25, $\frac{1}{3}$ HP