6.622 Power Electronics *Prof. David Perreault*

Lecture 9 - Magnetics 1

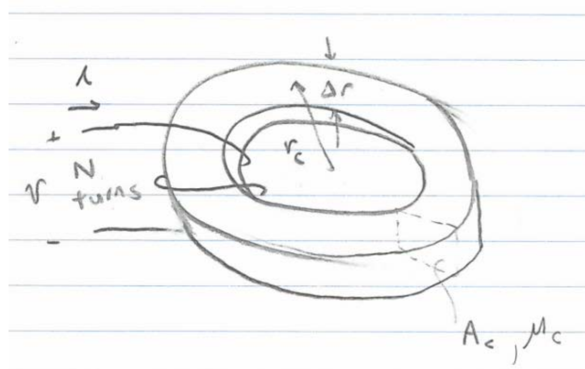
Ampere's Law $\oint H \cdot dl = \int J \cdot dA + \frac{d}{dt} \int \epsilon E \cdot dA \approx 0$
Faraday's Law $\oint E \cdot dl = -\frac{d}{dt} \int B \cdot A$
Flux continuity $\oint B \cdot da = 0$
H magnetic field intensity $(\frac{A}{M})$
B magnetic flux density (T) $1T = 10^4$gauss $= 1\frac{Joule}{A \cdot m^2}$
Material realtions $B = \mu H$, $\mu =$permeability $\frac{H}{m}$, $\mu_0 =$ permitivity of free space $(4\pi \times 10^{-7}\frac{H}{m})$
In general B = f(H) (or even more complicated, including history)

Consider design of an inductor
Assume

- $\mu_c >> \mu_0$
- $r_c >> \Delta r$

To find inductance

1. Find H (magnetic field)
2. Find Φ (magnetic flux)
3. Find $\lambda \rightarrow \lambda = Li$ (flux linkage)



1.
$$\int H \cdot dl = \int J \cdot dA \Rightarrow H \cdot l_c = N_i \Rightarrow H = \frac{N_i}{l_c}$$

2.
$$\Phi = B \cdot A_c = \mu H \cdot A_c \Rightarrow \Phi = N \cdot \frac{\mu_c A_c}{l_c} \cdot i$$

3. Flux linkage
$$\lambda = N\Phi = N^2 \frac{\mu_c A_c}{l_c} i$$

$$\lambda = Li \therefore L = N^2 \frac{\mu_c A_c}{l_c}$$

$$V = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt}$$

*Flux linkage λ is the net flux linking the whole winding, and is N times the flux in the core $\Rightarrow$ Explain flux linkage:

1. How much flux links the "soap bubble" surface defined by the N turns
2. Each turn sees a voltage $= \frac{d\Phi}{dt} = V_{\text{turn}}$

   Total voltage in all turns is $V = N \cdot V_{\text{turn}} = N\frac{d\Phi}{dt} = [N^2\frac{\mu_c A_c}{l_c}]\frac{di}{dt}$ $\lambda = \int V dt = [N^2\frac{\mu_c A_c}{l_c}]i$ $\therefore$ Total voltage is $\frac{d\lambda}{dt}$, $\lambda = N\Phi$

Notes

1. $L \propto N^2$ (Not N!)

   Some manufacturers will characterize a core with a specific inductance A_c (nH for 1 turn) so $L = N^2 A_L$ (in nH)

2. $L \propto \mu_c$ (material property which can vary a lot)

   $\rightarrow$ we need to do additional work for accurate and stable L value

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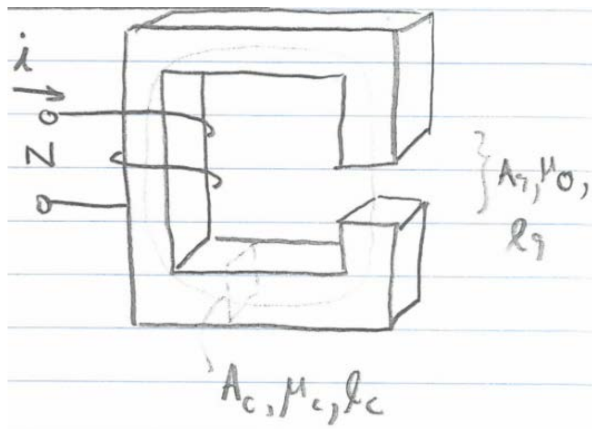
Consider a core with a gap in it

$$H_c l_c + H_g l_g = Ni$$

$$\Phi = \mu_c A_c H_c = \mu_0 A_g H_g$$



$$\Rightarrow H_c = \frac{\Phi}{\mu_c A_c}, H_g = \frac{\Phi}{\mu_0 A_g}$$

$$\therefore \Phi[\frac{l_c}{\mu_c A_c} + \frac{l_g}{mu_0 A_g}] = Ni$$

(Start here when introducing R)

$$\Phi = \frac{N}{[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g}]} i \qquad \lambda = N\Phi = \frac{N^2}{[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g}]} i$$

Since $\lambda = Li \Rightarrow$

$$\boxed{L = \frac{N^2}{\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g}}}$$

for $A_g \approx A_c$, when $\frac{l_g}{\mu_0} >> \frac{l_c}{\mu_c}$

$$W_m = \frac{1}{2} \int\int\int_{volume} B \cdot H dV$$

- inductance does not depend much on $\mu_c \Rightarrow$ insensitive!
- most energy is stored in the gap, not in the core! (H is small in core)

Lets step back and look at flux computation: "Reluctance models"

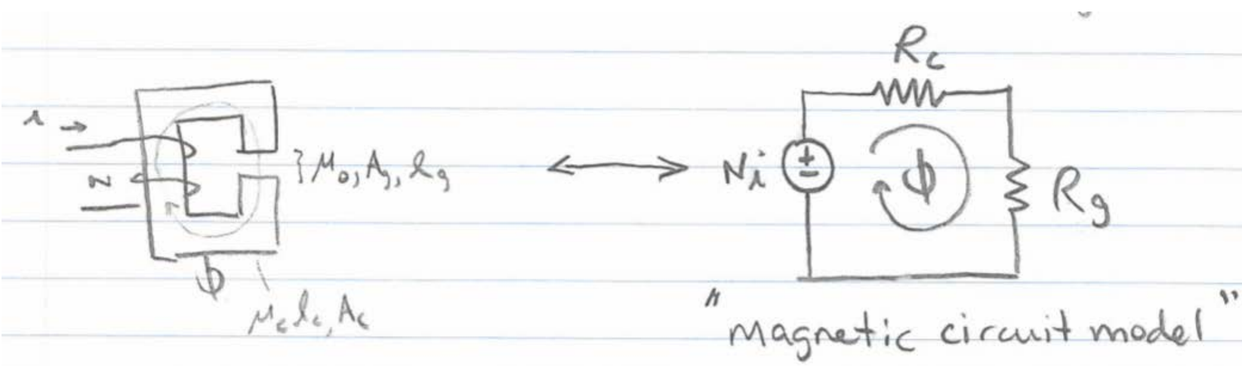
$$\Phi = \frac{Ni}{[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_c}]}$$

Similar in form to

$$``i" = \frac{``v"}{``R_1" + ``R_2"}$$

Define:

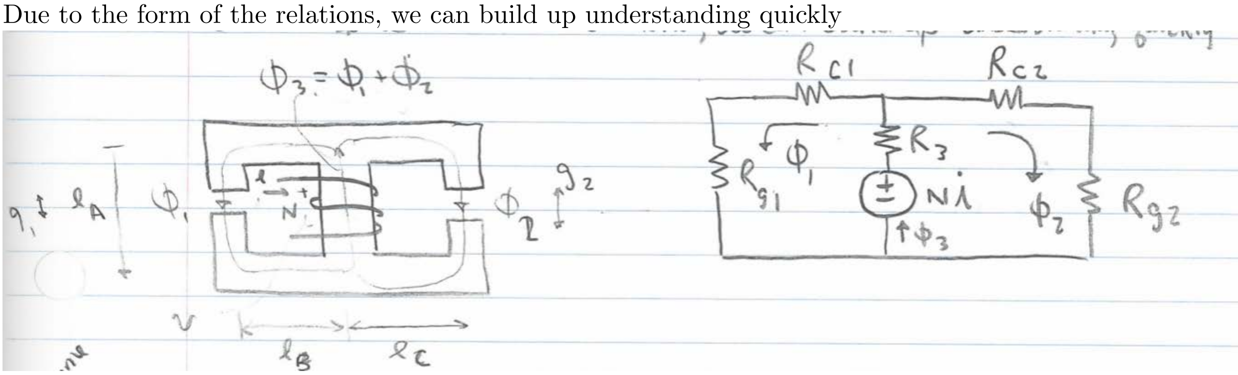
- Ni = Magnetomotivefoce "MMF"
- Φ = flux in magnetic circuit
- $R = \frac{l_x}{\mu_x A_x}$ reluctance of magnetic circuit element ("resistance" to magnetic flux μ is "conductiity" of magnetic flux)



Analogue of electrical circuit model for magnetics

Electric Circuit		Magnetic circuit
Voltage (EMF)		Magnetomotive Force (MMF)
Current	$\leftrightarrow$	flux
Resistance $R = \frac{l}{\sigma A}$		reluctance $R = \frac{l}{\mu A}$

Due to the form of the relations, we can build up understanding quickly



Assuming $A_c \approx A_g$

$$R_3 = \frac{l_a}{\mu_c A_c}, R_{g_1} = \frac{g_1}{\mu_0 A_c}, R_{g_2} = \frac{g_2}{\mu_0 A_c}$$

$$R_{c_1} = \frac{(2l_B + l_A - g_1)}{\mu_c A_c}, R_{c_2} = \frac{(2l_c + l_A - g_2)}{\mu_c A_c}$$

From the circuit model:

$$R_{\text{NET}} = R_3 + ((R_{c_1} + R_{g_1})||(R_{c_2} + R_{g_2}))$$

$$\Phi_3 = \frac{Ni}{R_{\text{NET}}} \Rightarrow \lambda_3 = \frac{N^2}{R_{\text{NET}}} i$$

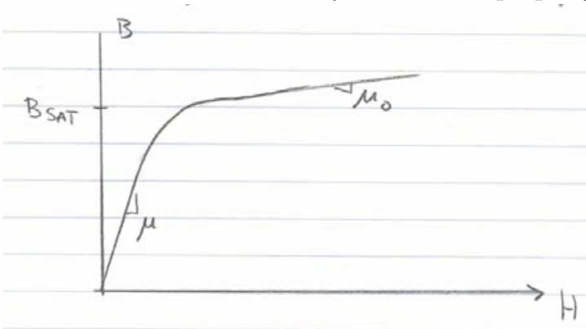
$$\therefore L = \frac{N^2}{R_{\text{NET}}}, V = [\frac{N^2}{R_{\text{NET}}}] \frac{di}{dt}$$

So the magnetic circuit model allows fast calculation of magnetic element behavior

- Main limitation is calculating "leakage" flux, as magnetic circuits are quite "leaky": μ_c $10^3 - 10^4 \mu_0$ typ., unlike electric circuits

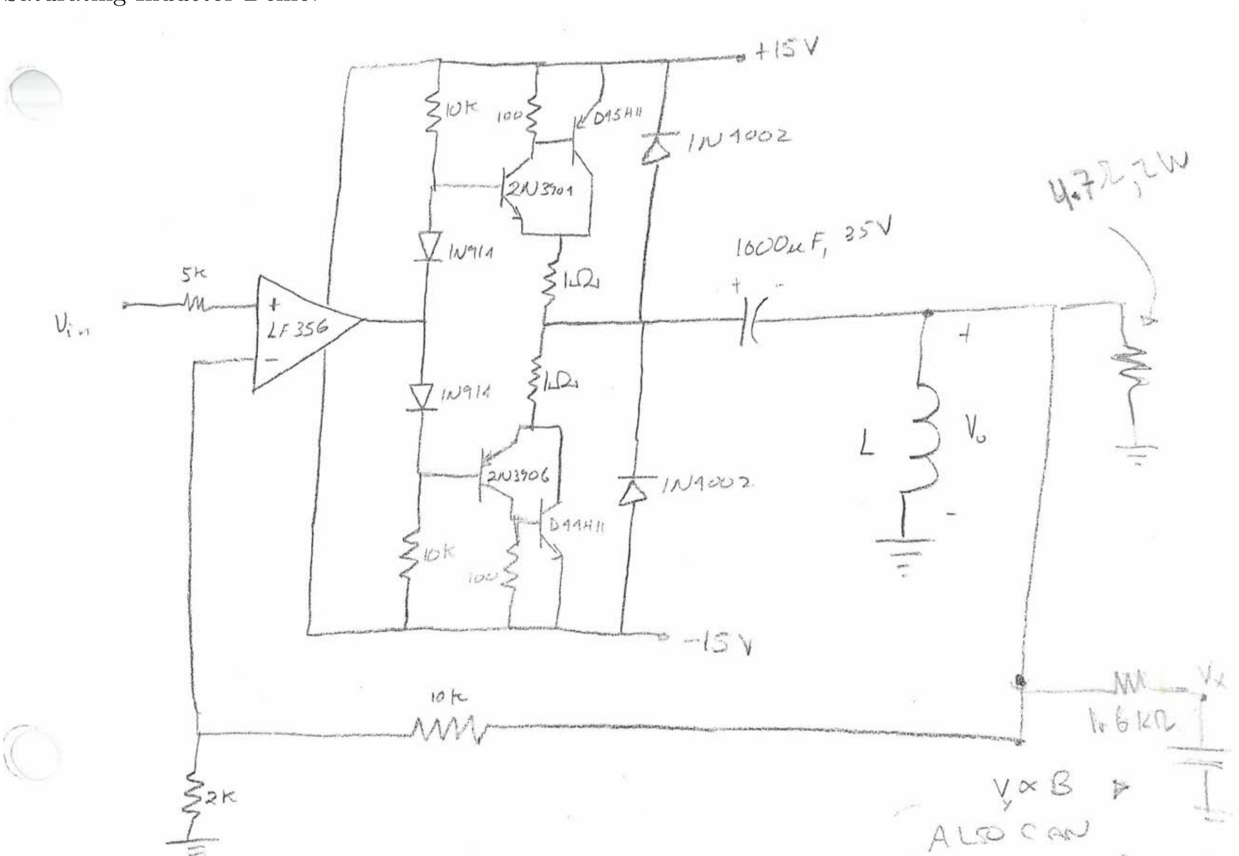
- for accuracy (in simple calculations) we need $\mu_c >> \mu_0, l_c >> \sqrt{A_c}, l_g << \sqrt{A_g}$

Note: These relations rely on material property $B = \mu H$. A slightly more refined model is $B = f(H)$



Above some flux density, material saturates $\frac{\delta B}{\delta H} \rightarrow \mu_0$
operate @ $B < B_{\text{sat}}$ (Iron μ $10^4\mu_0$, B_{sat} $2T$
Ferrite μ $10^3\mu_0$, B_{sat} $0.3T$)

Saturating Inductor Demo:



L: TX 39/20/13-2F2 core
$l_c = 84.9mm$
$A_c = 112mm^2$
$i_l = 3150nH$
$\mu_i = 1800\mu_0$
$N = 10$
$L = 0.315mH$ (0.33 mH measured)
V_in: Function generator, High Z
$300mV_pp - 2.1V_pp@2kHz$

Also can do B-H loop. Compare V_x (y) to i_L ($\propto$) in plot on scope.