6.334 Lecture Notes — Magnetics 1

Ampere's Law $\oint H \cdot dl = \int J \cdot dA + \cancel{\frac{d}{dt}\int \epsilon E \cdot dA}^{\approx 0}$

Faraday's Law $\oint E \cdot dl = -\frac{d}{dt}\int B \cdot dA$

Flux Continuity $\oint B \cdot dA = 0$

H magnetic field Intensity (A/m)

B magnetic flux density (T) $1\,T = 10^4\,\text{gauss} = 1\,\frac{\text{Joule}}{A\text{-}m^2}$

Material relations $B = \mu H$ μ = permeability (H/m)

μ_0 = perm of free space $(4\pi \times 10^{-7}\ H/m)$

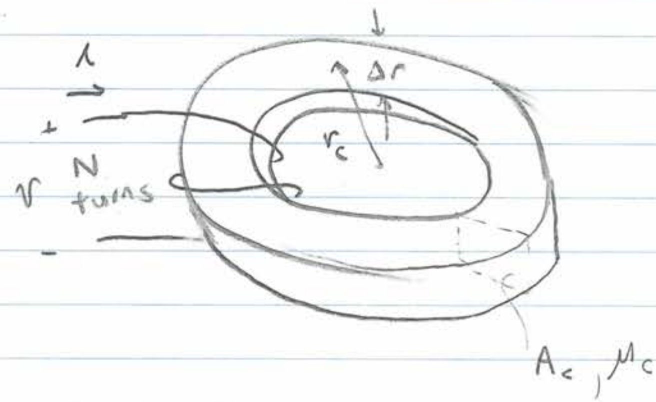
$B = \mu_0 H + M$

In general $B = f(H)$ (or even more complicated, including history)

Consider design of an inductor

Assume $\mu_c >> \mu_0$

$r_c >> \Delta r$



To find inductance

mag. field 1. Find H

mag flux 2. Find Φ

flux linkage 3. Find $\lambda \rightarrow \lambda = Li$

(1) $\int H \cdot dl = \int J \cdot dA \Rightarrow H \cdot l_c = Ni$

$\Rightarrow H = \frac{Ni}{l_c}$

(2) $\Phi = B \cdot A_c = \mu H \cdot A_c \Rightarrow \Phi = N \cdot \frac{\mu_c A_c}{l_c} \cdot i$

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③ Flux linkage $\lambda = N\Phi = N^2 \frac{\mu_c A_c}{l_c} i.$

$\lambda = L i \quad \therefore \quad L = N^2 \frac{\mu_c A_c}{l_c}$ $\boxed{V = \frac{d\lambda}{dt} = \frac{d}{dt}(Li) = L\frac{di}{dt}}$

⇒ Explain flux linkage : ① How much flux links the "soap bubble" surface defined by the N turns

② Each turn sees a voltage = $\frac{d\phi}{dt} = V_{TURN}$

total voltage all turns is $V = N \cdot V_{TURN} = N\frac{d\Phi}{dt} = \underbrace{\left[N^2 \frac{\mu_c A_c}{l_c}\right]}_{L} \frac{di}{dt}$

$$\lambda = \int V\,dt = \left[N^2 \frac{\mu_c A_c}{l_c}\right] i$$

∴ Total voltage is $d\lambda/dt$, $\lambda = N\phi$

* Flux linkage λ is the net flux linking the whole winding, and is N times the flux in the core

NOTES ① $L \propto N^2$ (Not N!)

Some manufacturers will characterize a core with a "specific inductance" A_L (nH for 1 turn) so $L = N^2 A_L$ (in nH)

② $L \propto \mu_c$ (material property which can vary a lot)

→ we need to do additional work for accurate + stable L value

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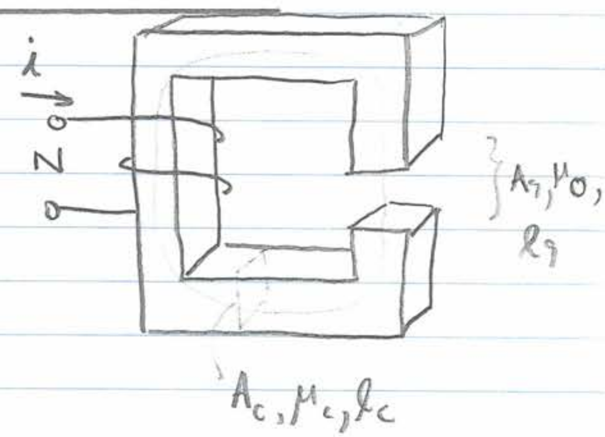
Consider a core with a gap in it



$$H_c l_c + H_g l_g = N i$$

$$\Phi = \mu_c A_c H_c = \mu_0 A_g H_g$$

$$\Rightarrow H_c = \frac{\Phi}{\mu_c A_c}, \quad H_g = \frac{\Phi}{\mu_0 A_g}$$

$$\therefore \Phi \left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g}\right] = N i$$

★ ← START HERE WHEN INTRODUCING $\mathcal{R}$

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$$\Phi = \frac{N}{\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g}\right]} i \qquad \lambda = N\Phi = \frac{N^2}{\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}\right]} i$$

Since $\lambda = L i \Rightarrow$
$$\boxed{L = \frac{N^2}{\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_g}}}$$

for $A_g \simeq A_c$, when $\frac{l_g}{\mu_0} >> \frac{l_c}{\mu_c}$

- inductance does <u>not</u> depend much on $\mu_c \Rightarrow$ insensitive!
- Most <u>energy</u> is stored in the <u>gap</u>, <u>not</u> in the core! (H is <u>small</u> in core)

$$W_m = \frac{1}{2}\iiint_{\text{Volume}} B \cdot H \, dV$$

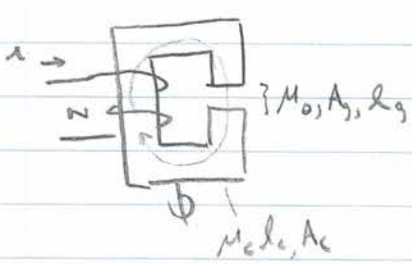
Lets step back and look at flux computation: "<u>Reluctance models</u>"

$$\Phi = \frac{N i}{\left[\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_0 A_c}\right]}$$ similar in form to $$\text{"}i\text{"} = \frac{\text{"}V\text{"}}{\text{"}R_1\text{"} + \text{"}R_2\text{"}}$$

Define: Ni = Magnetomotive force "MMF"

Φ = Flux in magnetic circuit

$\mathcal{R} = \frac{l_x}{\mu_x A_x}$ <u>reluctance</u> of magnetic circuit element
("resistance" to magnetic flux
μ is "conductivity" of magnetic flux)



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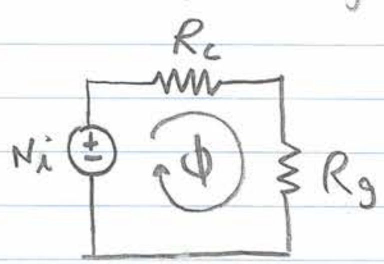

"magnetic circuit model"

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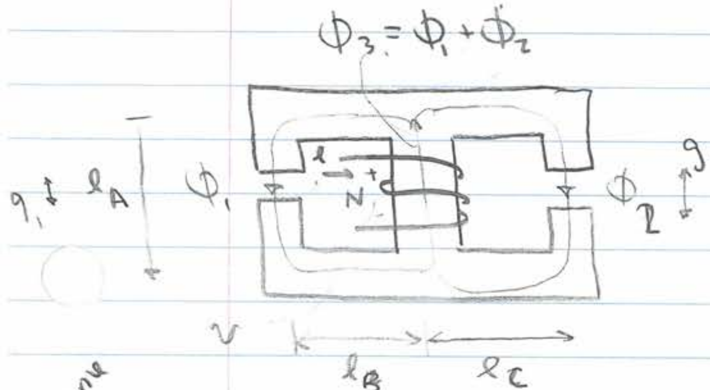
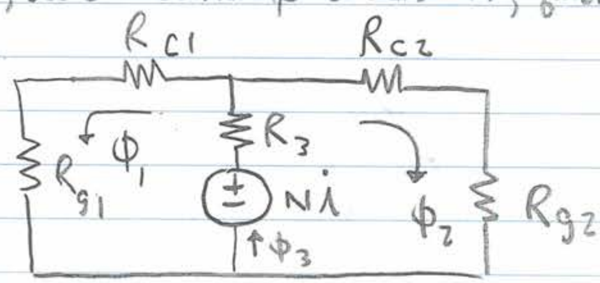
Analogue of Electric Circuit Model for Magnetics

Electric Circuit		Magnetic Circuit
Voltage (EMF)	$\longleftrightarrow$	Magnetomotive Force (MMF)
current		flux
Resistance $R = \frac{\ell}{\sigma A}$		reluctance $\mathcal{R} = \frac{\ell}{\mu A}$

Due to the form of the relations, we can build up understanding quickly





Assume $A_c \simeq A_g$

$$R_3 = \frac{\ell_A}{\mu_c A_c} \qquad R_{g1} = \frac{g_1}{\mu_0 A_c} \qquad R_{g2} = \frac{g_2}{\mu_0 A_c}$$

$$R_{c1} = \frac{(2\ell_B + \ell_A - g_1)}{\mu_c A_c} \qquad R_{c2} = \frac{(2\ell_c + \ell_A - g_2)}{\mu_c A_c}$$

From the circuit model:

$$\mathcal{R}_{NET} = \mathcal{R}_3 + \left((R_{c1} + R_{g1}) \,\|\, (R_{c2} + R_{g2})\right)$$

$$\Phi_3 = \frac{Ni}{R_{NET}} \Rightarrow \lambda_3 = \frac{N^2}{R_{NET}}\, i$$

$$\therefore\; L = \frac{N^2}{R_{NET}}, \quad V = \left[\frac{N^2}{R_{NET}}\right]\frac{di}{dt}$$

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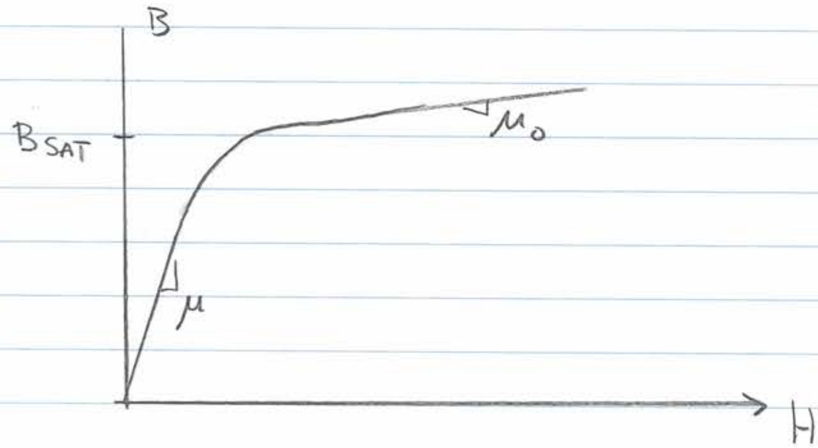
So the magnetic circuit model allows fast calculation of magnetic element behavior

→ Main limitation is calculating "leakage" flux, as magnetic circuits are quite "leaky": $\mu_c \sim 10^3 - 10^4 \mu_o$ typ., unlike electric circuits

→ for accuracy (in simple calculations) we need

$$\mu_c \gg \mu_o, \; l_c \gg \sqrt{A_c}, \; l_g \ll \sqrt{A_g}$$

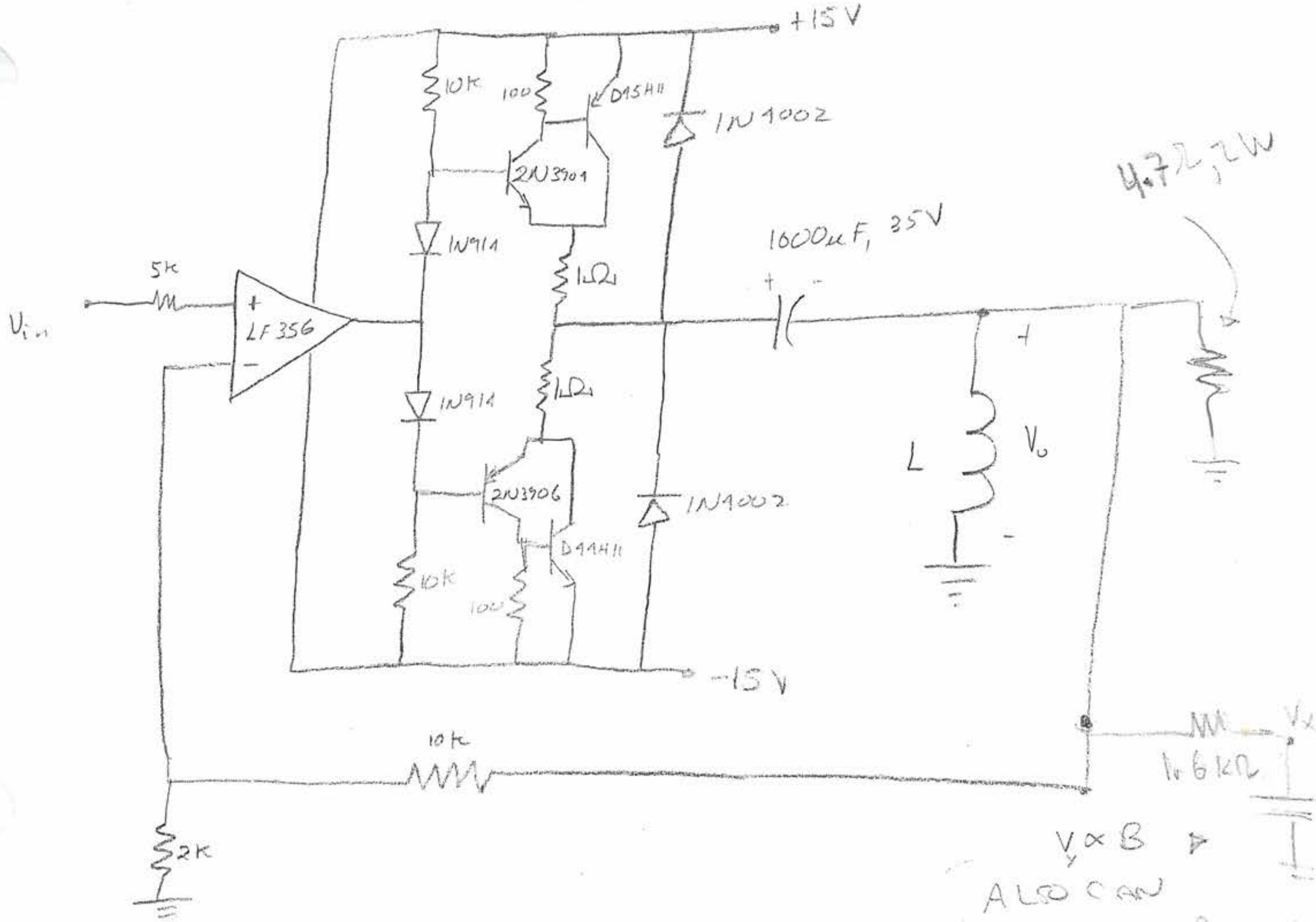
Note: These relations rely on material property $B = \mu H$.
A slightly more refined model is $B = f(H)$



Above some flux density, material saturates $\frac{\partial B}{\partial H} \rightarrow \mu_o$

- operate @ $B < B_{sat}$
(Iron $\mu \sim 10^4 \mu_o$, $B_{sat} \sim 2T$
Ferrite $\mu \sim 10^3 \mu_o$, $B_{sat} \sim 0.3T$)

SATURATING INDUCTOR DEMO 3/2/05 Sampanne D.



ALSO CAN DO B-H loop. Compare V_x to I_L in plot (y) (x) or scope

L: TX 39/20/13 - 3F3 core

$l_e = 84.9\,mm$

$A_e = 112\,mm^2$

$A_L = 3150\,nH$

$\mu_i = 1800\,\mu_0$

$N = 10$

$L = 0.315\,mH$ (0.33 mH measured)

set cutoff 1 decade below 2kHz.

V_{in}: Function generator, High Z

$300\,mV_{pp} - 2.1\,V_{pp}$ @ 2kHz