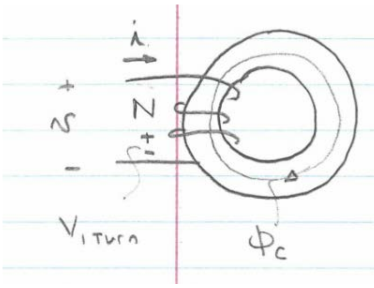
**6.622 Power Electronics** *Prof. David Perreault*

# Lecture 10 - Magnetics 2

## 1 Brief review:




- Given a magnetic structure, we can use Maxwell's equations to find the magnetic flux in the core. For magnetic materials with characteristic $B = \mu H$, $\Phi \propto N \cdot i$
- From Faraday's law, voltage on 1 turn of the winding is $V_{1\text{ turn}} = -\int_{1\text{ turn}} E \cdot dl = \frac{d\Phi_c}{dt}$ (Direction by Lenz's law)

- If the flux links N turns of a winding

$$V_{\text{total}} = N\frac{d\Phi_c}{dt}$$

Defining flux linkage $\lambda = N\Phi_c \Rightarrow V_{\text{total}} = \frac{d\lambda}{dt}$ $\{=\frac{d}{dt}(N\Phi_c)\}$

- From above, we have $\lambda \propto i \Rightarrow \lambda = Li$ {Proportionality constant is L, $\propto N^2$}

$\Rightarrow$ Can show DEMO of voltage buildup across turns of an inductor (circuit on last page. Can also be used for B-H loops, Hysteresis, ...)

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To rapidly calculate $\Phi$ in complicated structures (and find inductance, voltages, etc) we can use reluctance models (magnetic circuit models)

Magnetic circuits: an analogue of electric circuits for magnetics

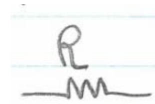
Electric | | Magnetic

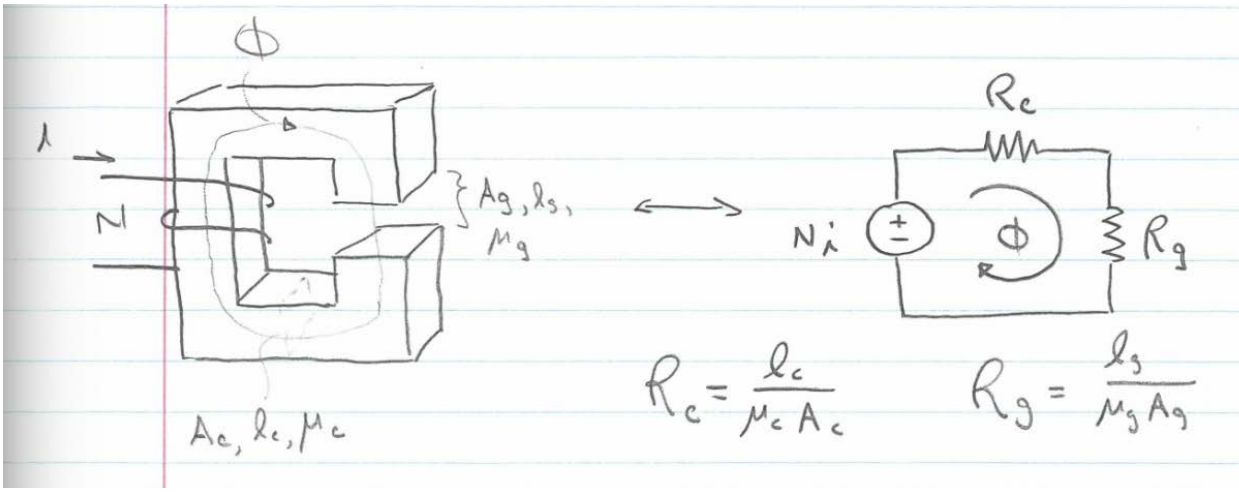

EMF (voltage) $\leftrightarrow$ Magneticmotiveforce/MMF


current $\leftrightarrow$ magnetic flux



Resistance $R = \frac{l_x}{\sigma_x A_x}$ $\leftrightarrow$ $R = \frac{l_x}{\mu_x A_x}$






Left side says: $A_c, l_c, \mu_c$
Right side says: $R_c = \frac{l_c}{\mu_c A_c}, R_g = \frac{l_g}{\mu_g A_g}$

$$\therefore \Phi = \frac{N_i}{R_c + R_g} = \frac{Ni}{\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}} = \frac{N^2}{R_{\text{net}}} \cdot i$$

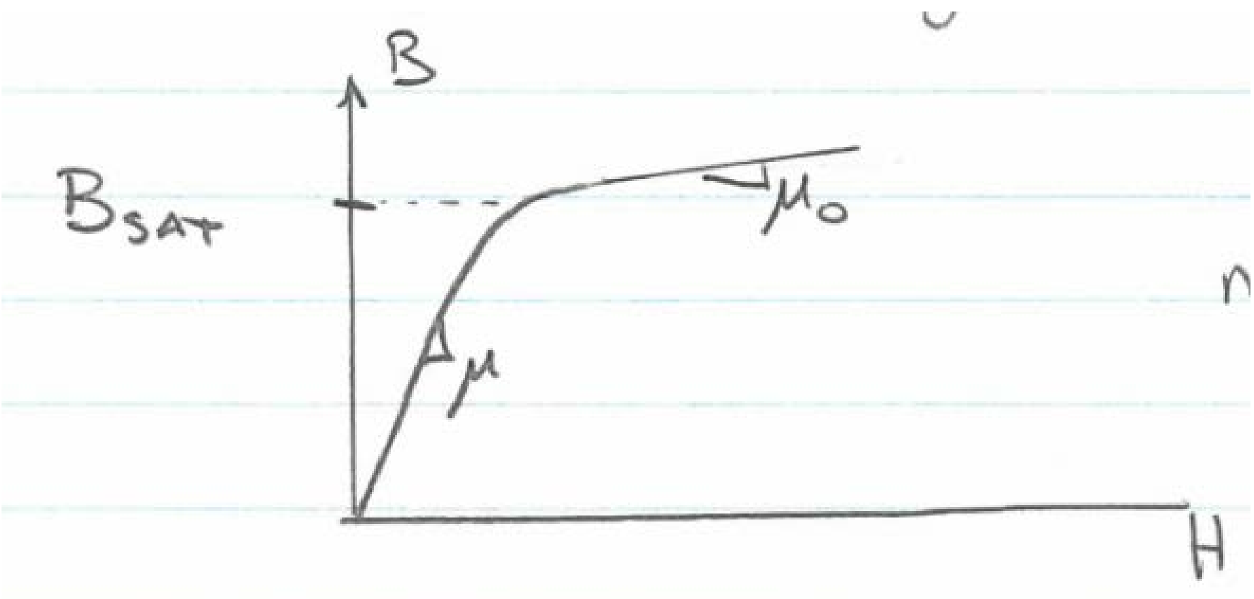
To find terminal voltage, find $\lambda$; $v = \frac{d\lambda}{dt}$

$$\lambda = N\Phi = [\frac{N^2}{\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}}] \cdot i = Li$$

(Note: $L \propto N^2$ and depends on $\mu$, geometry $L = \frac{N^2}{R_{\text{net}}} = A_l N^2$, which is the "specific inductance")

Notes:

1. These relations rely on material property B = $\mu H$. More generally, however, we see B = f(H)

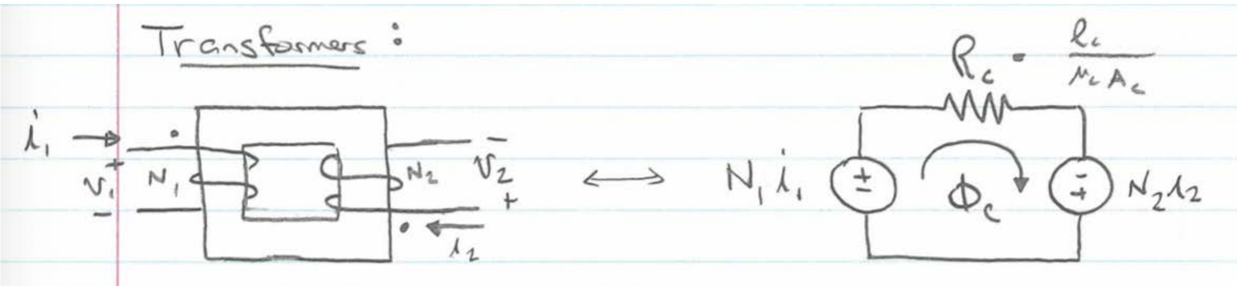



Above some flux density $B_{\text{SAT}}$, material saturates $\frac{\delta B}{\delta H} \rightarrow \mu_0$

* we must operate $B < B_{\text{SAT}}$ to use the simple model (Iron $B_{\text{SAT}}$ $2T$, $\mu$ $10^4\mu_0$)

2. this simple "lumped element" model assumes simple, well-defined paths through which flux flows. Because more accurate for $\mu_c >> \mu_0, l_c >> \sqrt{A_c}, l_g << \sqrt{A_c}, ...$

## 2 Transformers




* "Dot" convention: currents into the dots throw flux around the cone in the same direction.

$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi_c}{dt}$$

$$v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi_c}{dt}$$

$$v_2 = \frac{N_2}{N_1} v_1$$

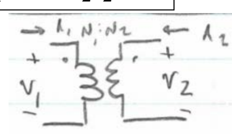
(Note polarity wrt dots!)

Currents:

$$\Phi_c = \frac{N_1 i_1 + N_2 i_2}{R_c} \Rightarrow N_1 i_1 + N_2 i_2 = \Phi_c R_c$$

If $\mu_c \to \infty, R_c \to 0$, so for finite $\Phi_c$:

$$N_1 i_1 = -N_2 i_2$$




⇒ These are the "ideal" transformer relations

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Consider nonidealities: (1) Magnetizing

Since $\mu_c < \infty$, $R_c > 0 \therefore N_1 i_1 + N_2 i_2 = \Phi_c R_c \neq \phi$ (There is an error in the current relation)

Consider $i_2 = \phi$ (open circuit secondary)

$\Phi = \frac{N_1 i_1}{R_c} \to V_1 = \frac{N_1^2}{R_c} \frac{di_2}{dt} = L_{\mu_1} \frac{di_1}{dt}$

Magnetizing inductance reflects the fact that there is some nonzero energy stored in the core ($\frac{1}{2} L_{\mu_1} i_{\mu_1}^2$) in the process of inducing voltage between windings.

Could place magnetizing L on other side of transformer with appropriate scaling for turns ratio $L_{\mu_2} = (\frac{N_2}{N_1})^2 L_{\mu_1}$

Magnetization of the core is the reason transformers can't work @ dc

$$V_1 = N_1 \frac{d\Phi}{dt} \Rightarrow \Phi_c = \frac{1}{N_1} \int V_1(t) dt$$

We require $\Phi_c < B_{\text{sat}} A_c$

$$\therefore \int V_1(t) dt < N_1 B_{\text{sat}} A_c$$

We have a fundamental volt-sec limit on the transformer to avoid saturation!

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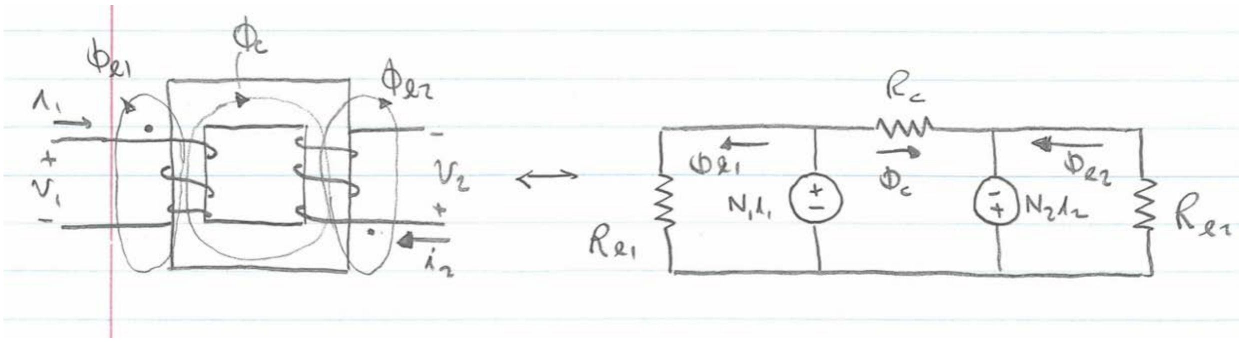
Leakage inductance

In reality, all flux does not follow the cone, so some flux from each winding does not link the other:




$$\Phi_{l1} = \frac{N_1 i_1}{R_{l1}}, \Phi_c = \frac{N_1 i_1 + N_2 i_2}{R_c}, Q_{l2} = \frac{N_2 i_2}{R_{l2}}$$

$$\lambda_1 = N_1(\Phi_{l1} + \Phi_c) = \frac{N_1^2}{R_c} i_1 + \frac{N_1^2}{R_{l1}} i_1 + \frac{N_1 N_2}{R_c} i_2$$

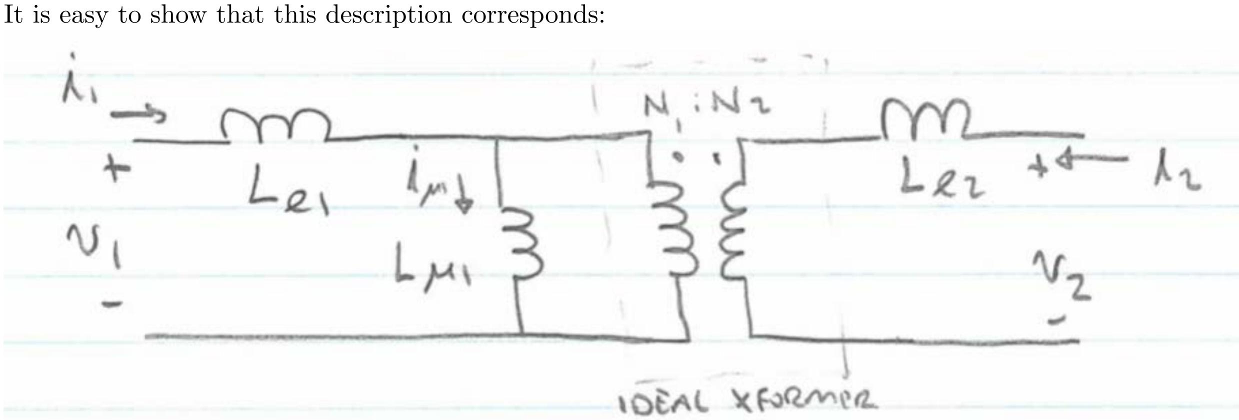
$$\lambda_2 = N_2(\Phi_c + \Phi_{l2}) = \frac{N_1 N_2}{R_c} i_1 + \frac{N_2^2}{R_c} i_2 + \frac{N_2^2}{R_{l2}} i_2$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{l1}} & \frac{N_1 N_2}{R_c} \\ \frac{N_1 N_2}{R_c} & \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{l2}} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_\mu \\ L_\mu & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{1}$$

$$\therefore \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_\mu \\ L_\mu & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{2}$$

Inductance matrix description

It is easy to show that this description corresponds:




where: $L_{l1} = \frac{N_1^2}{R_{l1}}$ and $L_{\mu_1} = \frac{N_1^2}{R_c}$ and $L_{l2} = \frac{N_2^2}{R_{l2}}$
and $L_{11} = L_{l1} + L_{\mu_1}$ and $L_{22} = L_{l2} + (\frac{N_2}{N_1})^2 L_{\mu_1}$ and $L_\mu = (\frac{N_2}{N_1}) L_{\mu_1}$

So: leakage adds "independent inductance" in series with transformer leads! Both magnetizing inductance $\rightarrow$ leakage inductance are important effects in practical transformers!