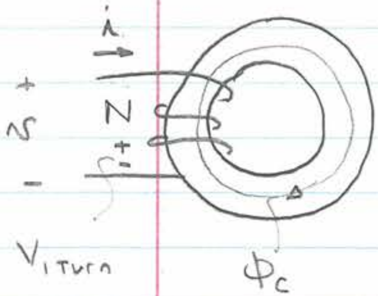
6.334 Lecture — Magnetics #2

Brief review: ★ Given a magnetic structure, we can use Maxwell's eqn's to find the magnetic flux in the core. For magnetic materials w/ characteristic $B = \mu H$, $\Phi \propto N \cdot i$



★ From Faraday's law, voltage on 1 turn of the winding is $V_{1\text{-turn}} = -\int_{1\text{-turn}} E \cdot dl = \frac{d\Phi_c}{dt}$ (Direction by Lenz's law)

★ If the flux links N turns of a winding

$$V_{Total} = N \frac{d\Phi_c}{dt}$$

Defining flux linkage $\lambda = N\Phi_c \Rightarrow V_{Total} = \frac{d\lambda}{dt}$ $\left\{ = \frac{d}{dt}(N\Phi_c) \right\}$

★ from above, we have $\lambda \propto i \Rightarrow \lambda = Li$ {Proportionality constant is L, $\propto N^2$}

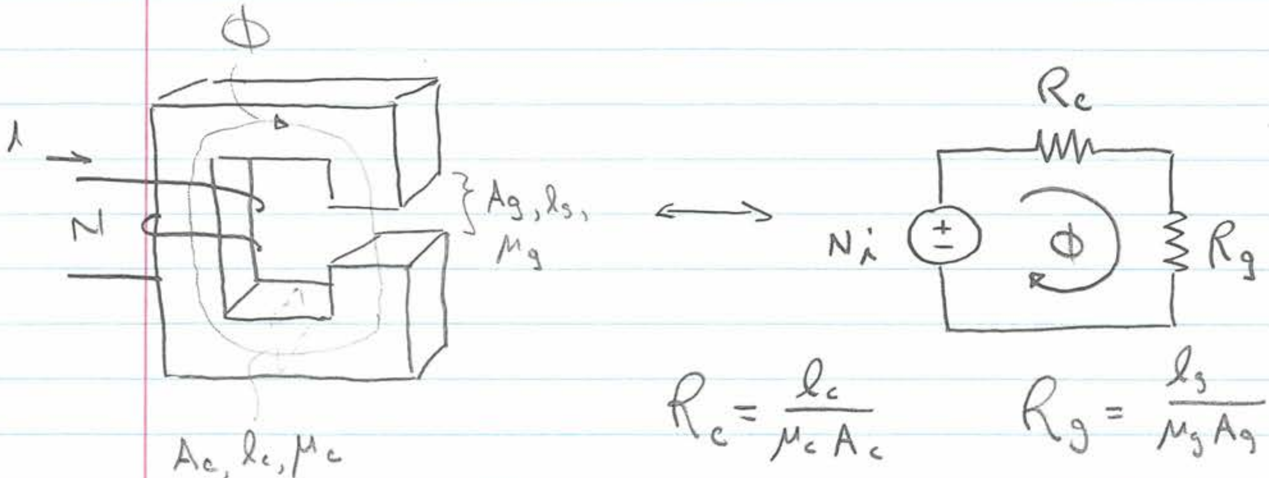
⇒ Can show DEMO of voltage buildup across turns of an inductor (Circuit on last page. Can also be used for B-H loops, Hysteresis, ...)

To rapidly calculate Φ in complicated structures (and find inductance, voltages, etc.) we can use reluctance models (Magnetic circuit models)

Magnetic Circuits: An analogue of electric circuits for magnetics

Electric		Magnetic
v EMF (voltage)	⟷	Magnetomotive Force / MMF Ni
i current	⟷	magnetic flux Φ
R Resistance $R = \frac{l_x}{\sigma_x A_x}$	⟷	Reluctance $\mathcal{R} = \frac{l_x}{\mu_x A_x}$ $\mathcal{R}$

6.334 Lecture — Magnetics # 2



$$R_c = \frac{l_c}{\mu_c A_c} \qquad R_g = \frac{l_g}{\mu_g A_g}$$

$$\therefore \Phi = \frac{Ni}{R_c + R_g} = \frac{Ni}{\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}} = \frac{N^2}{R_{NET}} \cdot i$$

To find terminal voltage, find λ; $v = \frac{d\lambda}{dt}$

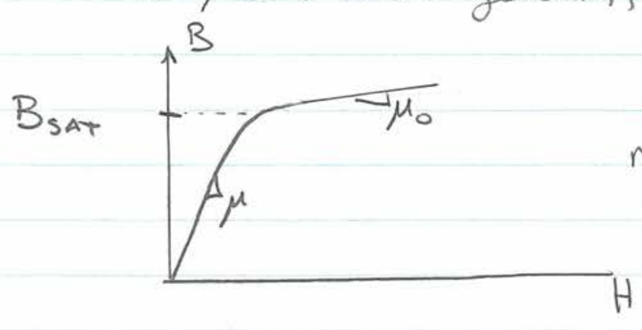
$$\lambda = N\Phi = \overbrace{\left[\frac{N^2}{\left(\frac{l_c}{\mu_c A_c} + \frac{l_g}{\mu_g A_g}\right)}\right]}^{L} \cdot i = Li$$

Note: $L \propto N^2$ + depends on μ, geometry

$L = \frac{N^2}{R_{NET}} = A_L N^2$

"specific inductance"

Notes: ① These relation rely on material property $B = \mu H$. More generally, however, we see $B = f(H)$



Above some flux density B_{SAT}, material saturates $\frac{\partial B}{\partial H} \to \mu_0$

★ we must operate $B < B_{SAT}$ to use the simple model

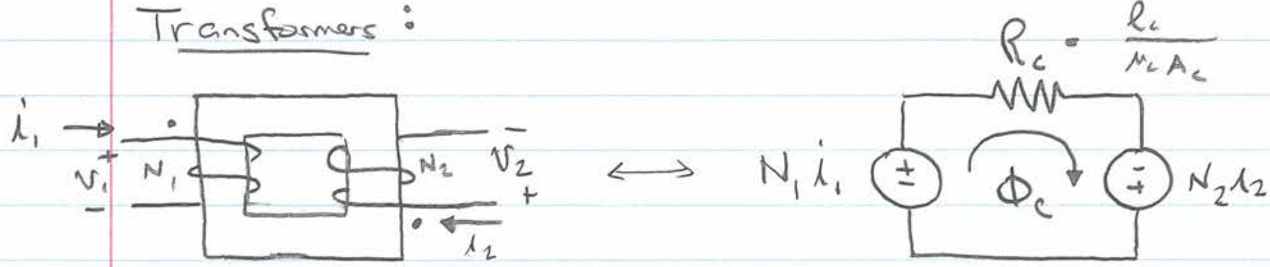
(Iron $B_{sat} \sim 2T$, $\mu \sim 10^4 \mu_0$)

② This simple "lumped element" model assumes simple, well-defined paths through which flux flows, Becomes more accurate for

$\mu_c >> \mu_0$, $l_c >> \sqrt{A_c}$, $l_g << \sqrt{A_c}$, ...

6.334 Lecture — Magnetics #2

Transformers:



★ "Dot" convention: currents into the dots throw flux around the core in the same direction

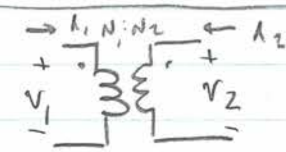
$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi_c}{dt}$$
$$v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi_c}{dt}$$
$$\boxed{v_2 = \frac{N_2}{N_1} v_1}$$

(Note polarity wrt dots!)

Currents: $\Phi_c = \frac{N_1 i_1 + N_2 i_2}{R_c} \Rightarrow N_1 i_1 + N_2 i_2 = \Phi_c R_c$

If $\mu_c \to \infty$, $R_c \to 0$, so for finite Φ_c: $\boxed{N_1 i_1 = -N_2 i_2}$

⇒ These are the "ideal" transformer relations



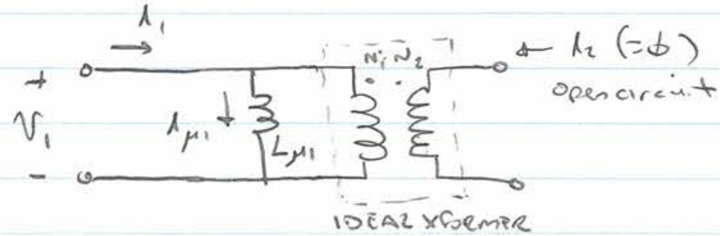
Consider nonidealities: (#1) Magnetizing

Since $\mu_c < \infty$, $R_c > 0$ $\therefore$ $N_1 i_1 + N_2 i_2 = \Phi_c R_c \neq 0$
(there is an error in the current relation)

Consider $i_2 = 0$ (open circuit secondary)

$$\Phi = \frac{N_1 i_1}{R_c} \longrightarrow v_1 = \frac{N_1^2}{R_c}\frac{di_1}{dt} = L_{\mu 1}\frac{di_1}{dt}$$

6.334 Lecture — Magnetics #2



Magnetizing Inductance reflects the fact that there is some nonzero energy stored in the core $\left(\frac{1}{2} L_{\mu 1} i_{\mu 1}^2\right)$ in the process of inducing voltage between windings

Could place magnetizing L on other side of transformer with appropriate scaling for turns ratio $L_{\mu 2} = \left(\frac{N_2}{N_1}\right)^2 L_{\mu 1}$

Magnetization of the core is the reason transformers can't work @ dc

$$V_1 = N_1 \frac{d\Phi}{dt} \Rightarrow \Phi_c = \frac{1}{N_1} \int V_1(t)\, dt$$

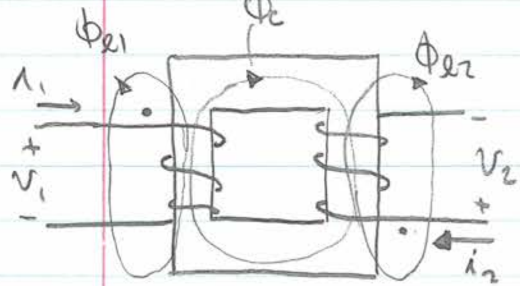
we require $\Phi_c < B_{SAT} A_c$

$$\therefore \boxed{\int V_1(t)\, dt < N_1 B_{SAT} A_c}$$

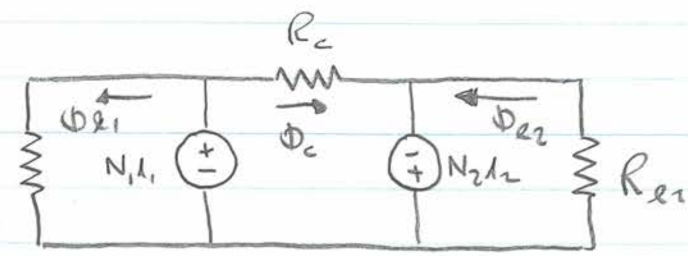
we have a fundamental Volt-sec limit on the transformer to avoid saturation!

Leakage Inductance

In reality, all flux does not follow the core, so some flux from each winding does not link the other:



$\longleftrightarrow$



6.334 Lecture — Magnetics #2

$$\Phi_{\ell 1} = \frac{N_1 i_1}{R_{\ell 1}} \qquad \Phi_c = \frac{N_1 i_1 + N_2 i_2}{R_c} \qquad \Phi_{\ell 2} = \frac{N_2 i_2}{R_{\ell 2}}$$

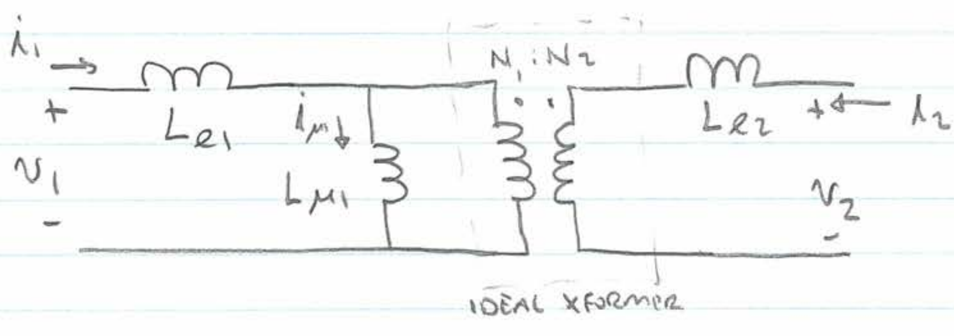
$$\lambda_1 = N_1(\phi_{\ell 1} + \phi_c) = \frac{N_1^2}{R_c} i_1 + \frac{N_1^2}{R_{\ell 1}} \cdot i_1 + \frac{N_1 N_2}{R_c} i_2$$

$$\lambda_2 = N_2(\Phi_c + \Phi_{\ell 2}) = \frac{N_1 N_2}{R_c} i_1 + \frac{N_2^2}{R_c} i_2 + \frac{N_2^2}{R_{\ell 2}} i_2$$

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{\ell 1}} & \frac{N_1 N_2}{R_c} \\ \frac{N_1 N_2}{R_c} & \frac{N_2^2}{R_c} + \frac{N_2^2}{R_{\ell 2}} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

$$\therefore \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix}$$ Inductance Matrix Description

It is easy to show that this description corresponds to a circuit model:



where: $L_{\ell 1} = \frac{N_1^2}{R_{\ell 1}}$ $\quad L_{\mu 1} = \frac{N_1^2}{R_c}$ $\quad L_{\ell 2} = \frac{N_2^2}{R_{\ell 2}}$

and $L_{11} = L_{\ell 1} + L_{\mu 1}$ $\quad L_{22} = L_{\ell 2} + \left(\frac{N_2}{N_1}\right)^2 L_{\mu 1}$ $\quad L_M = \left(\frac{N_2}{N_1}\right) L_{\mu 1}$

Both magnetizing inductance + leakage inductance are important effects in practical Xformers!

So: Leakage adds "independent inductors" in series w/ XFORMER Leads!