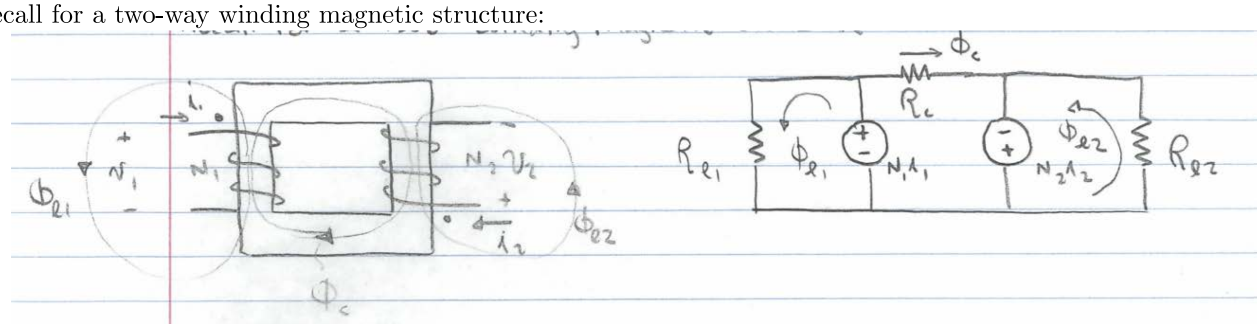
6.622 Power Electronics *Prof. David Perreault*

Lecture 11 - Magnetics 3

Recall for a two-way winding magnetic structure:



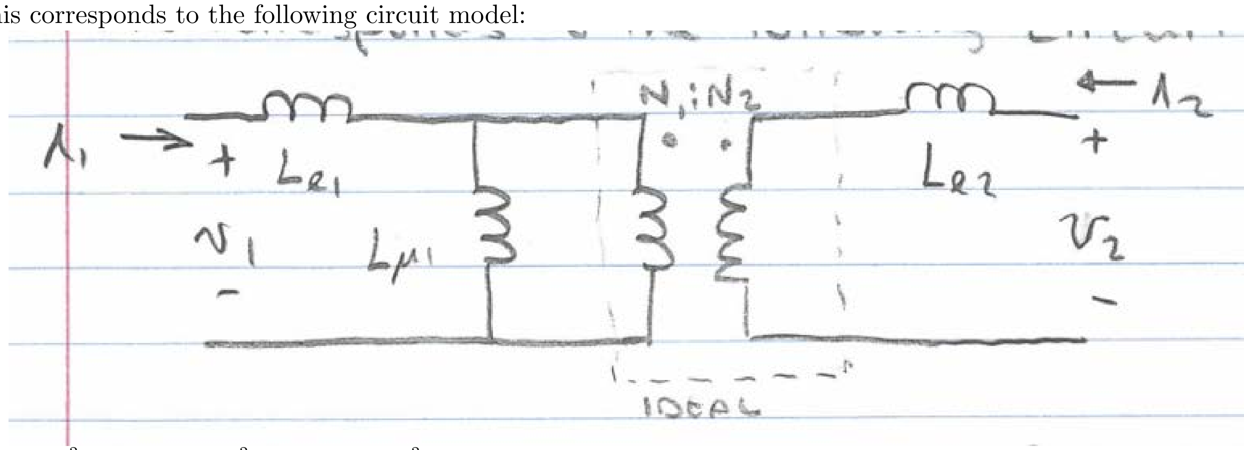
Note:

1. Directions of MMF sources are determined by the winding orientations. (Note dog convention)
2. Electrical behavior at the iwnding terminals is determined by flux linkage + Lenz's law (λ is $N\Phi$ from + side of MMF source)

Crunching the math we get:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{l1}} & \frac{N_1 N_2}{R_c} \\ \frac{N_1 N_2}{R_c} & \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{l2}} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_\mu \\ L_\mu & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \tag{1}$$

This corresponds to the following circuit model:



$L_{\mu_1} = \frac{N_1^2}{R_c}$ and $L_{l1} = \frac{N_1^2}{R_{l1}}$ and $L_{l2} = \frac{N_2^2}{R_{l2}}$

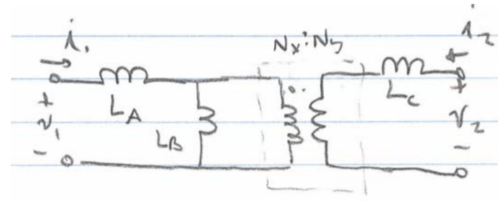
Also, we get $L_{11} = L_{\mu_1} + L_{l_1}$, $L_{22} = (\frac{N_1}{N_2})^2 L_{\mu_1} + L_{l_2}$, $L_\mu = \frac{N_2}{N_1} L_{\mu_1}$

Note that the inductance matrix has three quasi-independent parameters L_{11}, L_{22}, L_μ. These fully describe terminal behavior.

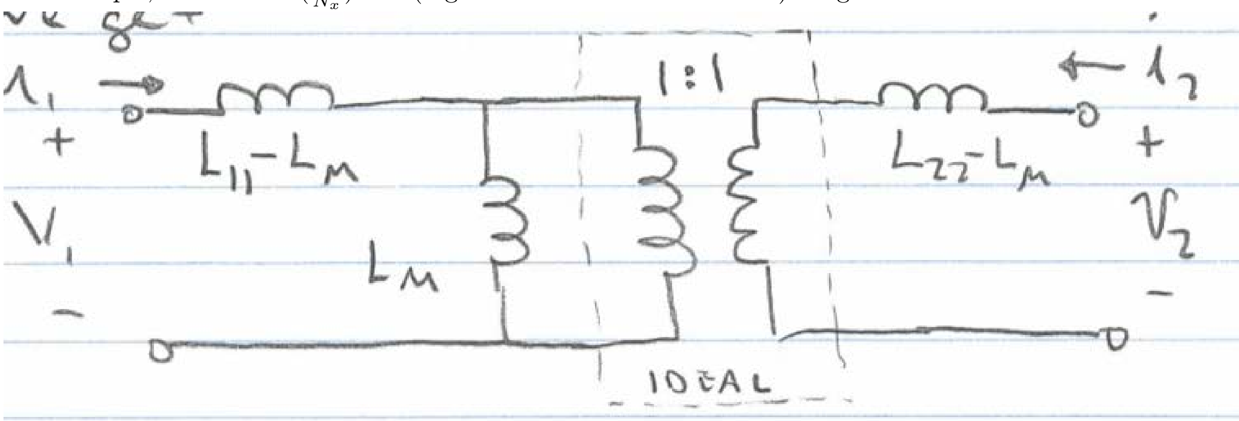
Our circuit model has 4 parameters: $L_{\mu_1}, L_{l1}, Ll2, \text{and} (\frac{N_2}{N_1})$. If we use the physical turns ratio, we get unique relations between the circuit model parameters and the inductance matrix parameters, and physical interpretations of their meanings.

However, if we don't know (or don't choose to use) the physical turns ratio, there are an infinite number of ways we can select the 4 circuit parameters to get the same terminal behavior.

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_\mu \\ L_\mu & L_{22} \end{bmatrix} \quad (2) \qquad \leftrightarrow$$



Choose one parameter arbitrarily, and solve for the other 3!
For example, if we select $(\frac{N_b}{N_x}) = 1$ (regardless of "real" turns ratio) we get



This gives us the <u>same</u> terminal behavior. We couldn't tell which one is "right" from v, i measurements, and can feel free to use whatever is most convenient in our design efforts.

$\Rightarrow$ many other useful circuilt models also exist.

Note: for a real magnetic system, there are physical limits on the parameter values.

In a 1-port system, $V = L\frac{di}{dt}$, we must have $L > 0$ (conservation of energy)

Similarly in a 2-port magnetic circuit (inverting the matrix)

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = L^{-1}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{L_{11}L_{22} - L_\mu^2}\begin{bmatrix} L_{22} & -L_\mu \\ -L_\mu & L_{11} \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \tag{3}$$

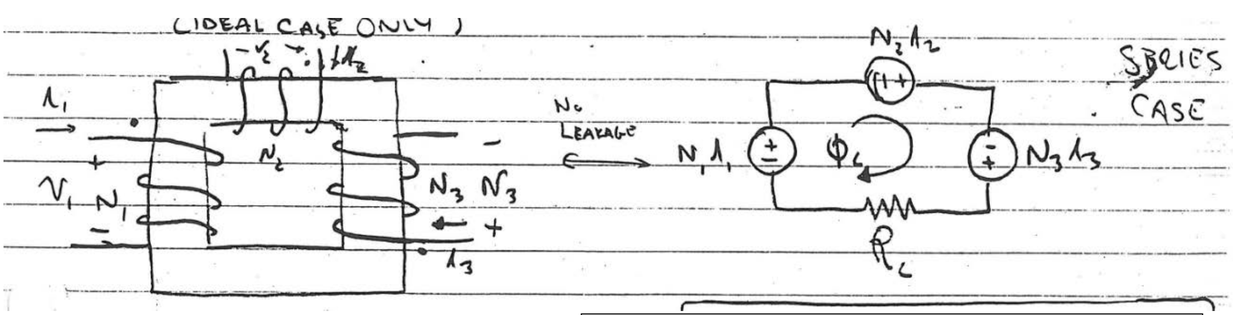
for a <u>real</u> system, we must have $|L_\mu| < \sqrt{L_{11}L_{22}}$

Otherwise, if $V_2 = 0$ (short) and we apply +v, $\frac{di_1}{dt} < 0$ and we would get energy out of the system forever!

In practice, $|L_\mu| = \sqrt{L_{11}L_{22}}$ is <u>perfect coupling</u> (no leakage)

Coupling coefficient $\kappa \triangleq \frac{L_\mu}{\sqrt{L_{11}L_{22}}}$, where $-1 < \kappa < 1$

Consider transformer with <u>more</u> than 2 windings: (ideal case only)



If $R_c \to 0$, no leakage ($\mu \to \infty$)

$$\boxed{N_1 i_1 + N_2 i_2 + N_3 i_3 = \Phi R_c = \oslash}$$

Also

$$v_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\Phi}{dt}$$

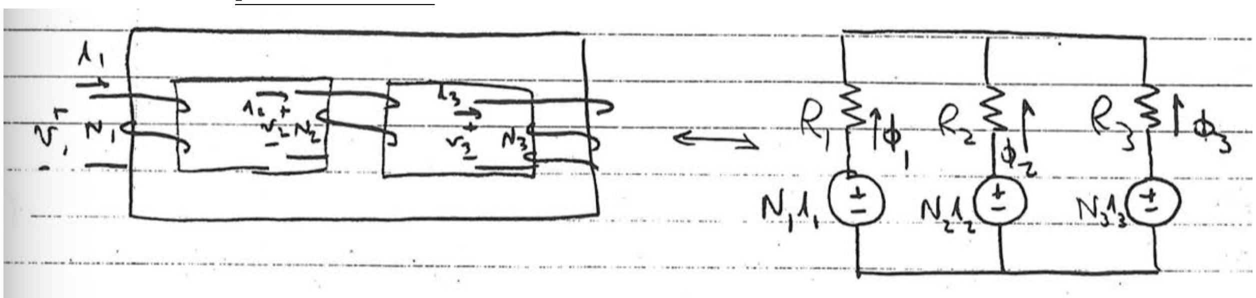
$$v_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\Phi}{dt}$$

$$v_3 = \frac{d\lambda_3}{dt} = N_3 \frac{d\Phi}{dt}$$

$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3}}$$

Note: as there is only one magnetic path, the dot convention is clear!

We can also have a parallel structure



If $\mu_c \to \infty, R_1, R_2, R_3 \to 0$

$\therefore$ from model, for $\Phi_1 - \Phi_3 < \infty$

$$\boxed{N_1 i_1 = N_2 i_2 = N_3 i_3}$$

Also

$$\Phi_1 + \Phi_2 + \Phi_3 = 0$$

$$\frac{\lambda_1}{N_1} + \frac{\lambda_2}{N_2} + \frac{\lambda_3}{N_3} = 0$$

$$\boxed{\therefore \frac{v_1}{N_1} + \frac{v_2}{N_2} + \frac{v_3}{N_3} = 0}$$

1. We get different relations in series + parallel cases
2. Parallel case: "Dot" convention is no longer a sufficient description
3. If we included nonidealities, we would get a 3 by 3 inductance matrix description

1 Multiwinding transformers (general case)

What happens as we add more windings? ⇒ we get larger <u>symmetric inductance matrices</u>

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \boxed{L_{11}} & L_\mu \\ \boxed{L_\mu} & \boxed{L_{22}} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} \quad (4)$$

⇒ 2 by 2 gives 3 <u>independent</u> parameters

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \boxed{L_{11}} & L_{12} \\ L_{13} & \\ \boxed{L_{21}} & \boxed{L_{22}} \\ L_{23} & \\ \boxed{L_{31}} & \boxed{L_{32}} \\ \boxed{L_{33}} & \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} \quad (5)$$

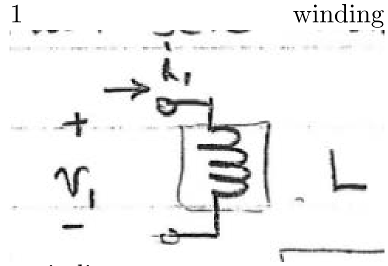
⇒ 3 by 3 gives 6 <u>independent</u> parameters

n-winding transformer ⇒ n by n gives $\frac{n(n+1)}{2}$ independent parameters. (Expanding from n-1 windings to n windigns adds n <u>more independent parameters</u>.)

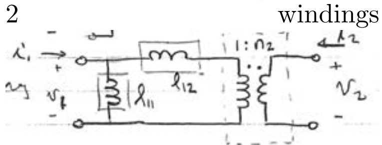
There are <u>many</u> circuit models that can capture the appropriate behavior. We will focus on the <u>extended cantilever model</u> as it

1. naturally gives the correct number of parameters for any number of windings
2. parameters can be readily extracted from device measurements (without severe numeral difficulties)

1 winding

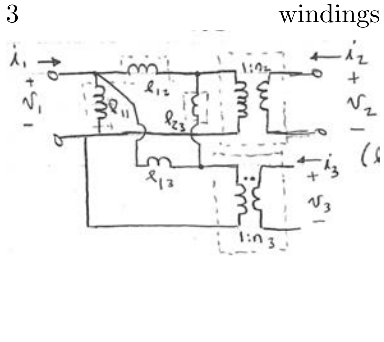
Cantilevel model:  (one parameter)

2 windings



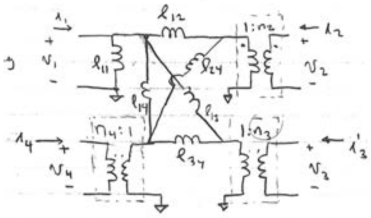
(like conventional model but chasing $L_{l_1} = 0$ and using other 3 as independent parameters)
adds 2 parameters = 3 params ($l_{11}, l_{12}, \frac{n_2}{1}$)

3 windings



adds 3 parameters (l_{13}, l_{23}, $\frac{n_3}{1}$) = 6 parameters

4 windings



4 new parameters ($\frac{n_4}{1}$, l_{14}, l_{24}, l_{34}) for <u>10</u> total

Generalizing from n-1 windings to n windings, we add:

- a transformer (turns ratio n_n)
- n-1 leakage inductances l_{mn} $m \in [1, n-1]$

for n new parameters (exactly the correct number of independent parameters!)

The correct values of the cantilever parameters can be extracted from straightforward measurements without high numerical sensitivity

1. l_{11} is found as inductance of port 1 with others open circuit (drive and measure voltage, current at port 1 with others open)
2. $n_2, ..., n_n$ apply a voltage source to port 1 and measure the scaling of voltages on other ports (with open circuit) (turns ratio is ratio of voltages)
3. l_{mn} drive a voltage on port m with all other ports shorted and measure current at port n (l_{mn} is determined for a specified ac frequency by the voltage to current ratio)

Note that there is a mapping between inductance matrix L elements $\{L_{ij}\}$, inverse inductance matrix $B = L^{-1}$ parameters $\{b_{ij}\}$ and the cantilever model parameters l_{ij}, n_j:

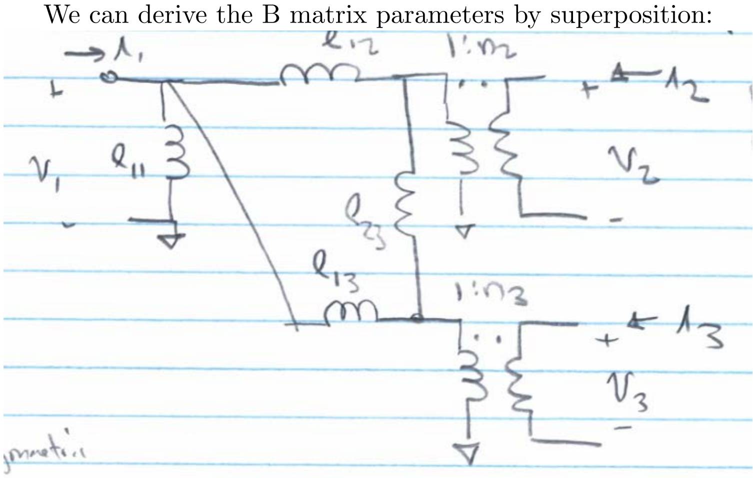
$$l_{11} = L_{11} \qquad n_j = \frac{L_{ij}}{L_{11}} \qquad l_{jk} = \frac{-1}{n_j n_k b_{jk}}$$

$$\leftrightarrow$$

$$b_{jk} = \frac{-1}{n_j n_k l_{jk}} j \neq k$$

$$b_{jj} = \frac{1}{n_j^2} \Sigma_{k=1}^{N} \frac{1}{l_{kj}}, l_{jj} = \begin{cases} \infty & \text{if } j \neq 1, \\ l_{11} & \text{if } j = 1. \end{cases}$$

We can derive the B matrix parameters by superposition:



Note: the PESC'98 paper has a typo in the equation for b_{jj}, missing a factor of $\frac{1}{n_j}$

$$i = B\lambda \text{ (where B is symmetric) : } \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \tag{6}$$

Superposition:

λ_1 only ($\lambda_2 = \lambda_3 = 0$)

$$i_1 = \frac{\lambda_1}{l_{11}} + \frac{\lambda_1}{l_{12}} + \frac{\lambda_1}{l_{13}}$$

$$i_2 = -\frac{\lambda_1}{n_2 l_{12}}$$

$$i_3 = -\frac{\lambda_1}{n_3 l_{13}}$$

λ_2 only ($\lambda_1 = \lambda_3 = 0$)

$$i_1 = -\frac{\lambda_2}{n_2 l_{12}}$$

$$i_2 = \frac{\lambda_2}{n_2^2 l_{12}} + \frac{\lambda_2}{n_2^2 l_{23}}$$

$$i_3 = -\frac{\lambda_2}{n_2 n_3 l_{23}}$$

λ_3 only ($\lambda_1 = \lambda_2 = 0$)

$$i_1 = -\frac{\lambda_3}{n_3 l_{13}}$$

$$i_2 = -\frac{\lambda_3}{n_2 n_3 l_{23}}$$

$$i_3 = \frac{\lambda_3}{n_3^2 l_{13}} + \frac{\lambda_3}{n_3^2 l_{23}}$$

For 3-winding case:

$$B = \begin{bmatrix} \frac{1}{l_{11}} + \frac{1}{l_{12}} + \frac{1}{l_{13}} & -\frac{1}{n_2 l_{12}} & -\frac{3}{n_3 l_{13}} \\ -\frac{1}{n_2 l_{12}} & \frac{1}{n_2^2 l_{12}} + \frac{1}{n_2^2 l_{23}} & -\frac{1}{n_3 l_{13}} \\ -\frac{3}{n_3 l_{13}} & -\frac{1}{n_3 l_{13}} & \frac{1}{n_3^2 l_{13}} + \frac{1}{n_3^2 l_{23}} \end{bmatrix} \tag{7}$$

Extending to n-winding:

$$b_{jj} = \frac{1}{n_j^2} \sum_{k=1}^{n} \frac{1}{l_{jk}} \text{ where we define } n_1 = 1, l_{jj} = 0 \text{ for } j \neq 1 \text{ and } l_{jk} = l_{kj}$$

$$\text{for j } \neq \text{ k } b_{jk} = -\frac{1}{n_j n_k l_{jk}}$$

We have $\lambda = Li$ where L is symmetric $L_{ji} = Lij$

From the inductance matrix, if we made $i_2 ... i_n = 0$ and only drive i_1:

$$\lambda_i = L_{11} i_1$$

$$\lambda_j = L_{j1} i_1 = L1ji_1, j \neq 1$$

From the cantilever circuit model with i_1 only and $i2 ... i_n = 0$:

$$\lambda_1 = l_{11} i_1$$

$$\lambda_j = n_j l_{11} i_1$$

By equating these we get

$$\boxed{l_{11} = L_{11} \qquad n_j = \frac{L_{1j}}{L_{11}}}$$

From the previous page we have

$$\boxed{l_{jk} = \frac{-1}{n_j n_k b_{jk}}, j \neq k}$$

So given inductance matrix parameters, we can determine the cantilever model parameters and vice versa, albeit with significant mathematical manipulations (e.g. matrix inversion).

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