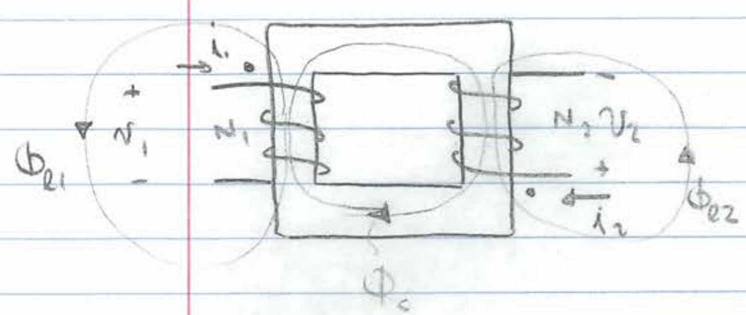
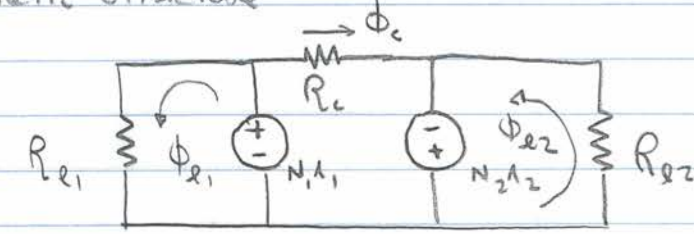
6.334 Lecture — Magnetics #3

Recall for a two-winding magnetic structure:





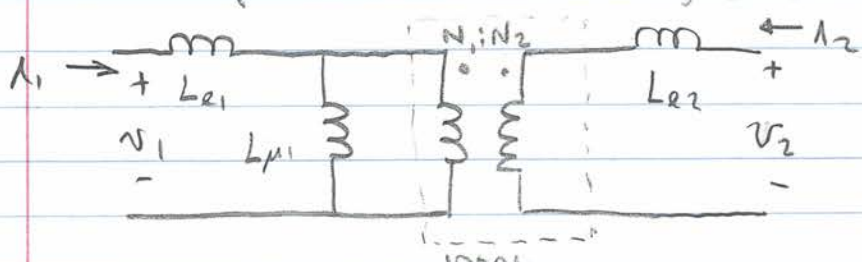
Note: ① Directions of MMF sources are determined by the winding orientations. (Note Dot convention)

② Electrical behavior at the winding terminals is determined by flux linkage + Lenz's law (λ is $N\phi$ from + side of MMF source)

Crunching the math we get:

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \frac{N_1^2}{R_c} + \frac{N_1^2}{R_{\ell 1}} & \frac{N_1 N_2}{R_c} \\ \frac{N_1 N_2}{R_c} & \frac{N_2^2}{R_c} + \frac{N_2^2}{R_{\ell 2}} \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \begin{bmatrix} \dot{i}_1 \\ \dot{i}_2 \end{bmatrix}$$

This corresponds to the following circuit model:



where $L_{\mu 1} = \frac{N_1^2}{R_c}$, $L_{\ell 1} = \frac{N_1^2}{R_{\ell 1}}$, $L_{\ell 2} = \frac{N_2^2}{R_{\ell 2}}$

Also, we get $L_{11} = L_{\mu 1} + L_{\ell 1}$, $L_{22} = \left(\frac{N_2}{N_1}\right)^2 L_{\mu 1} + L_{\ell 2}$, $L_M = \frac{N_2}{N_1} L_{\mu 1}$

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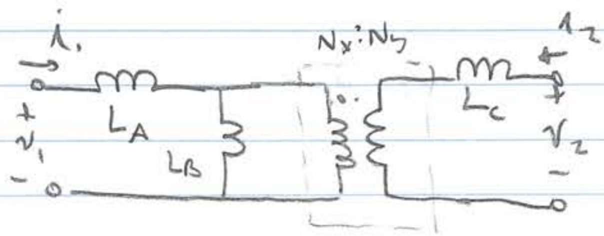
Note that the inductance matrix has three (quasi-) independent parameters L_{11}, L_{22}, L_M. These fully describe terminal behavior.

Our circuit model has 4 parameters: $L_{\mu 1}, L_{\ell 1}, L_{\ell 2}$ and $\left(\frac{N_2}{N_1}\right)$. If we use the physical turns ratio, we get unique relations between the circuit model parameters and the inductance matrix parameters, and physical interpretations of their meanings.

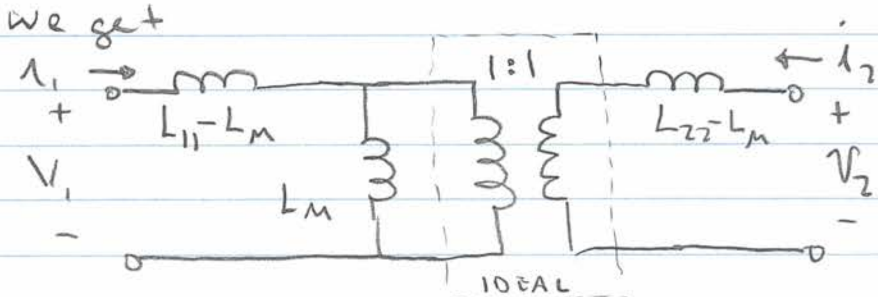
However, if we don't know (or don't choose to use) the physical turns ratio, there are an infinite # of ways we can select the 4 circuit parameters to get the same terminal behavior.

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_M \\ L_M & L_{22} \end{bmatrix} \longleftrightarrow$$



Choose one parameter arbitrarily, and solve for other 3 !

For example if we select $\left(\frac{N_y}{N_x}\right) = 1$ (regardless of "real" turns ratio) we get

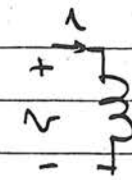


This gives us the same terminal behavior. We couldn't tell which one is "right" from v, i measurements, and can feel free to use whatever is most convenient in our design efforts.

$\Rightarrow$ many other useful circuit models also exist.

POWER ELECTRONICS NOTES - D. PERREAULT

Note: for a real magnetic system, there are physical limits on the parameter values.



In a 1-port system $V = L\frac{di}{dt}$, we must have $L > 0$
(cons. of energy)

Similarly in a 2-port magnetic circuit (inverting the matrix)

$$\frac{d}{dt}\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = L^{-1}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \frac{1}{L_{11}L_{22} - L_M^2}\begin{bmatrix} L_{22} & -L_M \\ -L_M & L_{11} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

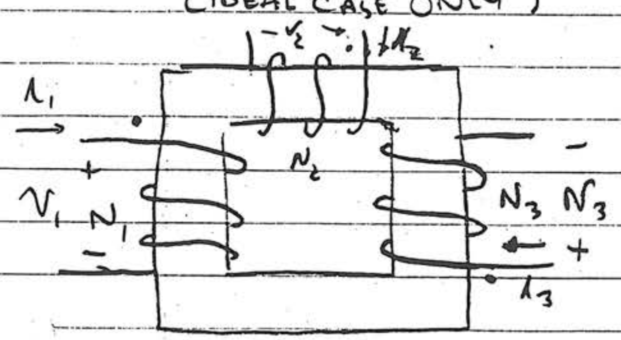
for a real system, we must have $|L_M| < \sqrt{L_{11}L_{22}}$.

Otherwise if $V_2 = 0$ (short) + we apply $+V_1$, $\frac{di_1}{dt} < 0$
and we would get energy out of the system forever!

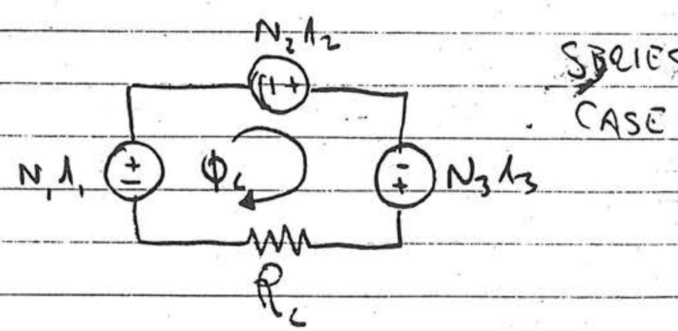
In practice, $L_M = \sqrt{L_{11}L_{22}}$ is perfect coupling (no leakage)

Coupling coeff. $K \triangleq \frac{L_M}{\sqrt{L_{11}L_{22}}}$ $-1 < k < 1$

★ CONSIDER TRANSFORMERS WITH MORE THAN 2 WINDINGS:
(IDEAL CASE ONLY)



No LEAKAGE



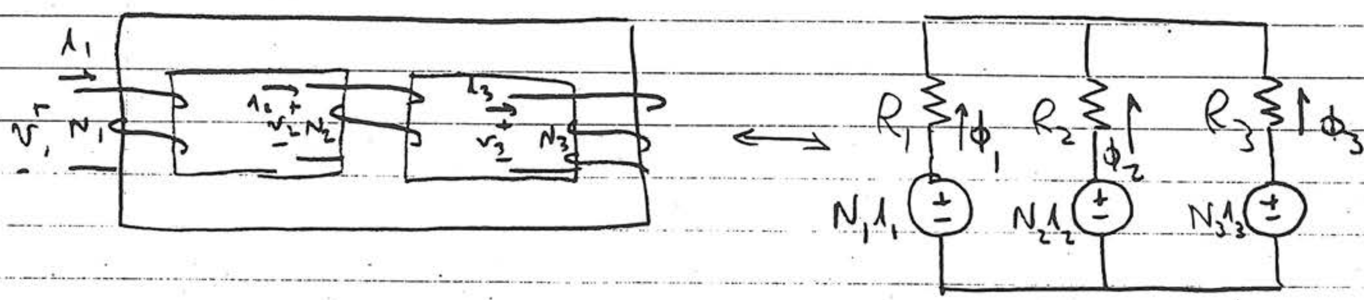
If $R_c \to 0$, no leakage $\boxed{N_1 i_1 + N_2 i_2 + N_3 i_3 = \phi R_c = 0}$
$(\mu \to \infty)$

POWER ELECTRONICS NOTES - D. PERREAULT

Also
$$v_1 = \frac{d\lambda_1}{dt} = N_1\frac{d\Phi}{dt}$$
$$v_2 = \frac{d\lambda_2}{dt} = N_2\frac{d\Phi}{dt}$$
$$v_3 = \frac{d\lambda_3}{dt} = N_3\frac{d\Phi}{dt}$$
$$\boxed{\frac{v_1}{N_1} = \frac{v_2}{N_2} = \frac{v_3}{N_3}}$$

Note: as there is only one magnetic path, the dot convention is clear!

WE CAN ALSO HAVE A PARALLEL STRUCTURE



If $\mu_c \to \infty$ $R_1, R_2, R_3 \to 0$

$\therefore$ from model, for $\phi_1 - \phi_3 < \infty$

$$\boxed{N_1\lambda_1 = N_2\lambda_2 = N_3\lambda_3}$$

Also $\Phi_1 + \Phi_2 + \Phi_3 = 0$

$$\frac{\lambda_1}{N_1} + \frac{\lambda_2}{N_2} + \frac{\lambda_3}{N_3} = 0$$

$$\therefore \quad \boxed{\frac{v_1}{N_1} + \frac{v_2}{N_2} + \frac{v_3}{N_3} = 0}$$

① WE GET DIFFERENT RELATIONS IN SERIES + PARALLEL CASES

② PARALLEL CASE: "DOT" CONVENTION IS NO LONGER A SUFFICIENT DESCRIPTION

③ If we included nonidealities, we would get a 3×3 inductance matrix description.

★ <u>multi-winding transformers</u> (general case)

What happens as we add more windings? ⇒ we get larger <u>symmetric</u> inductance matrices

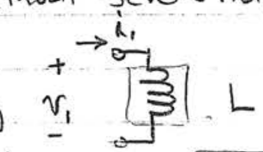
$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \boxed{L_{11}} & L_M \\ \boxed{L_M} & \boxed{L_{22}} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$ ⇒ 2×2 gives 3 <u>independent</u> parameters

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} \boxed{L_{11}} & L_{12} & L_{13} \\ \boxed{L_{21}} & \boxed{L_{22}} & L_{23} \\ \boxed{L_{31}} & \boxed{L_{32}} & \boxed{L_{33}} \end{bmatrix} \frac{d}{dt} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$ 3×3 gives 6 independent parameters

n-winding transformer ⇒ $n \times n$ gives $n(n+1)/2$ indep. params
(expanding from $n-1$ windings to n windings adds n <u>more</u> <u>indep. parameters</u>.)

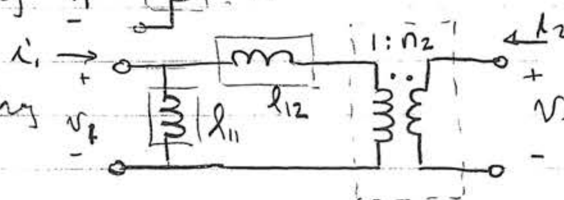
There are <u>many</u> circuit models that can capture the appropriate behavior. We will focus on the <u>Extended</u> <u>Cantilever Model</u> as it
① naturally gives the correct # of parameters for any # of windings
② parameters can be readily extracted from device measurements (without severe numerical difficulties)

cantilever model:

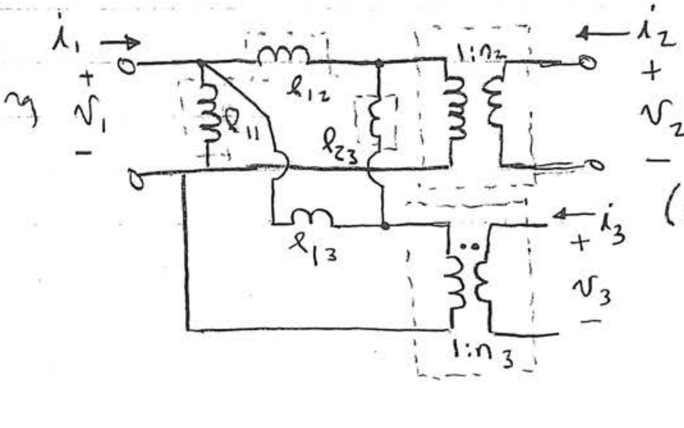
1 winding


(one parameter)

(like conventional model but choosing $L_{e1} = 0$ and using other 3 as independent parameters)

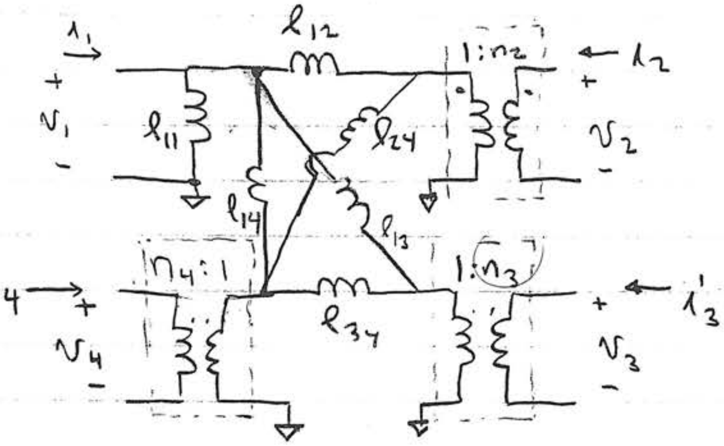
2 winding


adds 2 parameters = 3 params.
$(l_{11}, l_{12}, (\frac{n_2}{1}))$

3 winding


adds 3 parameters
$(l_{13}, l_{23}, (\frac{n_3}{1}))$
= 6 parameters

4-winding



4 new params ($\frac{n_4}{1}$, l_{14}, l_{24}, l_{34}) for 10 total.

Generalizing from $n-1$ windings to n windings, we add:
a transformer (turns ratio n_n)
$n-1$ leakage inductances l_{mn} $m \in [1, n-1]$
for n new parameters (exactly the correct # of independent params!)

The correct values of the cantilever parameters can be extracted from straightforward measurements without high numerical sensitivity

l_{11} is found as inductance of port 1 w/ others open circuit (drive & measure voltage, current at port 1 with others open)

$n_2, \dots, n_n$ apply a voltage source to port 1 and measure the scaling of voltages on other ports (with open circuit) (turns ratio is ratio of voltages.)

l_{mn} drive a voltage on port m with all other ports ~~port n~~ shorted and measure current at port n (l_{mn} is determined for a specified ac frequency by the voltage to current ratio)

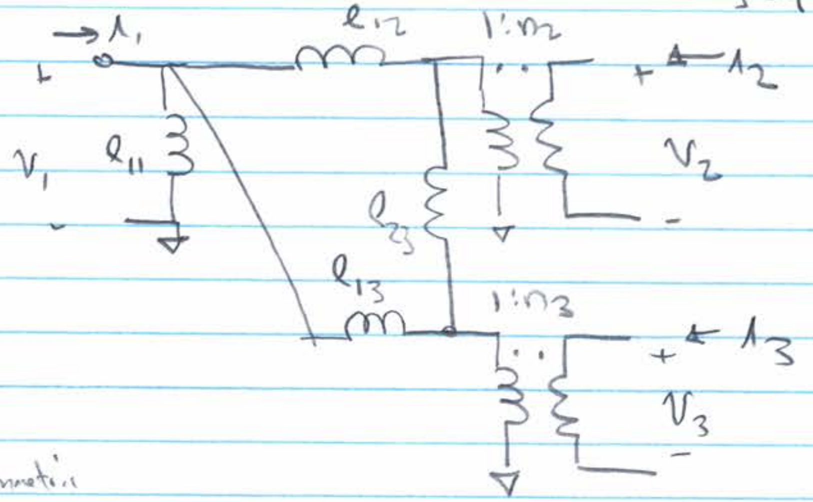
Note that there is a mapping between inductance matrix L elements $\{L_{ij}\}$, inverse inductance matrix $B = L^{-1}$ parameters $\{b_{ij}\}$ and the cantilever model params l_{ij}, n_j:

$$l_{11} = L_{11}$$
$$n_j = \frac{L_{1j}}{L_{11}}$$
$$l_{jk} = \frac{-1}{n_j n_k b_{jk}}$$

$\longleftrightarrow$

$$b_{jk} = \frac{-1}{n_j n_k l_{jk}} \quad j \neq k$$
$$b_{jj} = \frac{1}{n_j^2} \sum_{k=1}^{N} \frac{1}{l_{kj}}, \quad l_{jj} = \begin{cases} \infty & j \neq 1 \\ l_{11} & j = 1 \end{cases}$$

we can derive the B matrix parameters by superposition:



Note: the PESC'98 paper has a typo in the equation for b_{jj}, missing a factor of $\frac{1}{n_j}$

symmetric

$$i = B\lambda \;\therefore\; \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{12} & b_{22} & b_{32} \\ b_{13} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} \quad \text{where } b_{ij} = b_{ji}$$

superposition:

λ_1 only ($\lambda_2=\lambda_3=0$)	λ_2 only ($\lambda_1=\lambda_3=0$)	λ_3 only ($\lambda_1=\lambda_2=0$)
$i_1 = \frac{\lambda_1}{\ell_{11}} + \frac{\lambda_1}{\ell_{12}} + \frac{\lambda_1}{\ell_{13}}$	$i_1 = -\frac{\lambda_2}{n_2\ell_{12}}$	$i_1 = -\frac{\lambda_3}{n_3\ell_{13}}$
$i_2 = -\frac{\lambda_1}{n_2\ell_{12}}$	$i_2 = \frac{\lambda_2}{n_2^2\ell_{12}} + \frac{\lambda_2}{n_2^2\ell_{23}}$	$i_2 = -\frac{\lambda_3}{n_2 n_3\ell_{23}}$
$i_3 = -\frac{\lambda_1}{n_3\ell_{13}}$	$i_3 = -\frac{1}{n_2 n_3\ell_{23}}$	$i_3 = \frac{\lambda_3}{n_3^2\ell_{13}} + \frac{\lambda_3}{n_3^2\ell_{23}}$

for 3-winding case

$$B = \begin{bmatrix} \frac{1}{\ell_{11}} + \frac{1}{\ell_{12}} + \frac{1}{\ell_{13}} & -\frac{1}{n_2\ell_{12}} & -\frac{1}{n_3\ell_{13}} \\ -\frac{1}{n_2\ell_{12}} & \frac{1}{n_2^2\ell_{12}} + \frac{1}{n_2^2\ell_{23}} & -\frac{1}{n_2 n_3\ell_{23}} \\ -\frac{1}{n_3\ell_{13}} & -\frac{1}{n_2 n_3\ell_{23}} & \frac{1}{n_3^2\ell_{13}} + \frac{1}{n_3^2\ell_{23}} \end{bmatrix}$$

Extending to n-winding:

$$b_{jj} = \frac{1}{n_j^2}\sum_{k=1}^{n}\frac{1}{\ell_{jk}}$$

where we define $n_1 = 1$, $\frac{1}{\ell_{jj}} = 0$ for $j \neq 1$ and $\ell_{jk} = \ell_{kj}$

for $j \neq k$ $\quad b_{jk} = -\frac{1}{n_j n_k \ell_{jk}}$

we have $\lambda = L\dot{i}$ where L is symmetric $L_{ij} = L_{ji}$

from the inductance matrix, if we make $\dot{i}_2 \ldots \dot{i}_n = 0$ and only drive $\dot{i}_1$:

$$\lambda_1 = L_{11}\dot{i}_1$$
$$\lambda_j = L_{j1}\dot{i}_1 = L_{1j}\dot{i}_1 \quad j \neq 1$$

From the cantilever circuit model with $\dot{i}_1$ only and $\dot{i}_2 \ldots \dot{i}_n = 0$:

$$\lambda_1 = \ell_{11}\dot{i}_1$$

$$\lambda_j = n_j \ell_{11}\dot{i}_1$$

by equating these we get

$$\boxed{\begin{aligned} \ell_{11} &= L_{11} \\ n_j &= \frac{L_{1j}}{L_{11}} \end{aligned}}$$

from the previous page we have

$$\boxed{\ell_{jk} = -\frac{1}{n_j n_k b_{jk}} \quad j \neq k}$$

So given inductance matrix parameters we can determine the cantilever model parameters and vice versa, albeit with significant mathematical manipulations (e.g. matrix inversion)