6.622 Power Electronics *Prof. David Perreault*

Lecture 17 - Inverters 1

Fourier Series Review

Break up a periodic signal as a sum of harmonically-related sin + cosine terms (or as a sume of complex exponentials)

$$f(t) = \frac{b_0}{2} + \Sigma_{n=1}^{\infty} a_n sin(n\omega_0 t) + b_n cos(n\omega_0 t)$$

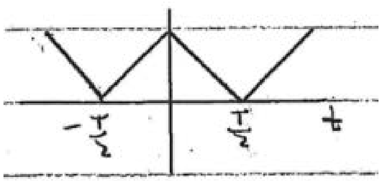
Coefficients can be calculated as:

$$a_n = \frac{2}{T} \int_{<T>} f(t) sin(n\omega_0 t) dt$$

$$b_n = \frac{2}{T} \int_{<T>} f(t) cos(n\omega_0 t) dt$$

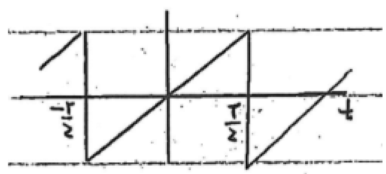
Some waveforms have special characteristics:

Even x(t)=x(-t)



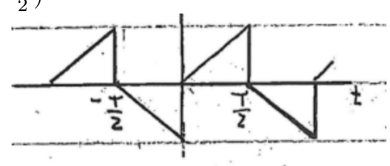
No sin terms a_n's = 0
Calculating a_n's are integrals of even function times (odd) sines, result is odd ∴ integral = 0

Odd x(t)=-x(-t)



No cos terms b_n's = 0
Calculating b_n's are integrals of odd function times (even) cosines, result is odd ∴ integral = 0

Half-wave symmetry x(t) = -x(t-$\frac{T}{2}$)



No even harmonics a_{2k}'s, b_{2k}'s = 0 where k is an integer
Calculating a_{2k}, b_{2k}'s are integrals where first half of integral exactly cancels 2nd half ∴ integral = 0

Aside: waveform decomposition

Note that we can decompose any waveform into even and odd components, or alternatively into "Half-wave symmetric" and "Half-wave repeating" components:

$$x(t) = x_{ev}(t) + x_{odd}(t)$$

where

$$x_{ev}(t) = \frac{x(t) + x(-t)}{2}$$

$$x_{odd}(t) = \frac{x(t) - x(-t)}{2}$$

x_{ev} has only the b_n terms of the fourier decomposition of $x(t)$, including b_0
x_{odd} has only the a_n terms of the fourier decomposition of x(t)

or

$$x(t) = x_{hws}(t) + x_{hwr}(t)$$

where

$$x_{hws}(t) = \frac{x(t) - x(t - \frac{T}{2})}{2}$$

$$x_{hwr}(t) = \frac{x(t) + x(t - \frac{T}{2})}{2}$$

x_{hws} is half-wave symmetric and has only the odd numbered fourier components of x(t), a_{2k+1}, b_{2k+1} for integer k.
x_{hwr} is "half-wave repeating" and comprises only the even numbered fourier components of x(t), a_{2k}, b_{2k} for integer k. X_{hwr} repeats every half-cycle of x(t)

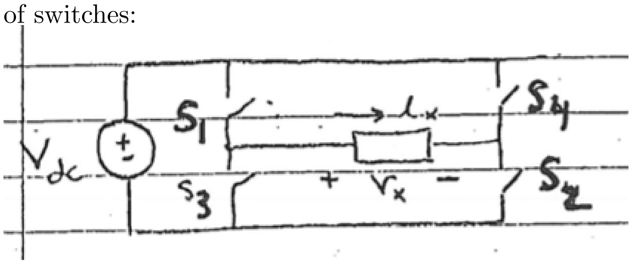
We can use these s to sort out different portions of a waveform content in manners that can be useful, and for thinking about how we might synthesize waveforms having desired properties (e.g. having no even harmonics)

Note that even and odd portions of a given waveform x(t) are orthogonal to each other, as are the HWS and HWR components:

$$\frac{1}{T}\int_{<T>} x_{ev}(t)x_{odd}(t) = 0 \qquad \frac{1}{T}\int_{<T>} x_{HWS}(t)x_{HWR}(t) = 0$$

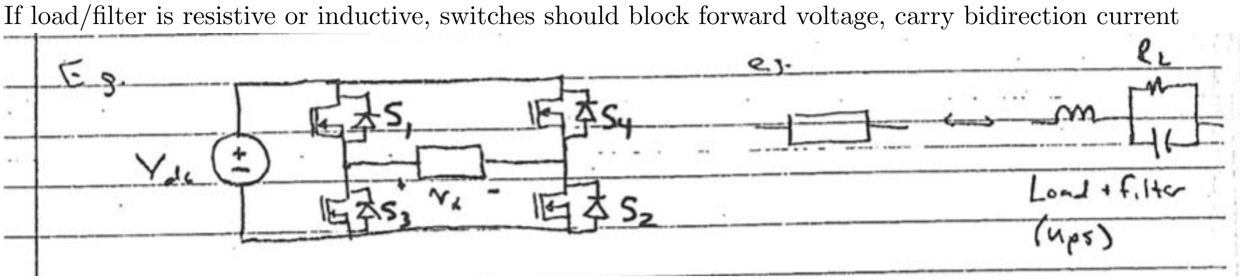
Inverter structure

Suppose one wants to create an AC waveform from a dc source. This can be accomplished with a bridge of switches:

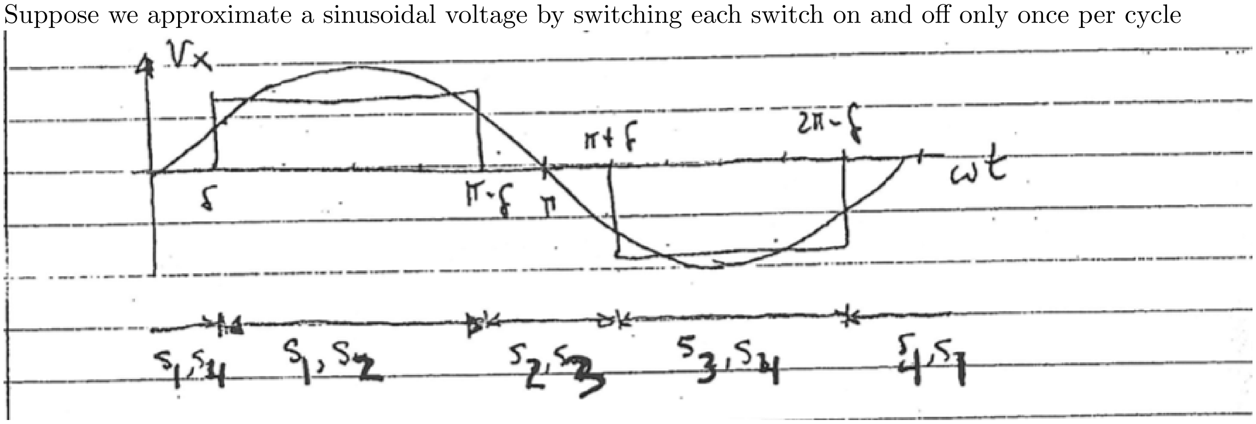


Switches on	v_x
S_1, S_2	$+v_{dc}$
S_3, S_4	-v_{dc}
S_1, S_4	0
S_2, S_3	0

If load/filter is resistive or inductive, switches should block forward voltage, carry bidirection current



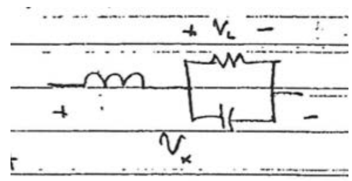
Suppose we approximate a sinusoidal voltage by switching each switch on and off only once per cycle



- v_x is odd (no b_k terms; synthesize sine wave)
- v_x is half-wave symmetric (no even harmonics, a_{2k}, b_{2k} terms = 0)

$$V_x(t) = \sum_{\text{n odd}} V_n sin(n\omega_0 t)$$

Ex. @ f = 0 → square wave $V_x(t) = \Sigma_{n,odd} \frac{2v_{dc}}{\pi n} sin(n\omega_0 t)$
fundamental, 3rd, 5th, 7th, etc...



If load filters out harmonics
V_L more pure than V_x, but difficult since harmonics are so close in frequency

What can we control by varying f?

1. Fundamental magnitude
2. Harmonic magnitudes

1. Control of fundamental

$$V_1 = \frac{2}{2\pi} \int_{<2\pi>} V_x(t) sin(\omega t) d(\omega t)$$
$$= \frac{2}{\pi} \int_f^{\pi - f} V_{dc} sin(\phi) d\phi$$

$$\boxed{V_1 = \frac{4V_{dc}}{\pi} cos(f)}$$

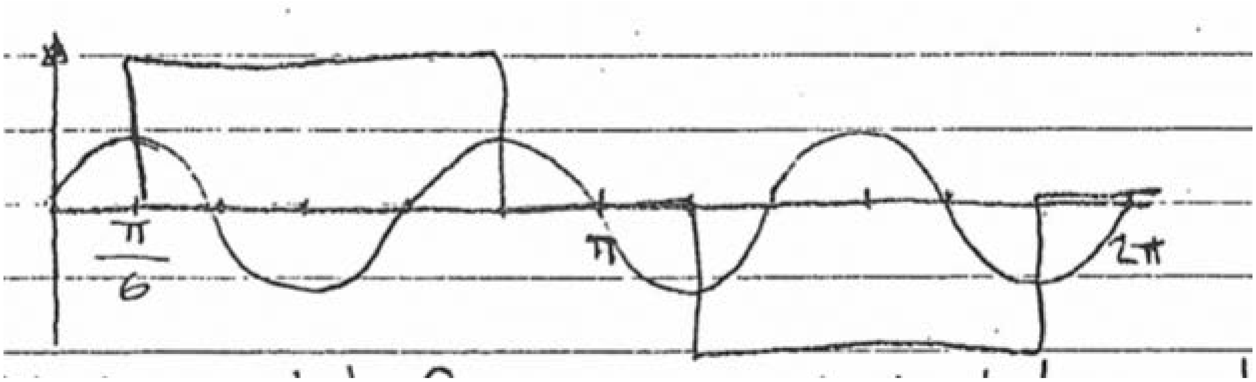
→ by control of f we can control the fundamental magnitude.

2. Can also control harmonics:

$$V_3 = \frac{2}{2\pi} \int_{<2\pi>} V_x(t) sin(w\omega t) d\omega t$$
$$= \frac{2v_{dc}}{\pi} \int_f^{\pi - f} sin(3\phi) d\phi$$
$$= \frac{2V_{dc}}{3\pi} cos(3\phi)|_{\pi - f}^{f}$$
$$= \frac{2V_{dc}}{2\pi} [cos(3f) - cos(3\pi - 3f)]$$
$$= \frac{4v_{dc}}{3\pi} cos(3f)$$

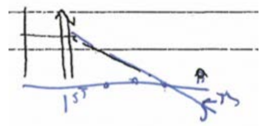
So if we choose f = $\frac{pi}{6} = 20°, V_3 \rightarrow 0$!
Then lowest harmonics will be the 5th (easier to filter)
However, we cannot control harmonics and fundamentals at the same time.
(Note: this value of f turns out to eliminate <u>all</u> triples (triple-n, 2n) harmonics! Thus we will have 5th, 7th, 11th, 13th)

This case



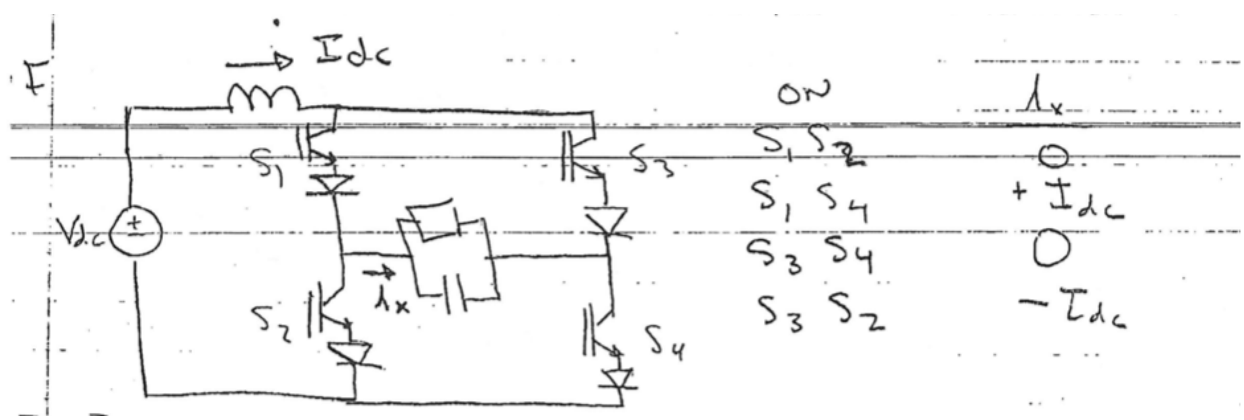
Any multiple of this frequency will also balance out to cancel exactly!

⇒ eliminating 3rds makes it easier to filter

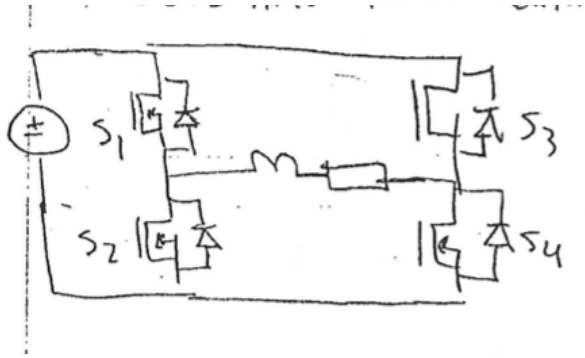


Note that there are other structures that can implement similar modulation and filtering:
Consider the topological dual

- Instead of dc voltage, use dc current (place large inductor on dc-side)
- instead of inductive filter, use capacitive filter (in parallel w/ loads)
- Instead of switches (carry bidirectional i → carry unidirection i), (carry unidirectional v → carry bidirection v)
- switch control is different:



Time
"Dead time" in switching different



In VSI:
S_1 off <u>before</u> S_2 on to avoid shorting V_{dc} $\Rightarrow$ "dead time" during which antiparallel diodes conduct

In CSI must turn S_1 on before S_3 off to prevent open circuits I_{dr}. Series switch diodes will pick up blocking of output.