SHOW APC INVERTER!



6.334 Lecture Notes — Inverters 1

## Fourier Series Review

Break up a periodic signal as a sum of harmonically-related sin + cosine terms (or as a sum of complex exponentials)

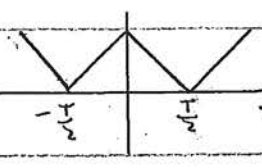
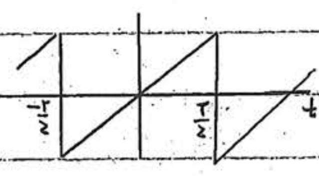
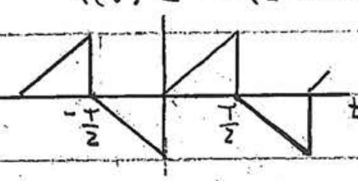
$$f(t) = \frac{b_0}{2} + \sum_{n=1}^{\infty} a_n \sin(n\omega_0 t) + b_n \cos(n\omega_0 t)$$

Coefficients can be calculated as:

$$a_n = \frac{2}{T}\int_{\langle T\rangle} f(t)\sin(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{T}\int_{\langle T\rangle} f(t)\cos(n\omega_0 t)\,dt$$

Some waveforms have special characteristics.

| Even $x(t) = x(-t)$ | Odd $x(t) = -x(-t)$ | Half-wave symmetry $x(t) = -x(t - T/2)$ |
|---|---|---|
|  |  |  |
| No sin terms $a_n$'s $= 0$ | No cos terms $b_n$'s $= 0$ | No even harmonics $a_{2k}$'s, $b_{2k}$'s $= 0$ $k \in I$ |
| calculating $a_n$'s are integrals of even fn. times (odd) sines; result is odd $\therefore$ integral $= 0$ | calculating $b_n$'s are integrals of odd fn times (even) cosines; result is odd $\therefore$ integral $= 0$ | Calculating $a_{2k}$, $b_{2k}$'s are integrals where 1st half of integral exactly cancels 2nd half $\therefore$ integral $= 0$ |

## Aside: Waveform decomposition {skip in lecture}

Note that we can decompose any waveform into even and odd components, or alternatively into "Half-wave Symmetric" and "Half-wave repeating" components:

$$x(t) = x_{ev}(t) + x_{odd}(t)$$

where $\begin{cases} x_{ev}(t) = \dfrac{x(t) + x(-t)}{2} \\ x_{odd}(t) = \dfrac{x(t) - x(-t)}{2} \end{cases}$

$x_{ev}$ has only the $b_n$ terms of the fourier decomposition of $x(t)$, including $b_0$

$x_{odd}$ has only the $a_n$ terms of the fourier decompostion of $x(t)$

or

$$x(t) = x_{hws}(t) + x_{hwr}(t)$$

where $x_{hws} = \dfrac{x(t) - x(t - T/2)}{2}$

$x_{hwr} = \dfrac{x(t) + x(t - T/2)}{2}$

$x_{hws}$ is half-wave symmetric and has only the odd numbered fourier components of $x(t)$, $a_{2k+1}, b_{2k+1}$ for $k \in I$

$x_{hwr}$ is "half-wave repeating" and comprises only the even numbered fourier components of $x(t)$, $a_{2k}$, $b_{2k}$ for $k \in I$. $x_{hwr}$ repeats every half-cycle of $x(t)$

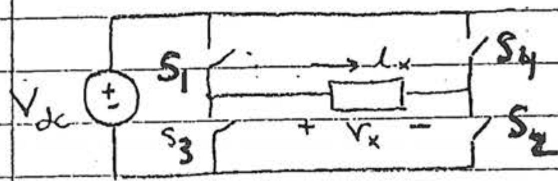
$\boxed{\text{We can use these decompositions to sort out different portions of a waveform content in manners that can be useful, and for thinking about how we might synthesize waveforms having desired properties (e.g. having no even harmonics)}}$

Note that even and odd portions of a given waveform $x(t)$ are orthogonal to each other, as are the HWS and HWR components:

$$\frac{1}{T}\int_{\langle T \rangle} x_{ev}(t)\, x_{odd}(t) = 0 \qquad \frac{1}{T}\int_{\langle T \rangle} x_{HWS} \cdot x_{HWR}\, dt = 0.$$

## Inverter Structure

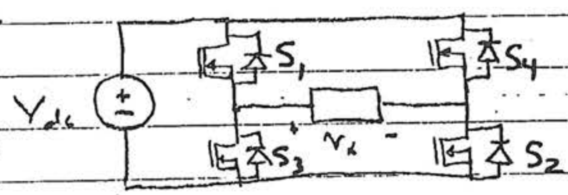
Suppose one wants to create an ac waveform from a dc source. This can be accomplished with a bridge of switches:



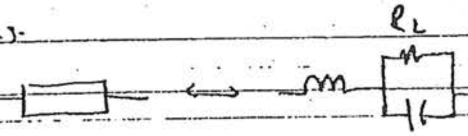

| Switches on | $V_x$ |
|---|---|
| $S_1, S_2$ | $+V_{dc}$ |
| $S_3, S_4$ | $-V_{dc}$ |
| $S_1, S_4$ | 0 |
| $S_2, S_3$ | 0 |

If load / filter is resistive or inductive, switches should block forward voltage + carry bidirectional current

E.g.

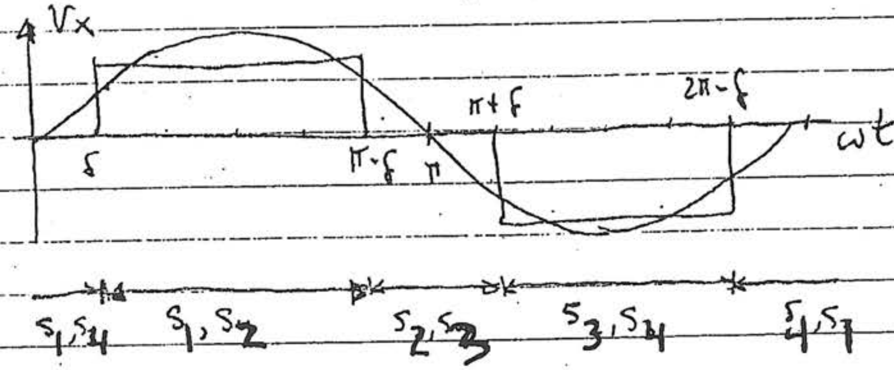



e.g.




Load + filter (ups)

Suppose we approximate a sinusoidal voltage by switching each switch on and off only once per cycle




- $V_x$ is odd (no $b_k$ terms; synthesizing sine wave)
- $V_x$ is half-wave symmetric (no even harmonics $a_{2k}, b_{2k}$ terms $= 0$)

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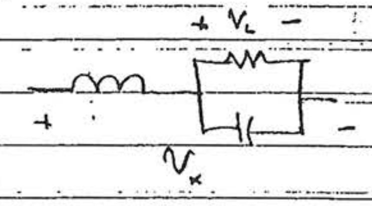
$$V_x(t) = \sum_{n \text{ odd}} V_n \sin(n\omega_o t)$$

Ex/ @ $\delta = 0$ → square wave $V_x(t) = \sum_{n, \text{odd}} \frac{4V_{dc}}{\pi n} \sin(n\omega_o t)$

fundamental, 3rd, 5th, 7th etc

If load filters out harmonics




$V_L$ more pure than $V_x$, but difficult since harmonics are so close in frequency

What can we control by varying $\delta$?

1. Fundamental Magnitude
2. Harmonic Magnitudes

1. Control of fundamental

$$V_1 = \frac{2}{2\pi} \int_{\langle 2\pi \rangle} V_x(t) \sin(\omega t)\, d(\omega t)$$

$$= \frac{2}{\pi} \int_{\delta}^{\pi - \delta} V_{dc} \sin(\varphi)\, d\varphi$$

$$\boxed{V_1 = \frac{4V_{dc}}{\pi} \cos(\delta)}$$

→ by control of $\delta$ we can control the fundamental magnitude.

2. Can also control harmonics:

$$V_3 = \frac{2}{2\pi} \int_{\langle 2\pi \rangle} V_x \sin(3\omega t)\, d\omega t$$

$$V_3 = \frac{2V_{dc}}{\pi}\int_{\delta}^{\pi-\delta} \sin(3\varphi)\,d\varphi$$

$$= \frac{2V_{dc}}{3\pi}\cos(3\varphi)\Big|_{\pi-\delta}^{\delta}$$

$$= \frac{2V_{dc}}{3\pi}\left[\cos(3\delta) - \cos(3\pi - 3\delta)\right]$$

$$= \frac{4V_{dc}}{3\pi}\cos(3\delta)$$

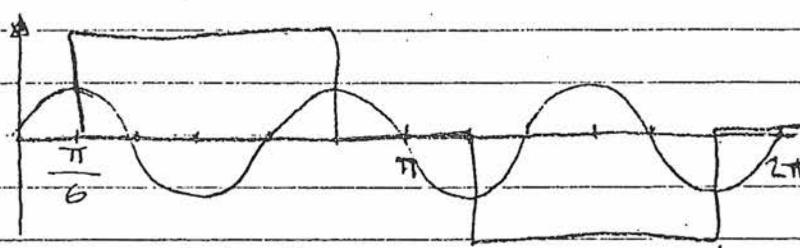
So if we choose $\delta = \frac{\pi}{6} = 30^\circ$, $V_3 \to 0$!

Then lowest harmonic will be the $5^{th}$ (easier to filter)

however, we cannot control harmonics + fundamental at the same time.

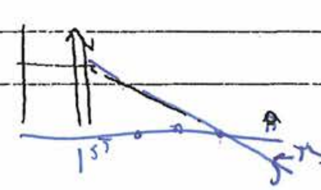
(note: this value of $\delta$ turns out to eliminate all triplen (triple-n, 3n) harmonics! Thus we will have $5^{th}$, $7^{th}$, $11^{th}$, $13^{th}$

This case




any multiple of this frequency will also balance out to cancel exactly!

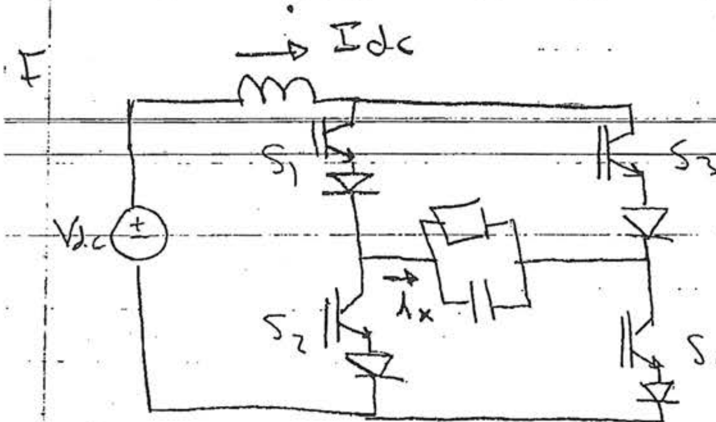
⇒ ELIMINATING 3RD MAKES IT EASIER TO FILTER



Note that there are other structures that can implement similar modulation + filtering:

CONSIDER THE TOPOLOGICAL DUAL

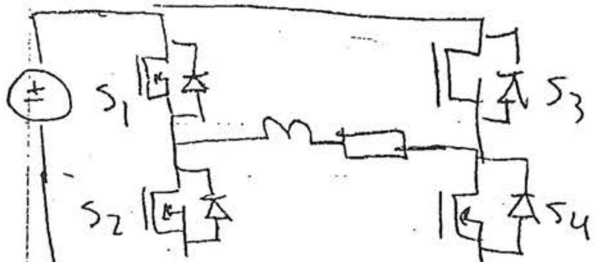
- instead of dc voltage, use dc current (place large inductor on dc-side)
- instead of inductive filter, use capacitive filter (in parallel w/ load)
- Instead of switches · carry bidirectional $i$, · block unidirectional $v$ ⇒ carry unidirectional $i$, block bidirectional $v$
- switch control is different:




| ON | $i_x$ |
| --- | --- |
| $S_1\ S_2$ | 0 |
| $S_1\ S_4$ | $+I_{dc}$ |
| $S_3\ S_4$ | 0 |
| $S_3\ S_2$ | $-I_{dc}$ |

TIME

- "DEAD TIME" IN SWITCHING DIFFERENT




IN VSI:
$S_1$ OFF BEFORE $S_2$ ON TO AVOID SHORTING $V_{DC}$.
⇒ "DEAD TIME" during which antiparallel diodes conduct

In CSI MUST TURN $S_1$ ON before $S_3$ off to prevent open circuiting $I_{dc}$. series switch diodes will pick up blocking of output.