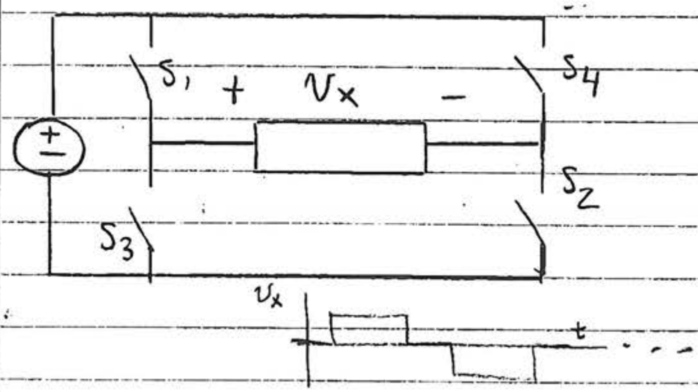
6.334 Lecture Notes — Inverters #2

FROM LAST TIME:

1Φ VSI inverter bridge structure



SWITCHES ON	V_x
$S_4 S_1$	0
$S_1 S_2$	$+V_{DC}$
$S_2 S_3$	0
$S_3 S_4$	$-V_{DC}$

- WE CAN SYNTHESIZE PULSING VOLTAGES $+, 0, -$ over time.
- Filter to get desired output waveform (e.g., sinusoid)
- Switching frequency limitations are often a practical constraint

★ ⇒ SHOW INVERTER MODULE FROM A TOYOTA PRIUS AS EXAMPLE

To reduce filtering requirements, we often use waveform symmetries to reduce unwanted content

$$f(t) = \frac{b_0}{2} + \sum_{n=1}^{\infty} a_n \sin(n\omega_0 t) + b_n \cos(n\omega_0 t)$$

$$a_n = \frac{2}{T}\int_{\langle T\rangle} f(t)\sin(n\omega_0 t)\,dt\ ,\quad b_n = \frac{2}{T}\int_{\langle T\rangle} f(t)\cos(n\omega_0 t)\,dt$$

→ for odd/even waves, synthesize odd/even patterns

⇒ for half-wave symmetric outputs $f(t) = -f(t-T/2)$ synthesize half-wave symmetric pulse patterns (a_{2k}, b_{2k} harmonics do not exist!)

NOTE: WE CAN EXTRACT COMPONENTS IN TIME

$$\underbrace{f_{HWS}}_{\text{HALF-WAVE SYMMETRIC}} = \frac{f(t)-f(t-T/2)}{2}\ ;\quad \underbrace{f_{HWR}}_{\text{HALF-WAVE REPEATING}} = \frac{f(t)+f(t-T/2)}{2}\ ;\quad f(t) = f_{HWS} + f_{HWR}$$

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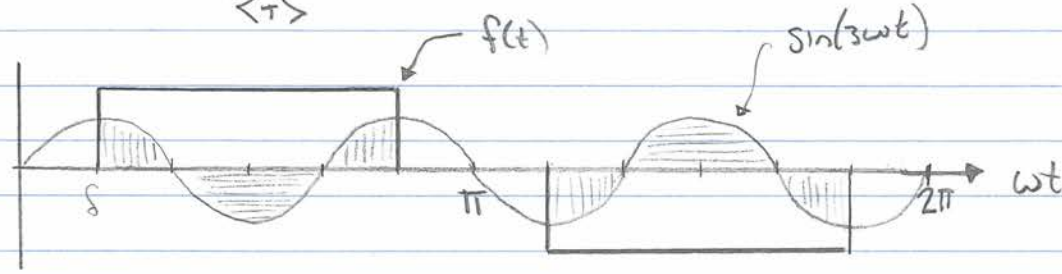
we have seen that by control of the width of one pulse per half cycle (and each device switching on/off once per full cycle) we can

- maintain half-wave symmetry (no even harmonics)
- eliminate triple-n harmonics (especially 3rd harmonic
- keep odd symmetry (no cosine components)

why? with only odd components, n^{th} harmonic amplitude is:

$$V_n = \frac{2}{T}\int_{\langle T\rangle} f(t)\sin(n\omega_o t)\,dt$$

$\delta = \frac{\pi}{6}$



Positive area of $f(t)\sin(3\omega t)$ cancels negative area $\therefore$ integral $= 0$!

⇒ we can introduce more notches in each half cycle (for more switching transitions in each cycle) so that higher harmonics (e.g. 5^{th} are cancelled) while not disturbing the nulling of the 3^{rd}. (see one example, next page)

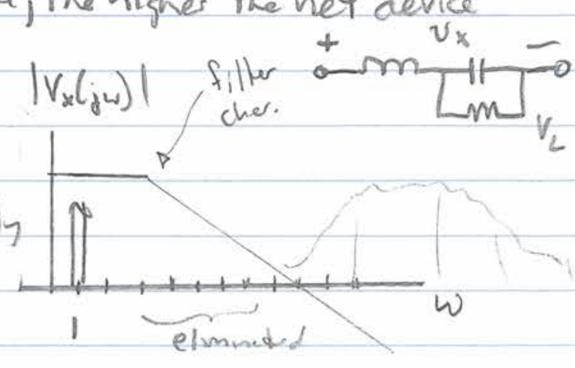
In general: Harmonic Elimination / Programmed PWM

→ can eliminate one odd harmonic for each pulse per half cycle (and each switching transition per a cycle)

→ more harmonics eliminated, the higher the net device switching frequency

→ precise timing required (μP)

→ As more low-order harmonics are eliminated, high-order harmonics actually increase, but these are more easily filtered!



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Note: see classic papers for more on this area:

1. H. S. Patel and R. G. Hoft, "Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters: Part I—Harmonic Elimination Techniques," *IEEE Trans. Industry Applications* IA-9 (3): 310–317 (May/June 1973).
2. H. S. Patel and R. G. Hoft, "Generalized Techniques of Harmonic Elimination and Voltage Control in Thyristor Inverters: Part II—Voltage Control Techniques," *IEEE Trans. Industry Applications* IA-10 (5): 666–673 (September/October 1974).

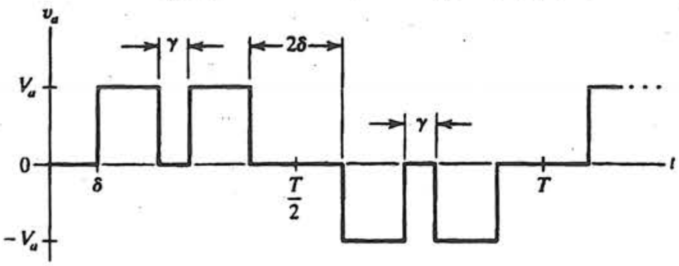

Figure 8.5 A tristate waveform containing notches of width γ.

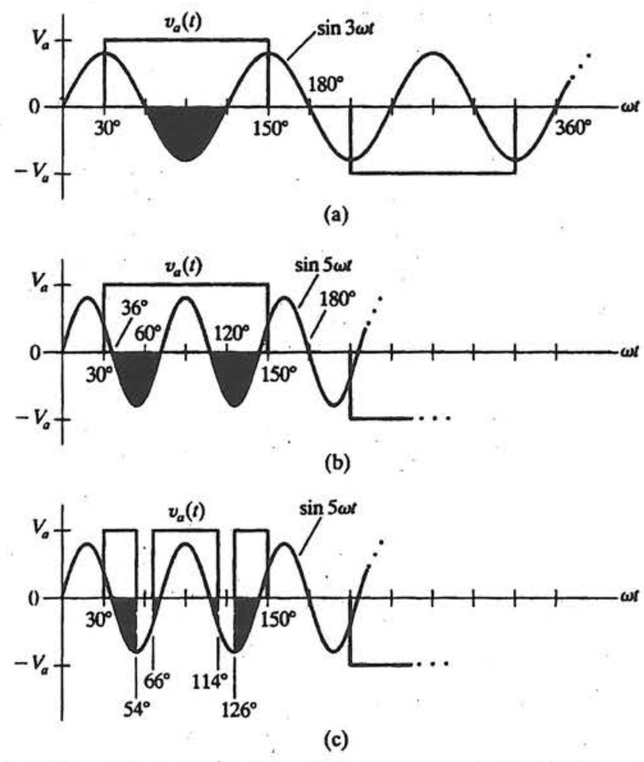

Figure 8.6 (a) Graphic representation of the integrand of *(8.14)*. The positive (lightly shaded) and negative (heavily shaded) areas are equal, canceling the third harmonic of the square wave. (b) The third-harmonic free waveform of (a) superimposed on a fifth-harmonic sine wave. (c) The square wave notched so that it is free from both third and fifth harmonics. A fifth-harmonic sine wave illustrates that the product of it and $v_a(t)$ has zero net area.

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★ Harmonic Cancellation

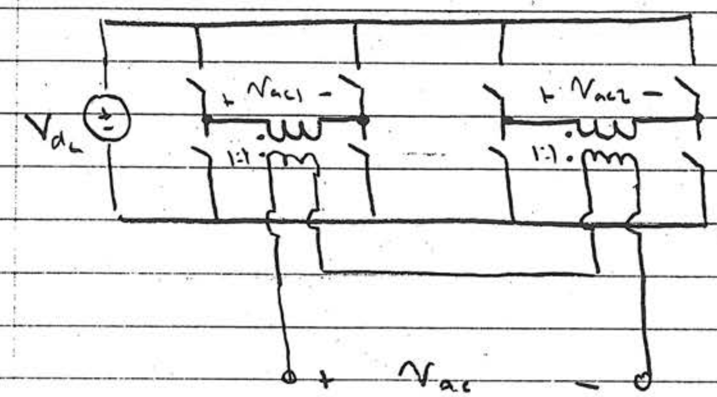
Add up time-shifted waveforms to cancel desired harmonic component(s)

If $x(t) = \sum A_n \sin(n\omega_o t + \phi_n)$

$\therefore x(t-t_1) = \sum A_n \sin(n\omega_o(t-t_1) + \phi_n)$

$= \sum A_n \sin(n\omega_o t + \underbrace{\phi_n - n\omega_o t_1}_{\phi_n'})$

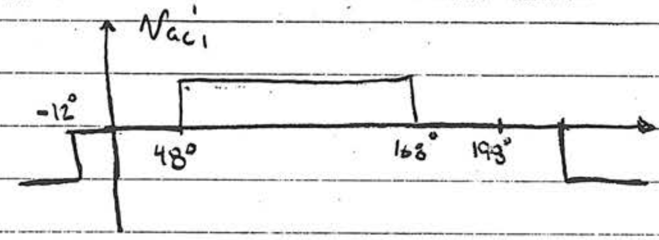
If we time-shift to change the fundamental by an angle $\Delta\theta_1 = -\omega_o t_1$, we shift the n^{th} harmonic by an angle $\Delta\phi_n = n\Delta\phi_1$



Sum 2 waveforms

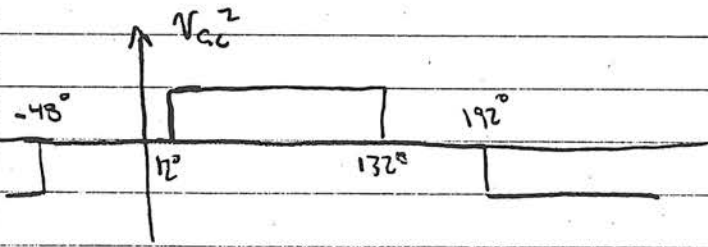
→ shift fundamentals by 36° (±18°)

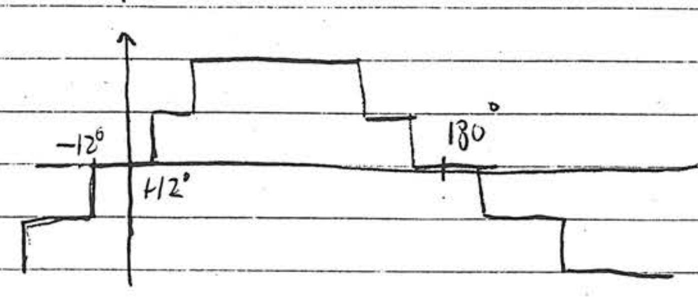
→ shifts 5th harmonics by 180° (±90°)



1st harm → shifted by +18°

Summed wave has no 5th harmonic





no third, 5th harmonic

by adding waveforms time shifted so that their 5th harmonics are 180° out of phase, the 5th harmonic is cancelled in the summed waveform!

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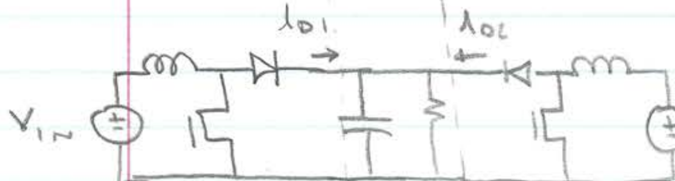
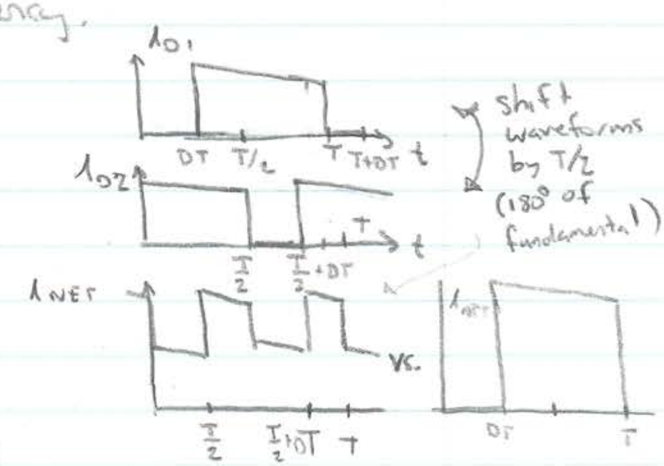
Harmonic Cancellation and Harmonic Elimination can also be applied in other forms of power converters.

For example, in dc-dc conversion, we have a desired frequency (dc) and undesired components (all ac ripple). We can suppress these, reducing ripple content + increasing fundamental ripple frequency.

e.g. Interleaving for cancellation





★ With two converters, fundamental ripple
1. frequency doubles!
2. Also, p-p current ripple (net) is half that of a single high-power unit.

We can interleave N identical converters by phase-shifting them by $\Delta t = T/N$ ($\omega_1 \Delta t = 2\pi/N$). The net ripple frequency in the input and output waveforms will ideally be at N times the individual switching frequency!

→ This trick is very widely used, including in the converters for most PC power supplies feeding the final low voltage to the microprocessor