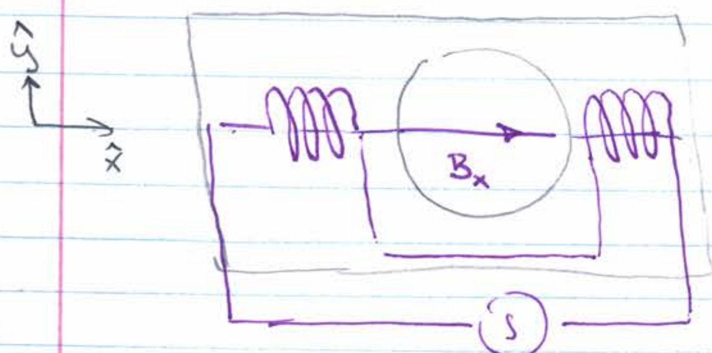
6.334 3-phase systems #1

The origins of polyphase

Suppose we wanted to build an ac machine where we wanted to use magnetic fields (or flux) from a winding on the stator to generate forces on the rotor;



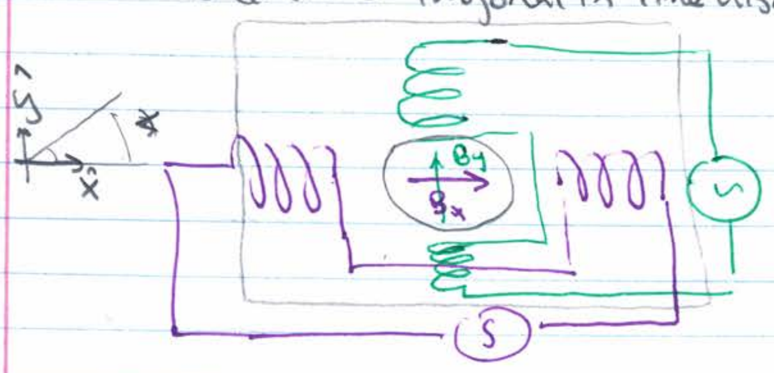
with an appropriate sinusoidal drive we'd get

$$\vec{B} = B_o \cos(\omega t)\,\hat{x}$$

This could generate a force on a rotor (e.g. to align with $\vec{B}$) (e.g. by magnetic poles attracting)

Unfortunately, if the rotor were already aligned with the $\hat{x}$ direction, we'd get no force or turning.

What if we added another winding orthogonal in space and made its drive orthogonal in time also?



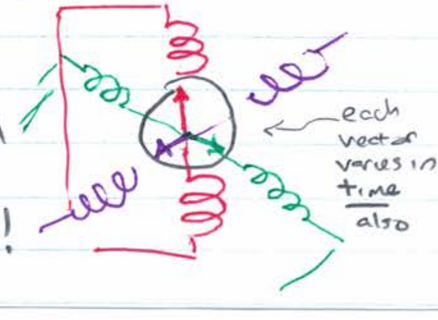
$$\hat{B} = \underline{B_o \cos(\omega t)}\,\hat{x} + \underline{B_o \sin(\omega t)}\,\hat{y}$$

$$|B| = B_o$$

$$\measuredangle B = \mathrm{ATAN}\left(\frac{\sin(\omega t)}{\cos(\omega t)}\right) = \omega t$$

★ we get a magnetic field (or flux) of constant magnitude that rotates in space! The rotor will be pulled rotationally at all conditions!

This type of characteristic was one that motivated early polyphase work (Tesla and others). Instead of 2 orthogonal phases, we also get this with 3 phases spaced in time + phase by $120°$!



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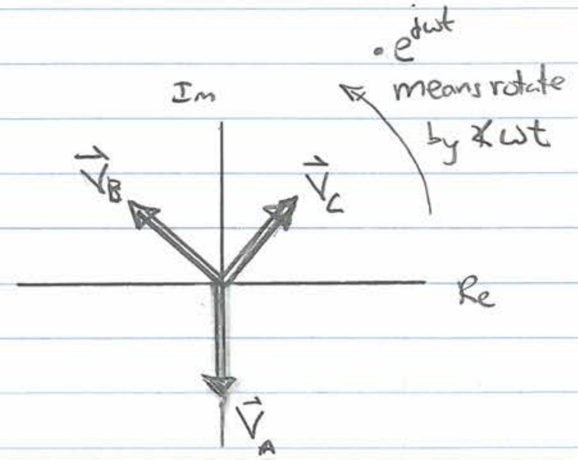
A three-phase voltage set is three sources separated by $120°$ in phase ($\frac{2\pi}{3}$ radians). The voltages sum to zero at all times.

e.g.
$$v_a = V_s \sin(\omega t) = Re\{\underbrace{V_s e^{-j\frac{\pi}{2}}}_{\vec{V}_A} \cdot e^{j\omega t}\}$$
$$v_b = V_s \sin(\omega t - \tfrac{2\pi}{3})$$
$$v_c = V_s \sin(\omega t + 2\pi/3)$$

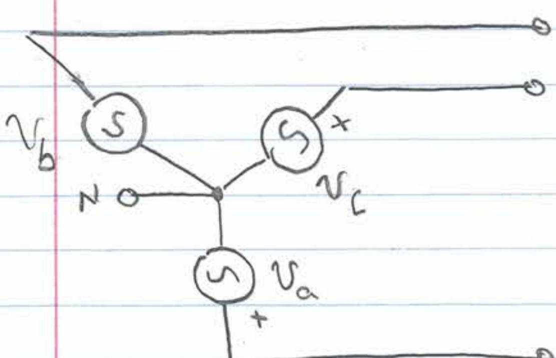
We can represent these as phasors

$$v_x(t) = Re\{\vec{V}_x e^{j\omega t}\}$$

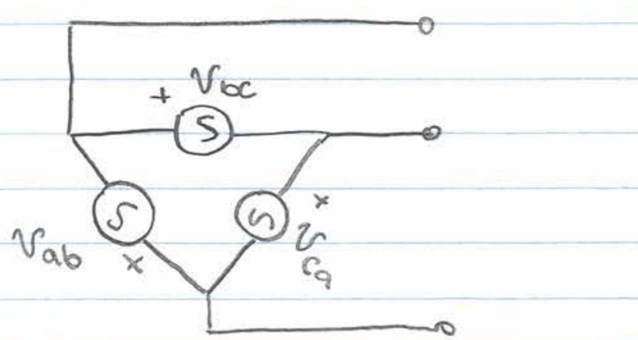


Connection of 3ϕ sources

Most common : Y connection



Δ connection (no neutral)



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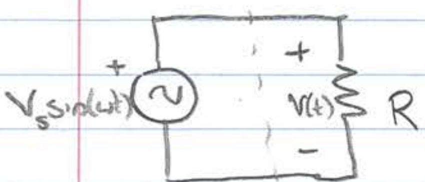
Why 3ϕ ?

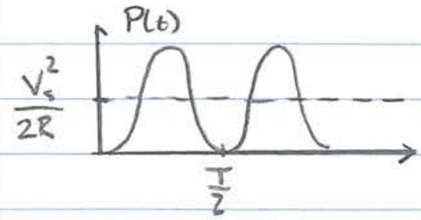
1. Consider power sourcing capability of 1ϕ (best case @ unity P.F.)



$$P(t) = \frac{v(t)^2}{R} = \frac{V_s^2}{R}\sin^2(\omega t)$$

$$= \frac{V_s^2}{2R}\left[1 - \cos(2\omega t)\right]$$

$T = \frac{1}{\omega}$

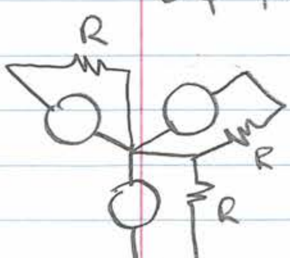


Even at unity power factor we get power pulsations between 0 and 2× average power at twice the line frequency. This makes sense, as we cannot draw power when $V \rightarrow 0$. This is bad for supplying power to machines, electronics, etc

→ torque pulsations in machine

→ $2 \times f_{line}$ energy storage needed to maintain const electronic load power

3ϕ power solves this fluctuation problem!



$$P_{TOTAL} = \frac{V_s^2}{R}\left[\sin^2(\omega t) + \sin^2\left(\omega t - \frac{2\pi}{3}\right) + \sin^2\left(\omega t + \frac{2\pi}{3}\right)\right]$$

$$= \frac{V_s^2}{2R}\left[3 - \underbrace{\cos(2\omega t) - \cos\left(2\omega t - \frac{4\pi}{3}\right) - \cos\left(2\omega t + \frac{4\pi}{3}\right)}_{\text{This } 3\phi \text{ set always adds to zero}}\right]$$

$$\boxed{P_{TOTAL} = \frac{3V_s^2}{2R}}$$

We can source constant power into a balanced 3ϕ load w/ no energy storage.

* Two-phase can also do this, as $\sin^2(\varphi) + \cos^2(\varphi) = 1$

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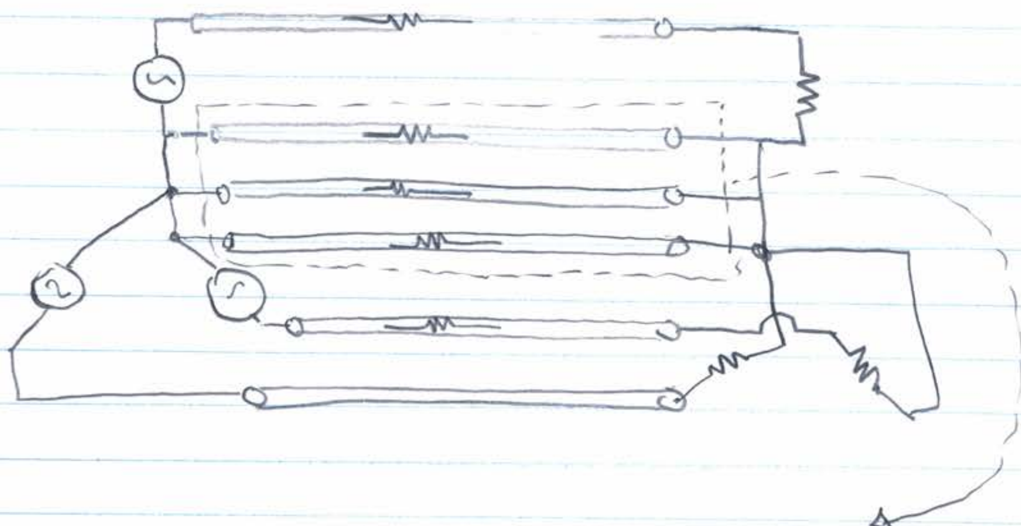
Consider distribution loss benefits of 3ϕ vs. 1ϕ or 2ϕ: If the load is far from the source



- Because the 3 phase voltages add to zero, if the loads are balanced (equal) the net currents in the return lines add to zero! Ideally, then we can remove the return lines (saving loss + $). At minimum we can use a single small return line to carry only the "imbalance" current (3-phase, 4-wire). This doesn't happen with 2-phase, since voltage vectors don't add to zero.

- Just as voltages (+currents) may add to zero in polyphase, Transformer **fluxes** add to zero with 3 phase legs, and we can build 3-phase transformers that are smaller and/or more efficient than 3 1-ϕ transformers would be

- Similar benefits can be applied to help improve electric machines

$\Rightarrow$ 3ϕ provides decided benefits compared to both 1ϕ and 2ϕ

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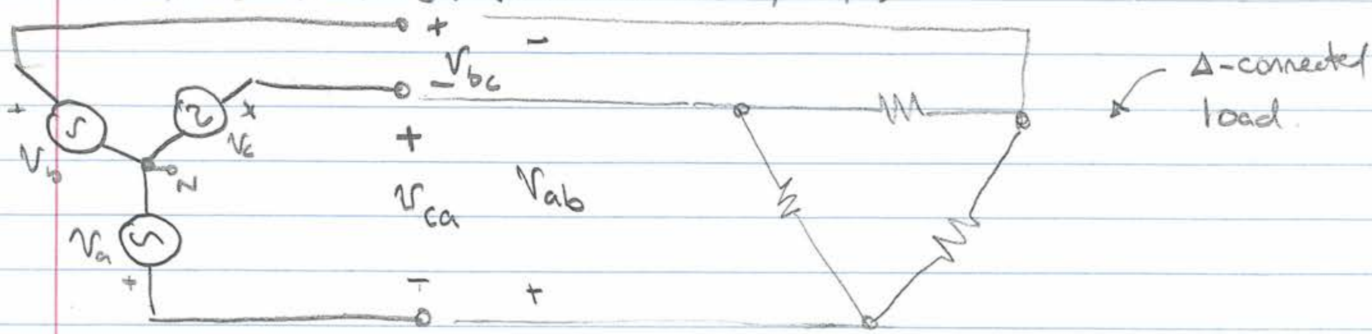
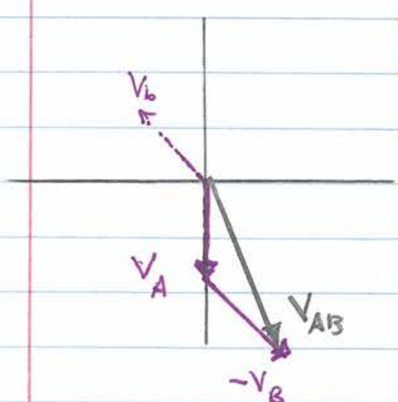
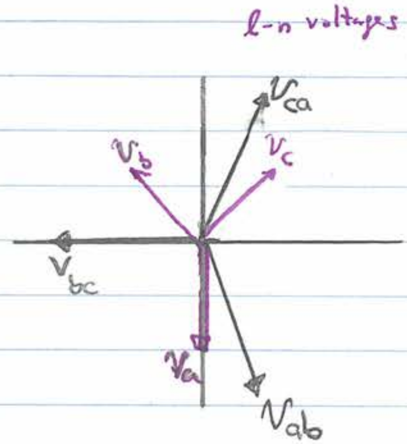
(2) With a 3ϕ set we can form a balanced 3ϕ load on line-to-line. (cannot do this w/ 2ϕ)



we can find line-to-line voltages from vector analysis

$$v_{ab} = v_a - v_b$$
$$= \sqrt{3}\, V_s \sin(\omega t + \frac{\pi}{6})$$





- l-l voltages are $\sqrt{3}$ larger in magnitude than l-n (120V l-n → 208V l-l)
- l-l voltages are shifted by $\frac{\pi}{6}$ (30°) from l-n voltages.

This can be handy depending on the desired load characteristic

★ Also, we can create balanced voltage sets with ANY phase shift by appropriate summing of voltages (e.g. w/ transformers)

⇒ Very handy for some designs.

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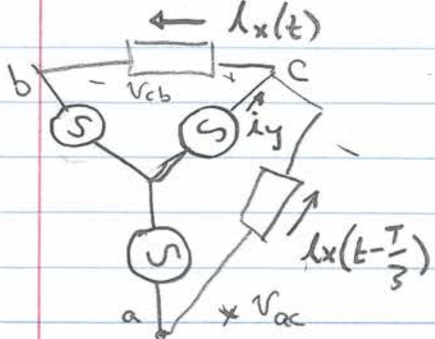
(3) 3ϕ systems allow cancellation of all triplen harmonics (harmonics that are multiples of 3 × fundamental frequency)

How? Consider a "Δ" connected load on a "Y" source



if load is balanced, l-l currents are the same but shifted in time by $T/3$

$$\therefore\ i_y = i_x(t) - i_x(t - T/3)$$

$T/3$ is $120°$ @ fundamental

$\therefore$ $3n$ harmonics of $i_x(t)$ dissappear in i_y by harmonic cancellation

That is, if $f(t) = \sum a_n \sin(n\omega t + \phi_n)$

$$f(t) - f(t - T/3) = \sum_n a_n \left[\underbrace{\sin(n\omega t + \phi_n) - \sin(n\omega t + \tfrac{nT}{3} - \phi_n)}_{\text{content cancels for } n \text{ any multiple of } 3} \right]$$

Since the current waveforms have fundamentals that differ by $120° = 2\pi/3$ radians, they are time shifted by $T/3$, and the triple-n harmonic content gets cancelled.

$\Rightarrow$ No even harmonics, 3n's gone $\Rightarrow$ $5^{th}, 7^{th}, 11^{th}, 13^{th} \ldots$ left