

Lecture 22 - 3Φ Systems 2

1 Review

Three-phase set separated by 120°

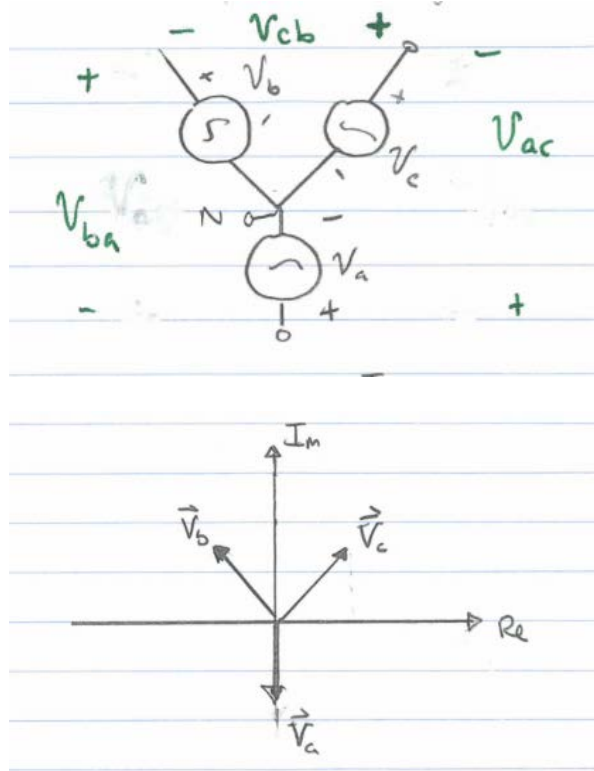
$$v_a = V_s \sin(\omega t)$$

$$v_b = V_s \sin(\omega t - \frac{2\pi}{3})$$

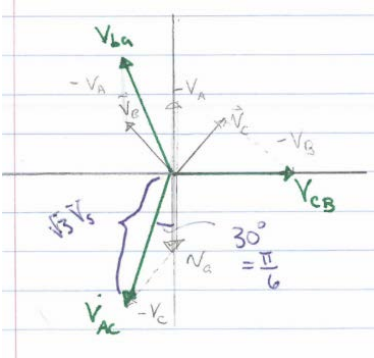
$$v_c = V_s \sin(\omega t + \frac{2\pi}{3})$$

Phasor notation:

$$v_x(t) = \text{Re} \left[\vec{V}_x e^{j\omega t} \right]$$



We can take vector differences to get line-line waveforms



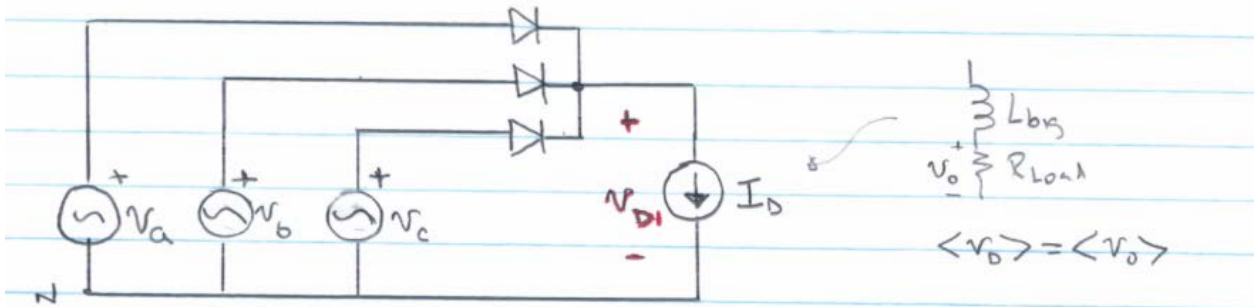
- $\sqrt{3}$ increase in magnitude for l-l
- 30° phase shift for l-l

So $120 V_{rms} \text{ l-n} \leftrightarrow 208 V_{rms} \text{ l-l} \Rightarrow$ Distribute 3Φ 208 l-l and break off l-n taps for different 1Φ branches

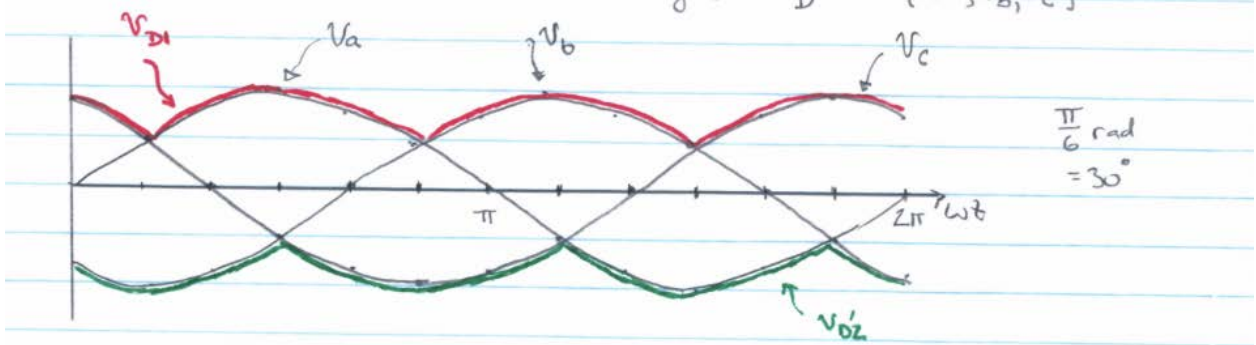
We can create sinusoids of any magnitude + phase by using transformers to select + add up parts of different l-n voltages! (as simple as vector addition...)

2 Consider diode rectification of 3-phase

The equivalent "half-wave" rectification is:

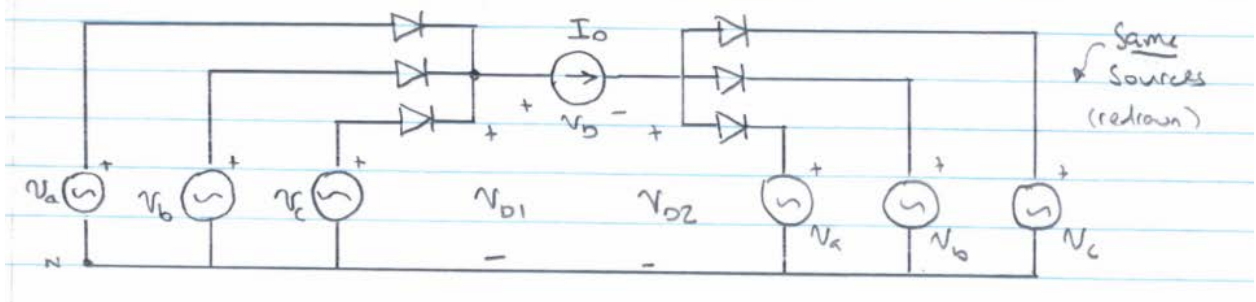


v_D is the diode "or" of the three line-to-neutral voltages $v_D = \max[v_a, v_b, v_c]$



If we had reversed the diodes, we'd get $v_{DZ} = \min[v_a, v_b, v_c]$

For "full wave" or "bridge" rectification, use 6 diodes for max + min:

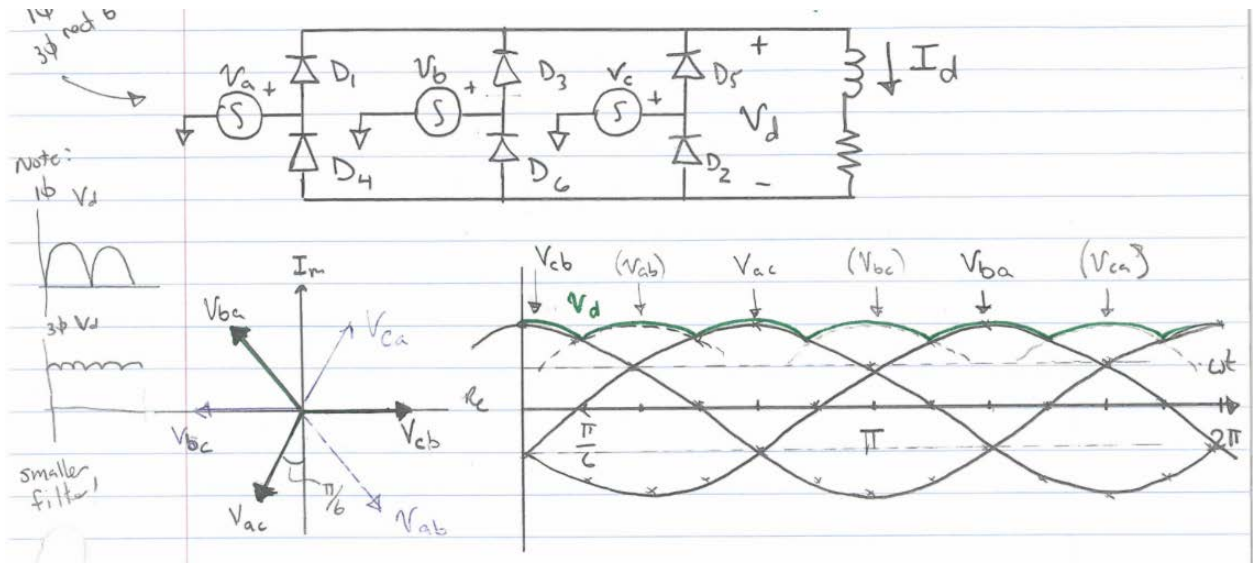


$$v_d = v_{D1} - v_{D2} = \max[v_a, v_b, v_c] - \min[v_a, v_b, v_c]$$

* this is the same as $v_d = \max[v_{ab}, v_{ac}, v_{bc}, v_{ba}, v_{ca}, v_{cb}]$

The 3Φ full-bridge rectifier can be drawn as follows:

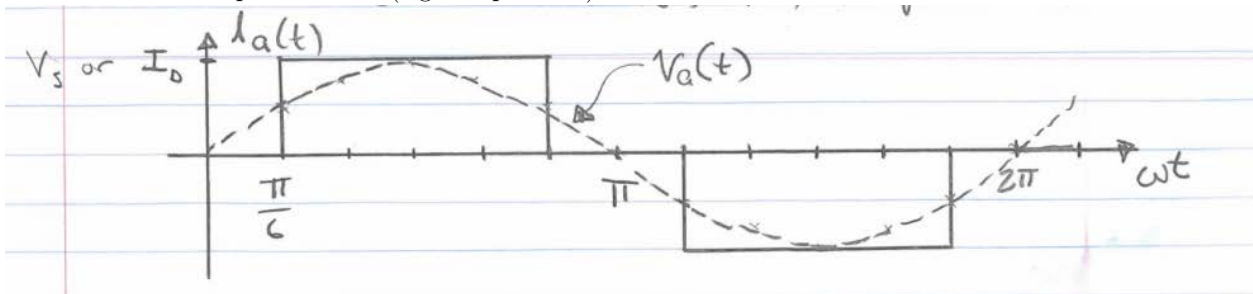
Note: 1Φ red 4 diodes, 3Φ red 6 diodes!



- v_d is $\max[v_{ab}, v_{ac}, v_{bc}, v_{ba}, v_{ca}, v_{cb}]$
- Phases named such that successive l-n maxima are $v_a, v_b, v_c, v_a, v_b, v_c, \dots$ so l-n max goes $v_{ab}, v_{ac}, v_{bc}, v_{ba}, v_{ca}, v_{cb}, \dots$
- Diodes numbered such that conduction sequence is: $\frac{D_6}{D_1}, \frac{D_1}{D_2}, \frac{D_2}{D_3}, \frac{D_3}{D_4}, \frac{D_4}{D_5}, \frac{D_5}{D_6}, \frac{D_6}{D_1}, \dots$

To simplify tracking behavior.

Consider waveforms + power factor (e.g. for phase A)



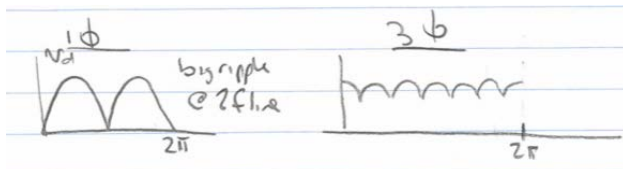
$$V_{a,rms} = \frac{V_s}{\sqrt{2}}; i_{a,rms} = \sqrt{\frac{2}{3}} I_d = \frac{2}{3} I_d$$

$$\langle P \rangle = \frac{1}{\pi} \int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I_d V_s \sin(\psi) d\psi = \frac{V_s I_d}{\pi} \cdot [\cos(\frac{\pi}{6}) - \cos(\frac{5\pi}{6})]$$

$$\langle P \rangle = \frac{V_s I_d \sqrt{3}}{\pi}$$

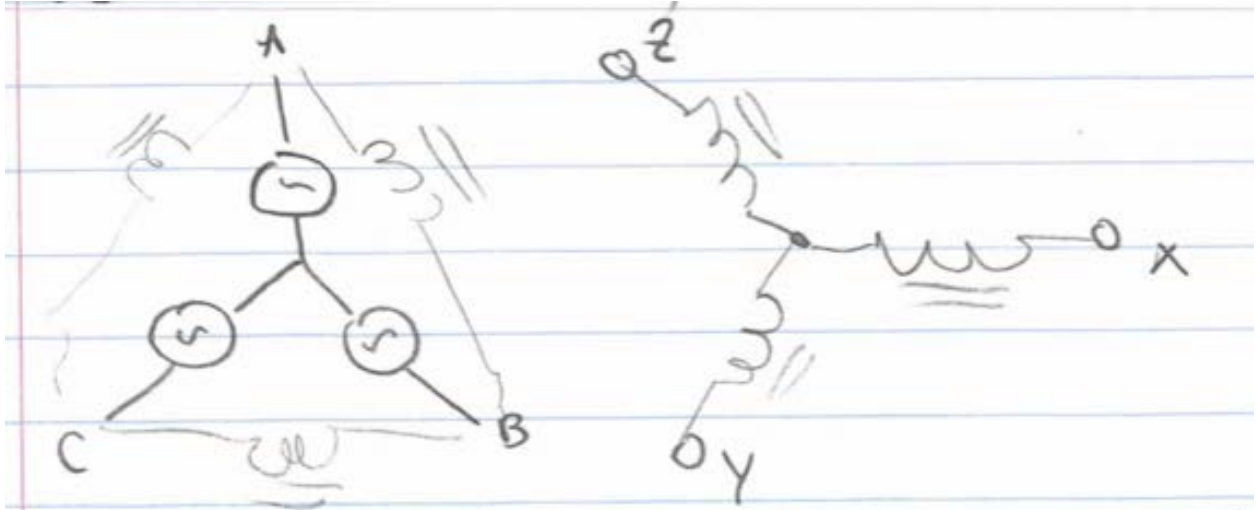
$$\Rightarrow K_p = \frac{\langle P \rangle}{V_{RMS} I_{RMS}} = \frac{V_s I_d \sqrt{3}}{\pi} \cdot \frac{\sqrt{2} \sqrt{3}}{V_s I_d \sqrt{2}} = \frac{3}{\pi} \approx 0.96$$

This compares favorably to a single-phase bridge rect ($K_p \approx 0.91$)
 Why better? : No triple harmonics!



Also: 4 diodes 6 diodes small ripple @ $6f_{line}$ $\Rightarrow 3\Phi$ wins!

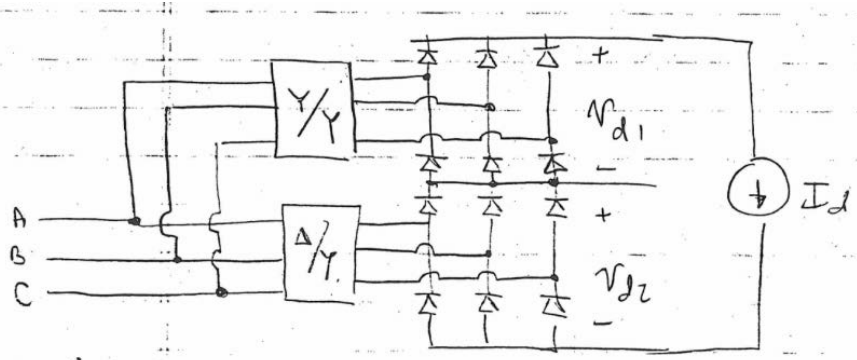
We can do even better (in terms of harmonics/ P.F.) with higher order rectifiers
 Using transformers, we can generate phase-shifted voltage sets



“Δ-Y” set \Rightarrow can create 30° phase shift $ABC \rightarrow XYZ$

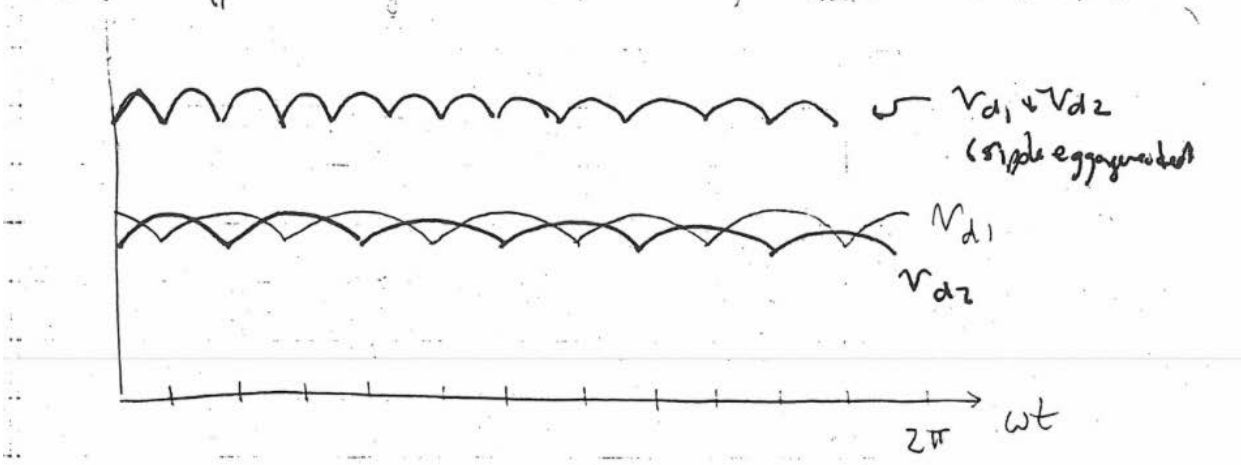
3 Higher-order rectifier

Suppose we use two phase-shifted transformer sets on series-stacked six-pulse bridges



The $\frac{Y}{Y}, \frac{\Delta}{Y}$ transformer sets generate equal voltage magnitudes, with a 30° phase difference between their 3Φ outputs

Since all voltages are isolated, constant current in the bridges \rightarrow the two six pulse bridges act independently. Since input waveforms shifted by 30° ($\frac{T}{12}$) and output ripple is at $6 \times$ input frequency ($T_{V_{out}} = \frac{T}{6}$) Output ripple voltage, correct shifted by $\frac{T_{V_{out}}}{2}$. (30° fund; is 180° 6th)



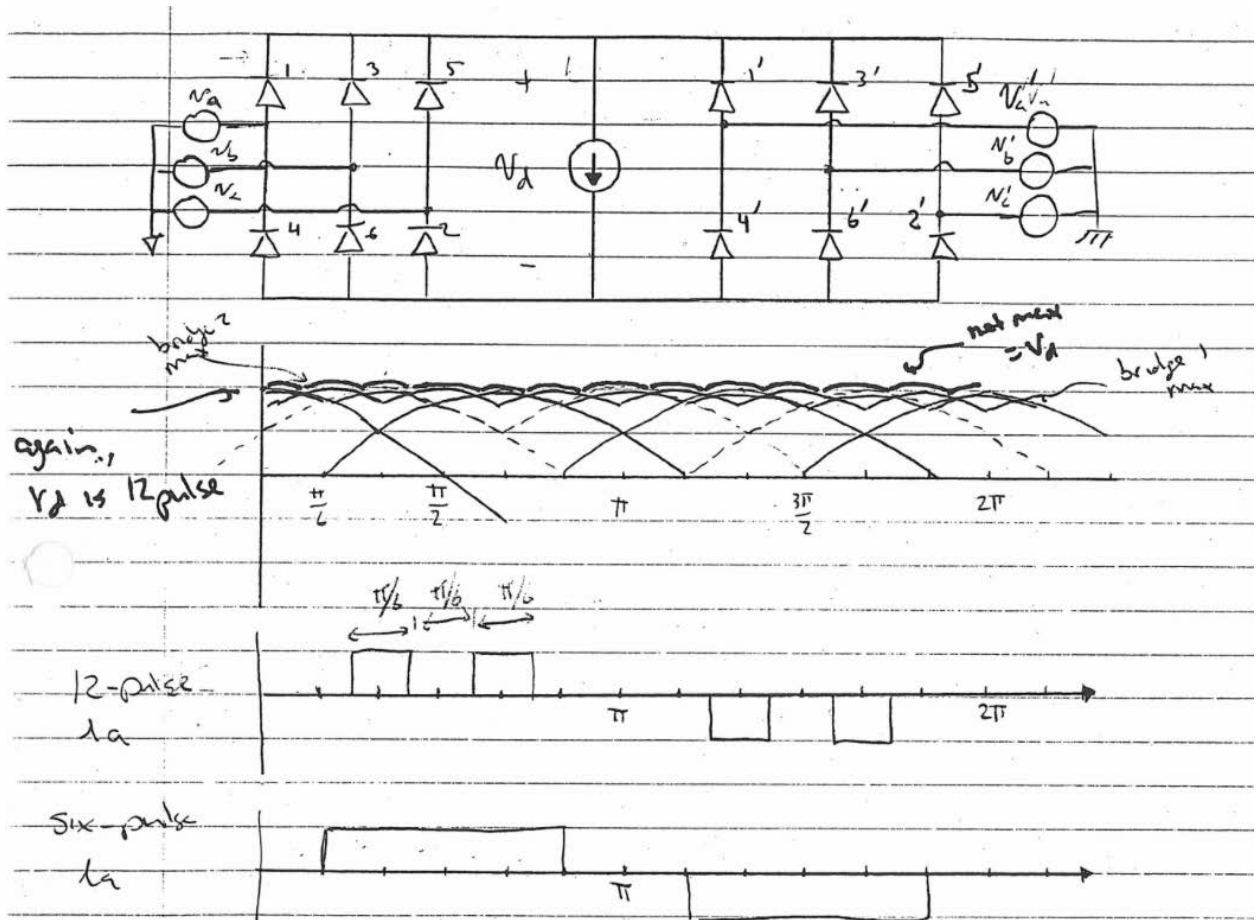
We have a 12-pulse rectifier:

- smaller ripple magnitude
- higher ripple frequency

For easier output filter

Net input current + power factor also improves

Consider parallel case (direct connection)



$$12 \text{ pulse } I_{d1,rms} = \sqrt{\frac{2\pi}{6} \cdot \frac{1}{2\pi} \cdot I_d^2} = \frac{I_d}{\sqrt{6}}$$

$$6 \text{ pulse } I_{d1,rms} = \sqrt{\frac{4\pi}{6} \cdot \frac{1}{2\pi} \cdot I_d^2} = \frac{I_d}{\sqrt{3}}$$

So 12 pulse, $I_{rms} \downarrow$ by $\sqrt{2}$, but twice as many devices

→ Each device carries the full current for $\frac{1}{2}$ the time!

(ohmic loss in devices, transformers, lines depend on RMS!)

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