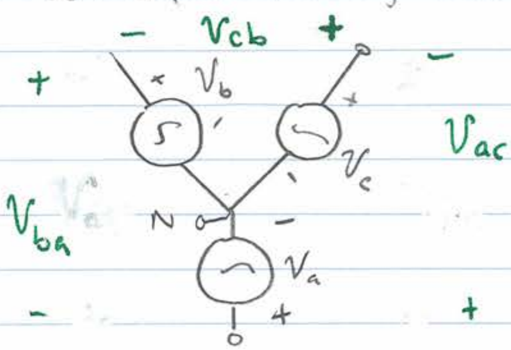
6.334 Lecture Notes — 3ϕ Systems #2

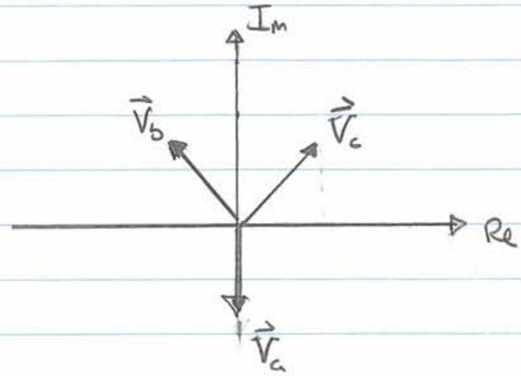
REVIEW: Three-phase set separated by 120°

$$v_a = V_s \sin(\omega t)$$
$$v_b = V_s \sin\left(\omega t - \frac{2\pi}{3}\right)$$
$$v_c = V_s \sin\left(\omega t + \frac{2\pi}{3}\right)$$

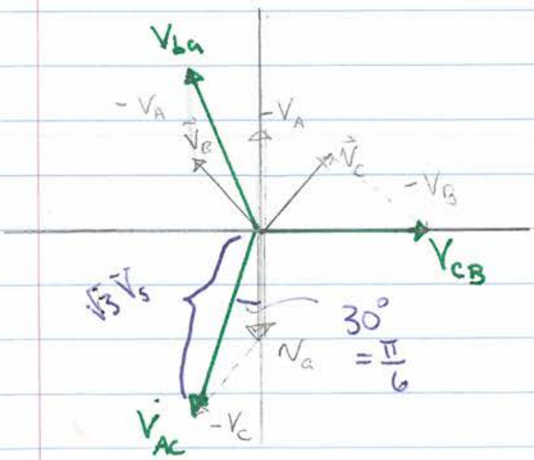


Phasor Notation:

$$v_x(t) = Re\left\{ \vec{V}_x e^{j\omega t} \right\}$$



WE CAN TAKE VECTOR DIFFERENCES TO GET line-line waveforms



- $\sqrt{3}$ increase in magnitude for l-l
- 30° Phase shift for l-l.

So: 120 V_{rms} l-n $\longleftrightarrow$ 208 V_{rms} l-l.

$\Rightarrow$ Distribute 3ϕ 208 l-l and break off l-n taps for different 1ϕ branches

We can create sinusoids of any magnitude + phase by using transformers to select + add up parts of different l-n voltages! (as simple as vector addition....)

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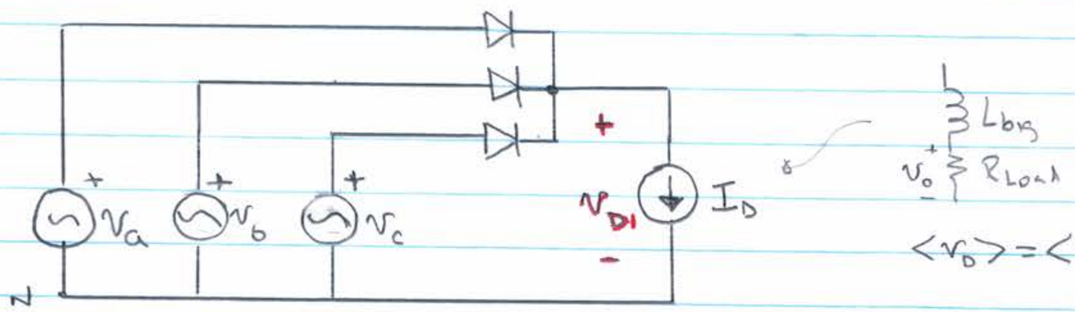
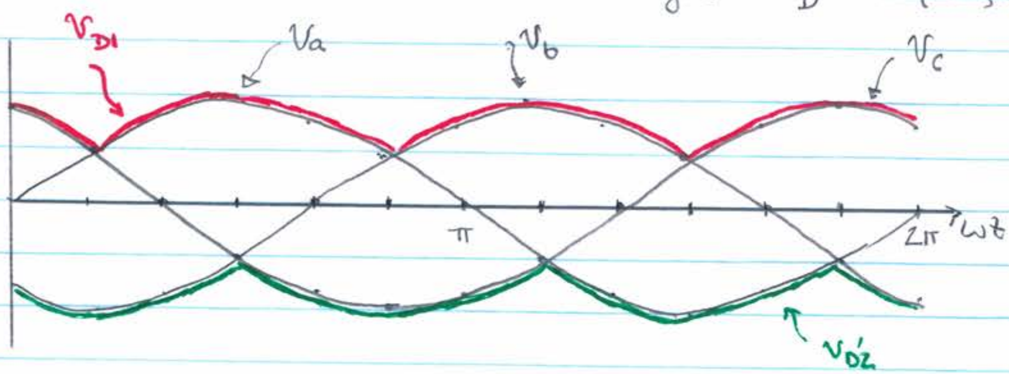
Consider diode rectification of 3-phase

The equivalent of "half-wave" rectification is:



$\langle v_D \rangle = \langle v_o \rangle$

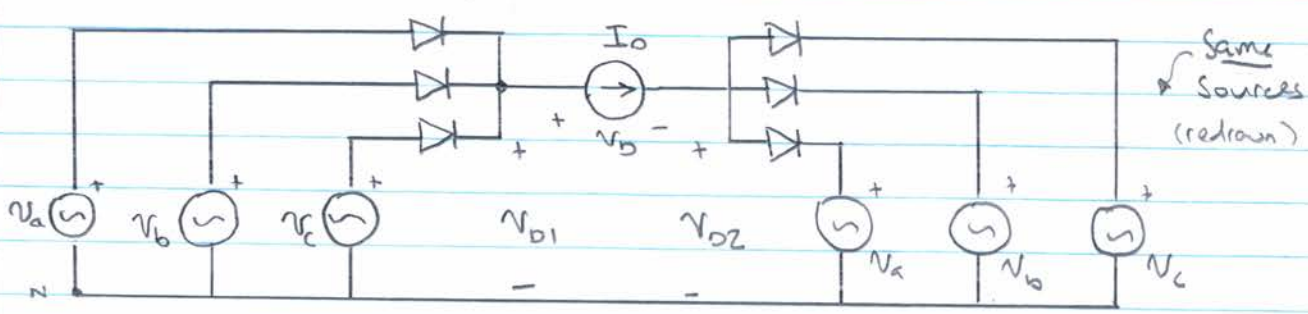
v_D is the diode "or" of the three line-to-neutral voltages $v_D = \max\{v_a, v_b, v_c\}$



$\frac{\pi}{6}$ rad $= 30^\circ$

If we had reversed the diodes, we'd get $v_{D2} = \min\{v_a, v_b, v_c\}$

For "full wave" or "bridge" rectification, use 6 diodes for max + min:



$$v_D = v_{D1} - v_{D2} = \max\{v_a, v_b, v_c\} - \min\{v_a, v_b, v_c\}$$

* This is the same as $v_D = \max\{v_{ab}, v_{ac}, v_{bc}, v_{ba}, v_{ca}, v_{cb}\}$

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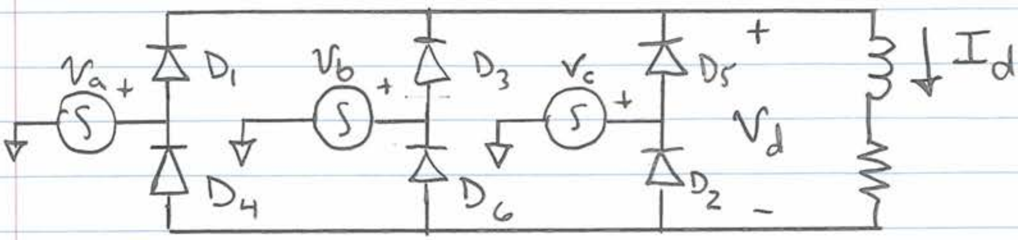
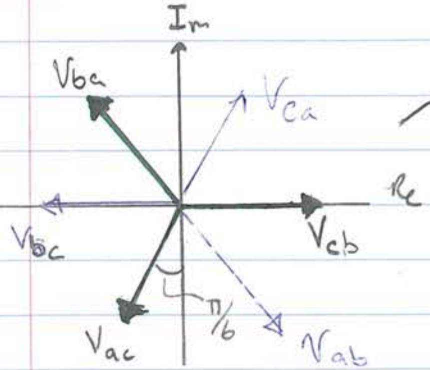
Note: 1ϕ rect 4 diodes / 3ϕ rect 6 diodes!

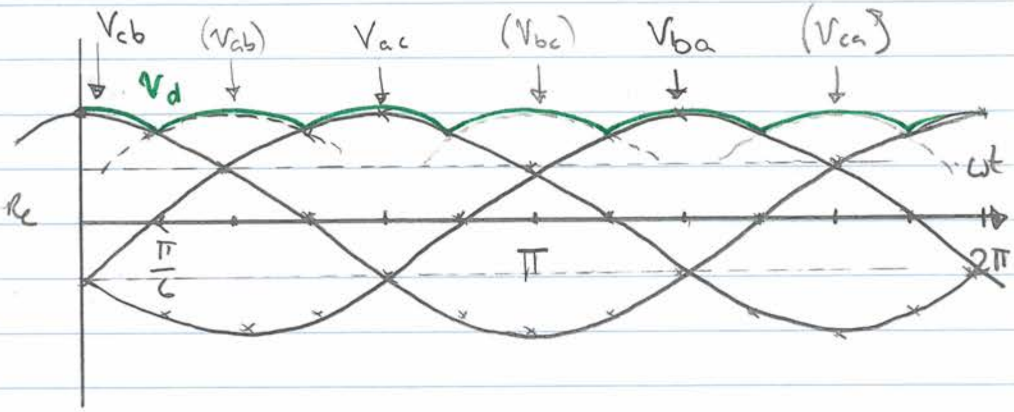
The 3ϕ Full-bridge rectifier can be drawn as follows:



Note:

smaller filter!



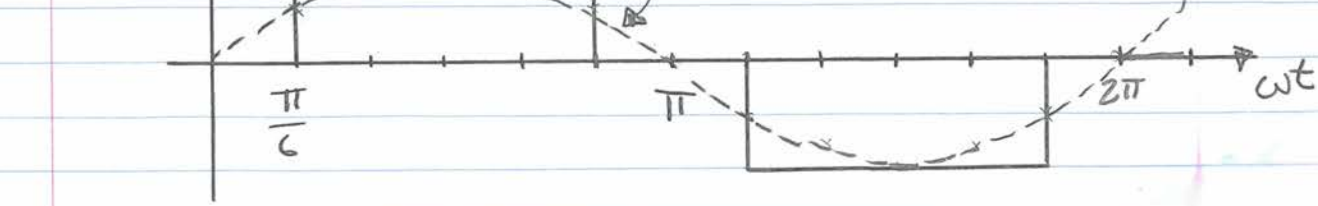


- V_d is $\max\{V_{cb}, V_{ac}, V_{ba}, V_{ab}, V_{bc}, V_{ca}\}$

TO SIMPLIFY TRACKING BEHAVIOR:

- Phases named such that successive l-n maxima are $V_a, V_b, V_c, V_a, V_b, V_c, \ldots$ so l-l max goes $V_{ab}, V_{ac}, V_{bc}, V_{ba}, V_{ca}, V_{cb}, \ldots$
- Diodes numbered such that conduction sequence is: $D_6/D_1, D_1/D_2, D_2/D_3, D_3/D_4, D_4/D_5, D_5/D_6, D_6/D_1 \ldots$

CONSIDER WAVEFORMS + POWER FACTOR (e.g. for phase A)



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$$V_{a,rms} = \frac{V_s}{\sqrt{2}} \qquad I_{a,rms} = \sqrt{\frac{2}{3} I_d^2} = \sqrt{\frac{2}{3}}\, I_d$$

$$\langle P \rangle = \frac{1}{\pi}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} I_d V_s \sin(\varphi)\, d\varphi = \frac{V_s I_d}{\pi}\cdot\left[\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{5\pi}{6}\right)\right]$$

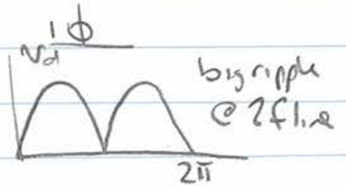
$$\langle P \rangle = \frac{V_s I_d \sqrt{3}}{\pi}$$

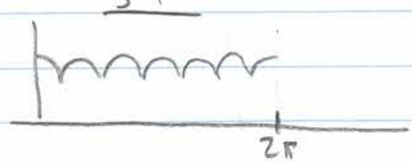
$$\Rightarrow \quad K_p = \frac{\langle P \rangle}{V_{rms} I_{rms}} = \frac{V_s I_d \sqrt{3}}{\pi}\cdot\frac{\sqrt{2}}{V_s}\,\frac{\sqrt{3}}{I_d}\,\frac{1}{\sqrt{2}} = \frac{3}{\pi} \approx .96$$

This compares favorably to a single-phase bridge rect ($K_p \approx 0.91$)

why better? : no triplen harmonics!

Also:



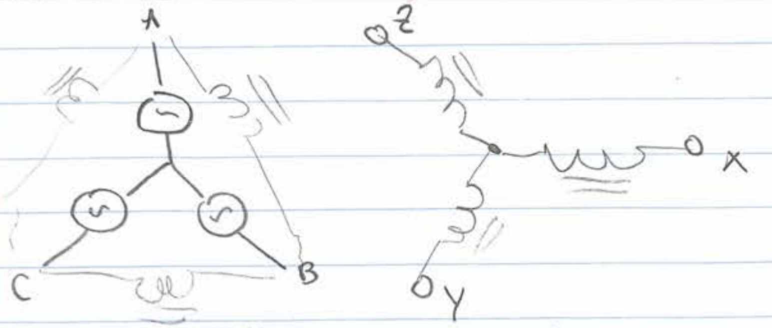


small ripple @ $6f_{line}$

4 diodes — 6 diodes — $\Rightarrow$ $\boxed{3\phi \text{ wins!}}$

WE CAN DO EVEN BETTER (IN TERMS OF HARMONICS/P.F.) WITH HIGHER-ORDER RECTIFIERS

USING TRANSFORMERS, WE CAN GENERATE PHASE-SHIFTED VOLTAGE SETS

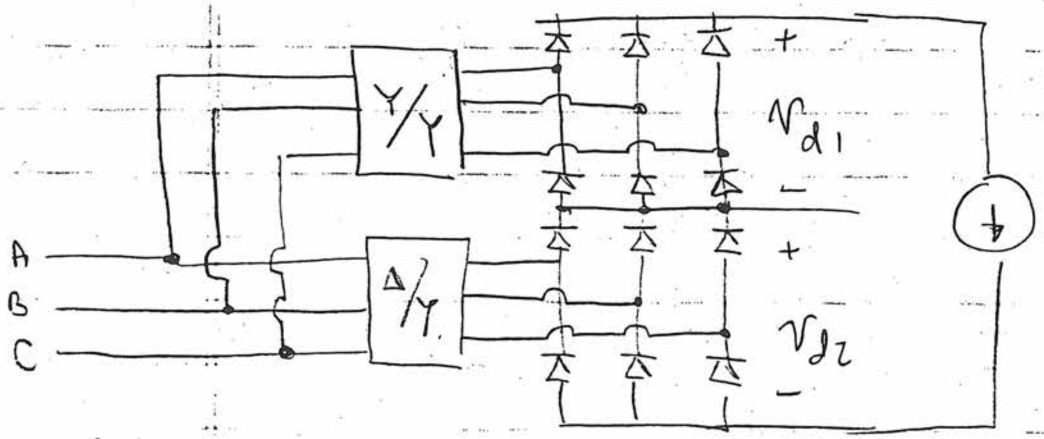


"Δ-Y" SET $\Rightarrow$ Can create 30° phase shift ABC → XYZ

Power Electronics Notes - D. Perreault

Higher-order rectifiers

Suppose we use two phase-shifted transformer sets on series-stacked six-pulse bridges

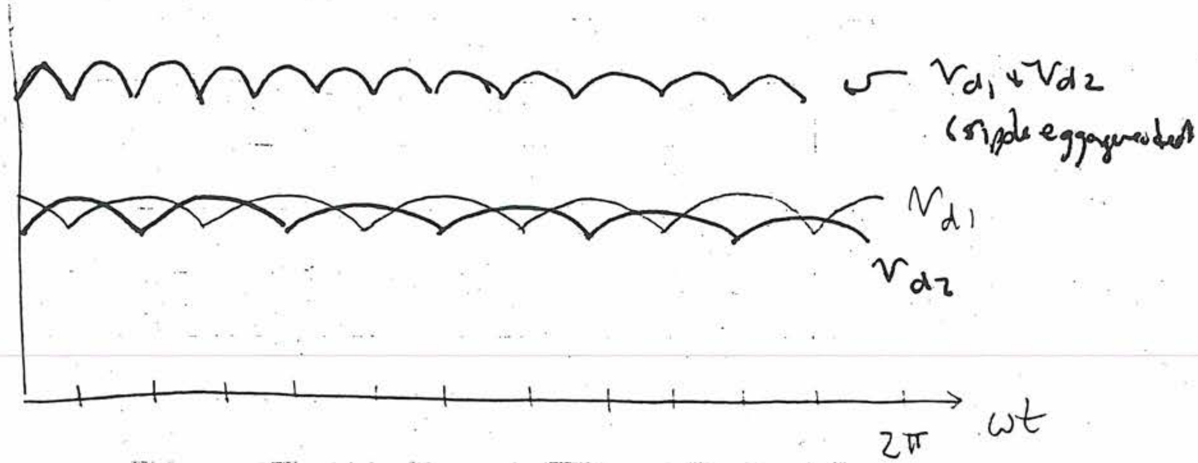


The Y/Y, Δ/Y transformer sets generate equal voltage magnitudes, with a 30° phase difference between their 3φ outputs

Since all voltages are isolated, constant current in the bridges → the two six pulse bridges act independently

Since input waveforms shifted by 30° $\left(\frac{T}{12}\right)$ and output ripple is at 6× input frequency $\left(T_{v_{out}} = \frac{T}{6}\right)$

Output ripple voltages are shifted by $T_{v_{out}}/2$ (30° fund, is 180° 6^{th})



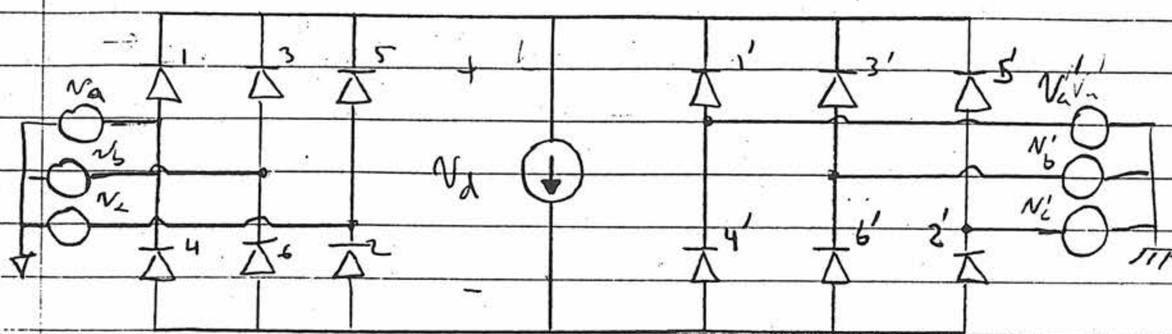
We have a 12-pulse rectifier! (fundamentals of ripple waves cancel)

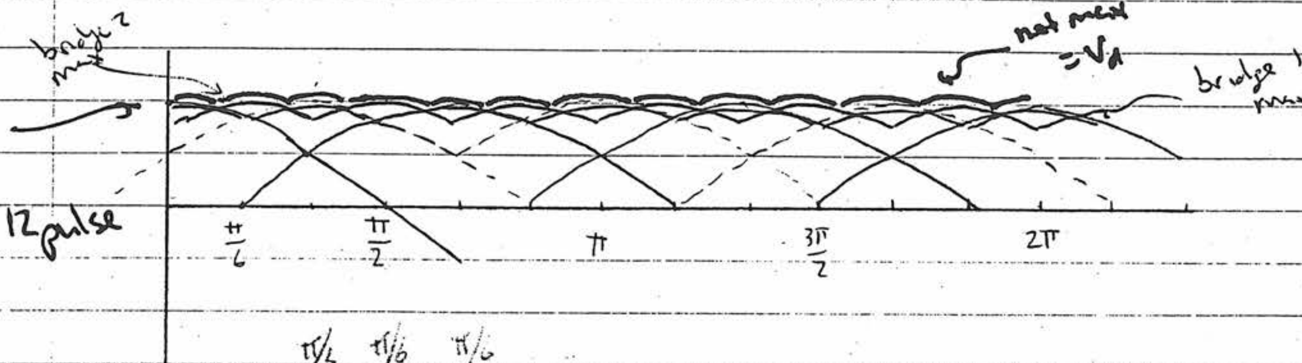
→ smaller ripple magnitude } easier output filter
→ higher ripple frequency }

→ net input current + power factor also improves

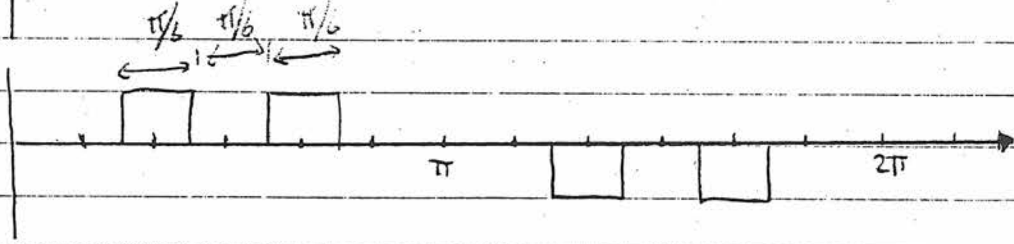
Power Electronics Notes – D. Perreault

consider parallel case (direct connection

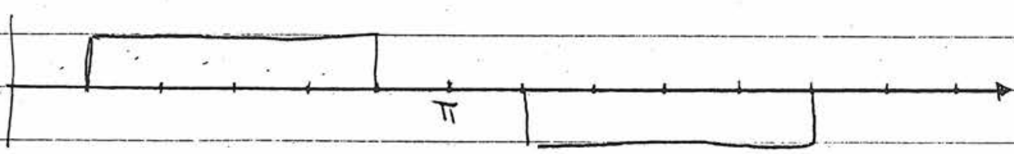




again,
v_d is 12 pulse



12-pulse
i_a



six-pulse
i_a

12 pulse $\quad I_{d1,rms} = \sqrt{\frac{2\pi}{6} \cdot \frac{1}{2\pi} \cdot I_d^2} = \frac{I_d}{\sqrt{6}}$

6 pulse $\quad I_{d1,rms} = \sqrt{\frac{4\pi}{6} \cdot \frac{1}{2\pi} I_d^2} = \frac{I_d}{\sqrt{3}}$

So 12 pulse, $I_{rms} \downarrow$ by $\sqrt{2}$, but twice as many devices

→ each device carries the full current for $\frac{1}{2}$ the time!

(ohmic loss in devices, xformers, lines depend on RMS!)